# Exercise 7

**1.11.2022**

## #1 Incomplete information

Consider an expert evaluating three different cars using three attributes (top speed, $CO_2$ emissions, and maintenance costs). The normalized attribute-specific values of the three cars are shown in the table below.

| | Top speed (km/h) | $CO_2$ emissions (g/km) | Maintenance costs (€/year) |
|---|---|---|---|
| x1 | 0 | 0.5 | 0.6 |
| x2 | 0.3 | 0.4 | 0.5 |
| x3 | 1 | 0.5 | 0.5 |

The expert states that changing $CO_2$ emissions from 250g/km (worst level) to 0g/km (best level) is more valuable than a change in maintenance costs from 1000€ to 0€ (worst to best), which is more valuable than changing top speed from 192km/h to 220km/h (worst to best).

a) Formulate the linear inequalities that result from the above preference statements.
b) Formulate the set of feasible attribute weights $S$ that are consistent with the expert's preferences. What are the extreme points of $S$?
c) Compute overall value intervals for the alternatives.
d) Establish dominance relations.

## #2 Sensitivity analysis

Consider three alternatives $A$=(100,100), $B$=(90,105) and $C$=(105,80). The DM's preferences are captured by value function $V(x)=x_1 + x_2$.

a) Which one of the alternatives has the highest value?
b) Suppose $x^P \in X$ is the alternative that maximizes $V(x)$ with the given attribute weights $w^*$ that are consistent with the DM's preference statements. Following methods proposed in the scientific literature, the DM conducts sensitivity analysis based on the idea that the *closest competitor* of $x^P$ is the alternative $x^{CC} \in X \backslash \{x^P\}$ which, loosely speaking, (i) does not maximize the overall value with $w^*$, but (ii) requires the "smallest" change in weights to become the value-maximizing alternative. That is, more formally,

$$x^{CC} = \arg \min_{x \in X \backslash \{x^P\}} \left\{ \min_{w \in S_w^0} (\|w - w^*\|_2 \mid V(x^P) = V(x)) \right\}.$$

Towards this end, the DM normalizes the value function such that $V^N(0,0)=0$ and $V^N(105,105)=1$. Which alternative is the closest competitor?

c) The value functions are unique up to positive affine transformations. The DM chooses to normalize the value function such that $V^N(0,0)=0$ and $V^N(1050,105)=1$. Which alternative is the closest competitor?