ELEC-E8116 Model-based control systems /exercises 8

1. Consider a simple integrator:

$$\dot{x}(t) = u(t)$$

Find an optimal control law that minimises a cost-function

$$J = \int_0^1 \left(x^2(t) + u^2(t) \right) dt$$

Further, consider the case, when the optimization horizon is infinite.

2. Consider the 1. order process $G(s) = \dfrac{1}{s-a}$, which has a realization

$$\begin{aligned} \dot{x}(t) &= ax(t) + u(t) \\ y(t) &= x(t) \end{aligned}$$

so that the state is the measured variable. It is desired to find the control, which minimizes the criterion

$$J = \frac{1}{2}\int_0^\infty \left(x^2 + Ru^2 \right) dt \qquad (R > 0)$$

Calculate the control and investigate the properties of the resulting closed-loop system.

3. In solving the discrete-time LQ problem an essential step is to find a "first control step" by minimizing the cont

$$J_{N-1} = \frac{1}{2} x_{N-1}^T Q x_{N-1} + \frac{1}{2} u_{N-1}^T R u_{N-1} + \frac{1}{2}\left(Ax_{N-1} + Bu_{N-1} \right)^T S_N \left(Ax_{N-1} + Bu_{N-1} \right)$$

Do it.

4. The discrete time LQ problem and its solution can be given as

$$x_{k+1} = A_k x_k + B_k u_k, \quad k > i$$

$$J_i = \frac{1}{2} x_N^T S_N x_N + \frac{1}{2} \sum_{k=i}^{N-1} \left(x_k^T Q_k x_k + u_k^T R_k u_k \right), \quad \text{(final state free)}$$

$$S_N \geq 0, \quad Q_k \geq 0, \quad R_k > 0$$

$$S_k = \left(A_k - B_k K_k \right)^T S_{k+1} \left(A_k - B_k K_k \right) + K_k^T R_k K_k + Q_k$$

$$K_k = \left(B_k^T S_{k+1} B_k + R_k \right)^{-1} B_k^T S_{k+1} A_k, \quad k < N$$

$$u_k^* = -K_k x_k, \quad k < N$$

$$J_i^* = \frac{1}{2} x_i^T S_i x_i$$

Show that the Riccati equation can also be written in the form

$$S_k = A_k^T \left[S_{k+1} - S_{k+1} B_k \left(B_k^T S_{k+1} B_k + R_k \right)^{-1} B_k^T S_{k+1} \right] A_k + Q_k, \; k < N, \; S_N \text{ given}$$

(The "Joseph-stabilized form" of the Riccati equation)