ELEC-E8116 Model-based control systems
exercises 6

Problem 1. Consider a SISO-system in a one-degree-of-freedom control configuration. The connection between the real and nominal system is

$$G_0(s) = G(s)(1 + \Delta_G(s))$$

By using the Nyquist stability criterion derive a condition to the system to be robustly stable.

Problem 2. Consider the first order process

$$G_P(s) = \frac{k}{\tau s + 1} e^{-\theta s}$$

with parameter uncertainties such that $2 \leq k, \theta, \tau \leq 3$. The system is modelled with

$$G_0(s) = G(s)(1 + \Delta_G(s))$$

in which the nominal model is chosen to be the first order model without delay

$$G(s) = \frac{\bar{k}}{\bar{\tau} s + 1} = \frac{2.5}{2.5s + 1}$$

Discuss possible candidates for the function $\Delta_G(s)$.

Problem 3. Consider the process described in Exercise 5, Problem 1 with the exception that the process model is uncertain. The true system is

$$G_0(s) = G(s)(1 + \Delta_G(s))$$

in which the relative uncertainty has been modeled as

$$\Delta_G(s) = \frac{10s + 0.33}{(10/5.25)s + 1}$$

Is the controlled (closed loop) system robustly stable?

Problem 4. Let a closed-loop SISO-system be stable. Prove that the maximum delay that can be added to the process without causing closed-loop instability is

$$\theta_{\max} = PM / \omega_c$$

where PM is the phase margin of the (original) system and ω_c is the gain crossover frequency.

Problem 5. Let the weight of the sensitivity function be given as

$$\frac{1}{W_s} = A\frac{\dfrac{s}{A\omega_0}+1}{\dfrac{s}{B\omega_0}+1}, \quad 0 < A << 1, B >> 1$$

Sketch a schema for the magnitude plot of the frequency response and investigate its characteristics. What is the slope in the inceasing part of the curve? What is the magnitude at frequency ω_0?

Generate a second order model, where the slope is twice as large as in the previous case. Investigate again the characteristics. What is the magnitude at frequency ω_0?

Problem 6. Consider the angular frequencies $\omega_B, \omega_c, \omega_{BT}$ which are used to define the bandwidth of a controlled system. State the definitions. Prove that when the phase margin is less than 90 degrees ($PM < \pi / 2$) it holds $\omega_B < \omega_c < \omega_{BT}$. Interpretations?

Problem 7. Consider a SISO-system. The maximum values of the sensitivity and complementary functions are denoted M_S and M_T , respectively. Let the gain and phase margins of a closed-loop system be *GM* (gain margin) and *PM* (phase margin). Prove that

$$GM \geq \frac{M_S}{M_S - 1} \qquad PM \geq 2\arcsin\left(\frac{1}{2M_S}\right) \geq \frac{1}{M_S} \ \text{[rad]}$$

$$GM \geq 1 + \frac{1}{M_T} \qquad PM \geq 2\arcsin\left(\frac{1}{2M_T}\right) \geq \frac{1}{M_T} \ \text{[rad]}$$