

Equation collection, can be used in the exam.

$$(P^{-1} + H'R^{-1}H)^{-1} = P - PH'(HPH' + R)^{-1}HP \quad (1.3.3-11)$$

where P is $n \times n$, H is $m \times n$, and R is $m \times m$.

An alternative version of the above is

$$(A + BCB')^{-1} = A^{-1} - A^{-1}B(B'A^{-1}B + C^{-1})^{-1}B'A^{-1} \quad (1.3.3-12)$$

$$f_x(x) \triangleq \frac{\partial f}{\partial x} = [\nabla_x f(x)]' = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{\partial x_1} & \dots & \frac{\partial f_m}{\partial x_n} \end{bmatrix} \quad (1.3.5-3)$$

$$\phi_{xx}(x) \triangleq \frac{\partial^2 \phi(x)}{\partial x^2} = \nabla_x \nabla_x' \phi(x) = \begin{bmatrix} \frac{\partial^2 \phi}{\partial x_1 \partial x_1} & \dots & \frac{\partial^2 \phi}{\partial x_1 \partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 \phi}{\partial x_n \partial x_1} & \dots & \frac{\partial^2 \phi}{\partial x_n \partial x_n} \end{bmatrix} \quad (1.3.5-6)$$

Define the *stacked vector*

$$y \triangleq \begin{bmatrix} x \\ z \end{bmatrix} \quad (3.2.1-1)$$

The notation

$$y \sim \mathcal{N}[\bar{y}, P_{yy}] \quad (3.2.1-2)$$

will indicate that the variable y is **normally (Gaussian) distributed** with mean

$$\bar{y} = \begin{bmatrix} \bar{x} \\ \bar{z} \end{bmatrix} \quad (3.2.1-3)$$

and covariance matrix (assumed nonsingular)

$$P_{yy} = \begin{bmatrix} P_{xx} & P_{xz} \\ P_{zx} & P_{zz} \end{bmatrix} \quad (3.2.1-4)$$

For x and z jointly Gaussian, as assumed in (3.2.1-2), the **conditional mean** is

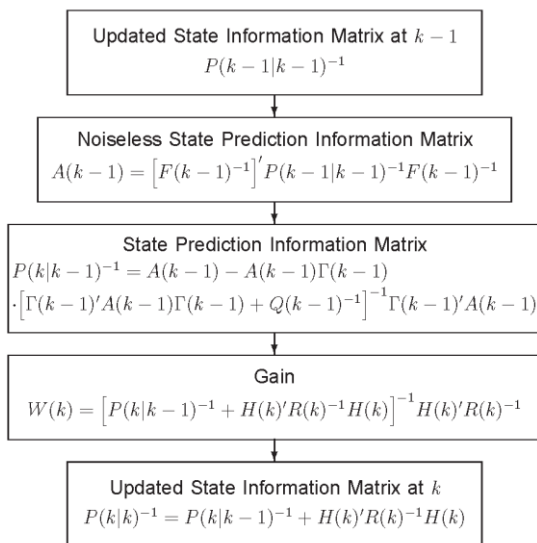
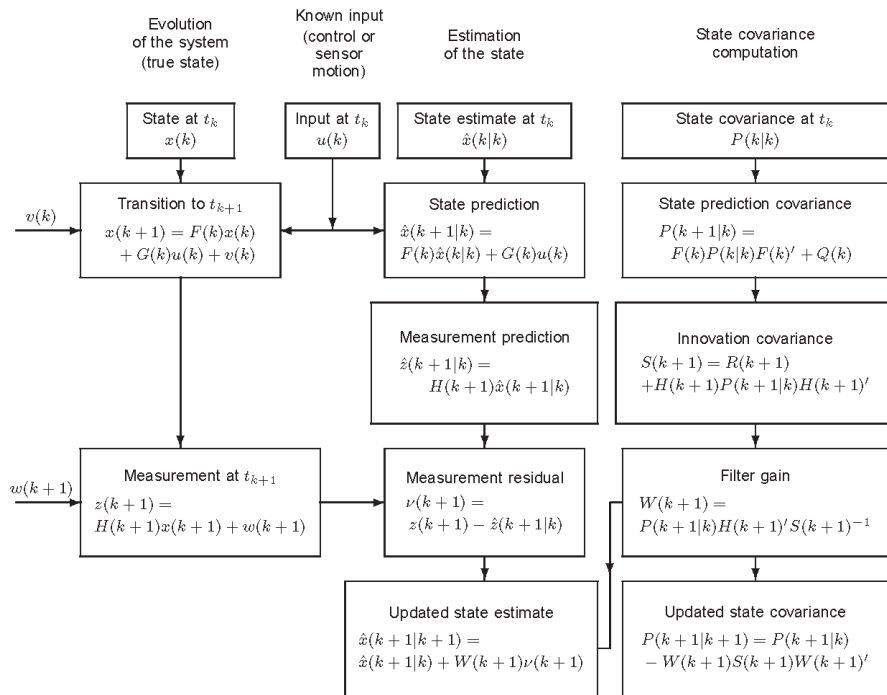
$$\hat{x} \triangleq E[x|z] = \bar{x} + P_{xz}P_{zz}^{-1}(z - \bar{z}) \quad (3.2.1-7)$$

and the corresponding **conditional covariance matrix** is

$$P_{xx|z} \triangleq E[(x - \hat{x})(x - \hat{x})'|z] = P_{xx} - P_{xz}P_{zz}^{-1}P_{zx} \quad (3.2.1-8)$$

$$\hat{x}(k) = [H^{k'}(R^k)^{-1}H^k]^{-1}H^{k'}(R^k)^{-1}z^k \quad (3.4.1-9)$$

$$P(k) = [H^{k'}(R^k)^{-1}H^k]^{-1} \quad (3.4.1-15)$$



Information filter state prediction

$$\hat{y}(k|k-1) = (F(k-1))^{-1}\{[I - P(k-1|k-1)^{-1}F(k-1)^{-1}\Gamma(k-1)] \cdot [\Gamma(k-1)'A(k-1)\Gamma(k-1) + Q(k-1)^{-1}]^{-1} \cdot \Gamma(k-1)'(F(k-1))^{-1}\hat{y}(k-1|k-1)\} \quad (7.2.3-6)$$

Information filter state update

$$\hat{y}(k|k) = \hat{y}(k|k-1) + H(k)'R(k)^{-1}z(k) \quad (7.2.3-8)$$

$$\hat{x}(k|N) = \hat{x}(k|k) + C(k)[\hat{x}(k+1|N) - \hat{x}(k+1|k)] \quad (8.6.2-16)$$

where the **smoother gain** is

$$C(k) = P(k|k)F(k)'[F(k)P(k|k)F(k)' + Q(k)]^{-1} \quad (8.6.2-17)$$

or

$$C(k) = P(k|k)F(k)'P(k+1|k)^{-1} \quad (8.6.2-18)$$

$$P(k|N) = P(k|k) + C(k)[P(k+1|N) - P(k+1|k)]C(k)' \quad k = N-1, \dots, 0 \quad (8.6.2-29)$$

