ELEC-E8101 Digital and Optimal Control
Exercise 8
Autumn 2022

1
Your manager has given a task for you. Your assignment is to replace the old analog PID controller with a new digital PID controller. The transfer function of the continuous time process is

$$G(s) = \frac{e^{-0.7s}}{s^2+0.8s+0.5}$$

The old analog PID controller is

$$G_{PID}(s) = K\left(1 + \frac{1}{T_I s} + T_D s\right)$$

where

$$K = 1, T_I = 1.5,\ T_D = 1.$$

Define a discrete PID controller approximation for the continuous-time PID controller

a) so that the theoretical "text book" PID controller is discretized by using backward difference approximation.
b) so that the theoretical "text book" PID controller is discretized by using Tustin approximation.
c) so that the practical continuous PID controller (use N = 10)

$$G_{PID}(s) = K\left((Y_{ref}(s) - Y(s)) + \frac{1}{T_I s}(Y_{ref}(s) - Y(s)) - \frac{T_D s}{1 + T_D s/N} Y(s) \right)$$

is discretized by replacing the integral by sum (standard Euler integration) and using backward difference approximation in the derivative action.

What are the facts that affect the choosing of the sampling rate h? What value of h would you choose? Justify. By simulations, compare how the different approximations work. In the simulations use the continuous time process and the discrete PID controllers which you have defined in a) - c). Compare the control signals and the responses of the controlled systems. Use different reference signals. What is the best approximation? Try using different sampling periods in the discrete controllers. What happens if the sampling period changes?

2
A continuous system with transfer function

$$G(s) = \frac{1}{s(10s+1)}$$

is controlled by a discrete system with ZOH

$$u[k] = -0.5u[k-1] + 13(e[k] - 0.88e[k-1])$$

sample time being $T_s = 1$.

Check if the closed-loop system fulfills the specifications

- Step response overshoot: $M_P < 16\%$
- Step response settling time (1%): $t_s < 10s$
- Steady-state error to unit ramp: $e_{ss} < 1$