Tasoaalto tyhjiössä

μ_0, ε_0

$\rho = 0, \bar{J} = 0$

$$\nabla \cdot \bar{D} = 0 \quad (\bar{D} = \varepsilon_0 \bar{E})$$

$$\nabla \times \bar{E} = -j\omega\mu_0 \bar{H}$$

$$\nabla \times \bar{H} = j\omega\varepsilon_0 \bar{E}$$

$$\underbrace{\nabla \times (\nabla \times \bar{E})}_{\nabla(\nabla\cdot\bar{E}) - \nabla^2 \bar{E}} = -j\omega\mu_0 j\omega\varepsilon_0 \bar{E} = \omega^2 \mu_0 \varepsilon_0 \bar{E}$$

$$\underbrace{\nabla(\nabla\cdot\bar{E})}_{0}$$

$$\nabla^2 \bar{E}(\bar{r}) + \underbrace{\omega^2 \mu_0 \varepsilon_0}_{k^2} \bar{E}(\bar{r}) = 0$$

$$\bar{E}(z) = \bar{u} E_+ e^{-jkz}$$

$$\bar{H}(z) = \bar{u} \times \bar{u}_z \frac{E_+}{\eta} e^{-jkz}$$

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$\frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$$

$$\bar{E} = \bar{E}_r + j\bar{E}_i \Rightarrow \bar{E}(t) = \bar{E}_r \cos\omega t - \bar{E}_i \sin\omega t$$

Materian vaikutus

$$\varepsilon = \varepsilon_r \varepsilon_0 \qquad \mu = \mu_r \mu_0 \qquad \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$k = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0}\underbrace{\sqrt{\mu_r\varepsilon_r}}_{n}$$

$$e^{-jkz}$$

$$k\lambda = 2\pi \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}} = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0}} \cdot \frac{1}{n}$$

$$\frac{2\pi c}{2\pi f} = \lambda_0$$

$$\lambda = \frac{\lambda_0}{n}$$

Aallonpituus $\lambda = \frac{c}{nf}$

Taitekerroin $n = \sqrt{\mu_r \varepsilon_r}$

Häviöt

$$\nabla \times \bar{H} = \bar{J} + j\omega\varepsilon\bar{E}$$
$$= \sigma\bar{E} + j\omega\varepsilon\bar{E}$$
$$= j\omega\left(\varepsilon + \frac{\sigma}{j\omega}\right)\bar{E}$$

ε

$\mu \quad \sigma$

Johtavuus

$$= j\omega\underbrace{\left(\varepsilon - j\frac{\sigma}{\omega}\right)}\bar{E} \qquad \bar{J} = \sigma\bar{E}$$

KOMPLEKSINEN PERMITTIIVISYYS !

$$\Rightarrow \quad \varepsilon = \varepsilon' - j\varepsilon'' \qquad \varepsilon = \varepsilon_r\varepsilon_0 = \left(\varepsilon_r' - j\varepsilon_r''\right)\varepsilon_0$$

$\uparrow \frac{\sigma}{\omega\varepsilon_0}$

MUISTISÄÄNTÖ!

$$\varepsilon_r'' = 60\lambda\sigma$$

λ: m, σ: S/m

$$[\cdot\cdot] = \frac{A}{Vm}\cdot s\cdot\frac{Vm}{As} = 1$$

=> Aaltoluku on kompleksinen

$$k = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu(\varepsilon' - j\varepsilon'')} = k_r - jk_i$$

k_i: $-\mathrm{Im}\{k\}$

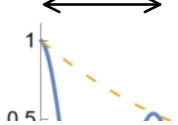
$$e^{-jkz} = e^{-j(k_r - jk_i)z} = e^{-jk_rz}\cdot\underbrace{e^{-j(-jk_i)z}}_{e^{-k_iz}}$$

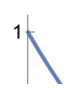
$$\mathrm{Re}\left\{e^{-jk_rz}\,e^{-k_iz}\,e^{j\omega t}\right\} = e^{-k_iz}\cos(\omega t - k_rz)$$

Aallon vaimennus

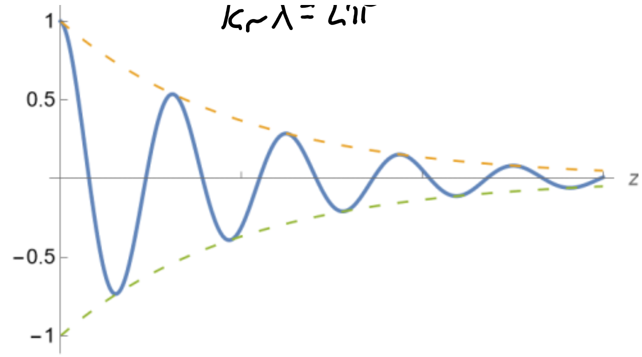
λ

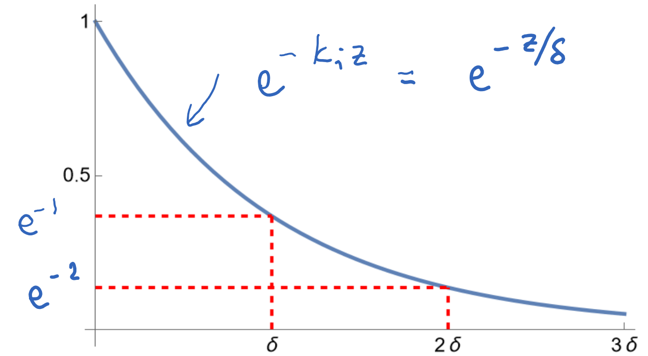
$k_r\lambda = 2\pi$





$e^{-k_iz} \quad e^{-z/\delta}$





Tunkeutumissyvyys δ

$$\delta = \frac{1}{k_i}$$

MAGNEETTIKENTTÄ JA TEHO

$$\bar{E} = \bar{u}\, E_+\, e^{-jkz} \qquad \bar{H} = \bar{u}_z \times \bar{u}\, \frac{E_+}{\eta}\, e^{-jkz}$$

$$\bar{S} = \frac{1}{2}\bar{E}\times\bar{H}^* = \frac{1}{2}\underbrace{\bar{u}\times(\bar{u}_z\times\bar{u})}_{\bar{u}_z \underbrace{\bar{u}\cdot\bar{u}}_{1} - \underbrace{\bar{u}_z\cdot\bar{u}}_{0} u} E_+ e^{-jkz}\left(\frac{E_+}{\eta}e^{-jkz}\right)^*$$

$(ab)^* = a^* b^*$

$e^{-jk_r z}\cdot e^{-k_i z}$

$$= \frac{1}{2}\bar{u}_z\, E_+\, e^{-j(k_r - jk_i)z}\, \frac{E_+^*}{\eta^*}\, e^{+jk^* z}$$

$e^{+j(k_r + jk_i)z} = e^{+jk_r z}\cdot e^{-k_i z}$

$$= \frac{1}{2}\bar{u}_z\, \frac{|E_+|^2}{\eta^*}\, e^{-jk_r z}\, e^{-k_i z}\, e^{+jk_r z}\, e^{-k_i z}$$

$e^{a}\cdot e^{-a} = 1$

$$= \bar{u}_z\, \frac{|E_+|^2}{2\eta^*}\, e^{-2k_i z} \quad \Rightarrow \langle \bar{S}(t)\rangle = \mathrm{Re}\{\bar{S}\}$$

TEHON VAIMENEMINEN

Hyvän eristeen approksimaatio

$\varepsilon' - j\varepsilon''$

$k = \omega\sqrt{\mu\varepsilon} = k_r - jk_i$ ($k_i \leftarrow 1/\delta$)

$\varepsilon' \gg \varepsilon''$

$$\Rightarrow k = \omega\sqrt{\mu(\varepsilon' - j\varepsilon'')} = \omega\sqrt{\mu\varepsilon'}\sqrt{1 - j\varepsilon''/\varepsilon'}$$

$$\sqrt{1 - j\varepsilon''/\varepsilon'} \approx 1 - j\frac{\varepsilon''}{2\varepsilon'}$$

$$= \omega\sqrt{\mu\varepsilon'} - j\omega\sqrt{\mu\varepsilon'}\frac{\varepsilon''}{2\varepsilon'}$$

$\varepsilon'' = \frac{\sigma}{\omega}$

$$\omega\sqrt{\mu}\frac{\sigma}{\omega}\frac{1}{2\sqrt{\varepsilon'}} = \frac{\sigma\eta}{2}, \quad \eta = \sqrt{\frac{\mu}{\varepsilon'}}$$

Hyvän johteen approksimaatio

$\varepsilon'' \gg \varepsilon'$

$$k = k_r - jk_i$$

$$= \omega\sqrt{\mu(\varepsilon' - j\varepsilon'')} = \omega\sqrt{\mu}\sqrt{\varepsilon''}\sqrt{-j}$$

($\varepsilon'' = \sigma/\omega$, $\sqrt{-j} = \frac{1-j}{\sqrt{2}}$)

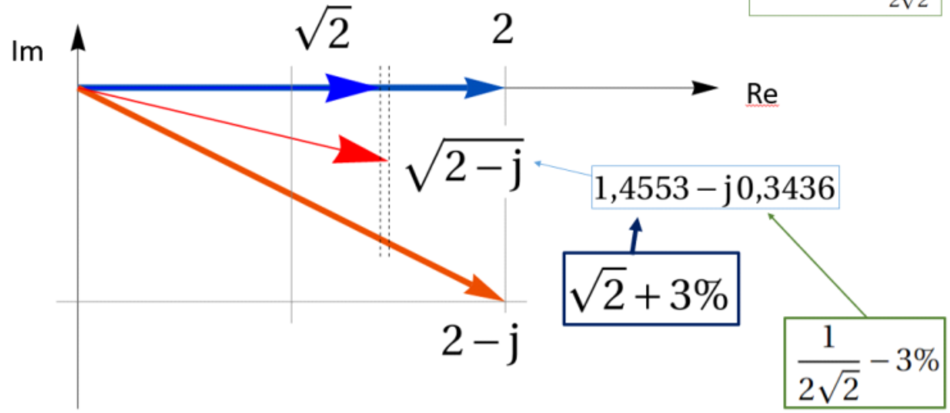
$$= \omega\sqrt{\mu}\sqrt{\sigma/\omega}\frac{1-j}{\sqrt{2}} = (1-j)\sqrt{\frac{\mu\sigma\omega}{2}}$$

$$\delta = \frac{1}{k_i} = \sqrt{\frac{2}{\omega\mu\sigma}} = \frac{1}{\sqrt{\pi f\mu\sigma}}$$

Approksimointia: $\sqrt{\varepsilon_r' - \mathrm{j}\varepsilon_r''} \approx \sqrt{\varepsilon_r'} - \mathrm{j}\dfrac{\varepsilon_r''}{2\sqrt{\varepsilon_r'}}$

jos $\varepsilon_r'' \ll \varepsilon_r'$

$\sqrt{2-\mathrm{j}} \approx \sqrt{2} - \dfrac{\mathrm{j}}{2\sqrt{2}}$



Kupari: $\sigma \approx 58 \cdot 10^6 \frac{S}{m}$ $\qquad f = 10^9\,Hz$

$$\delta = \frac{1}{\sqrt{\pi \cdot 4\pi \cdot 10^{-7} \cdot 10^9 \cdot 58 \cdot 10^6}} \quad \frac{1}{\sqrt{\frac{1}{s}\frac{Vs}{Am}\frac{A}{Vm}}} \approx \frac{1}{0{,}5 \cdot 10^6}\,m = 2\,\mu m$$

$2\pi \cdot 10^4 \cdot 8 \approx 5 \cdot 10^5$

Tunkeutumissyvyys (δ) häviöllisessä väliaineessa

$$\delta = \frac{-1}{k_0\,\mathrm{Im}\left\{\sqrt{\varepsilon_r' - \mathrm{j}\varepsilon_r''}\right\}}$$

Jos taas hyvä eriste (johtavuustermi on pieni):

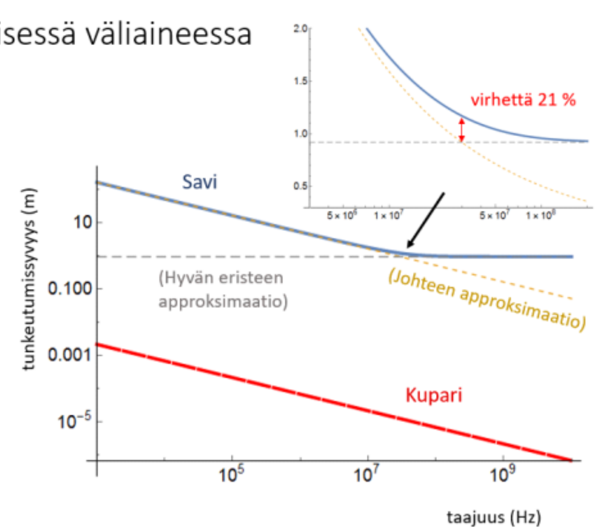
$$\delta \approx \frac{2}{\sigma\eta}$$

Savi

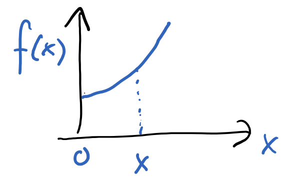
$\varepsilon_r' = 3$

$\sigma = 0{,}01\,\mathrm{S/m}$

$\eta = \sqrt{\dfrac{\mu_0}{\varepsilon_r'\varepsilon_0}} \approx 218\,\Omega$



Taylorin sarja (Maclaurinin sarja)



$$f(x) = f(0) + x f'(0) + \frac{1}{2} x^2 f''(0) + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} x^n f^{(n)}(0)$$

$$\sqrt{1+x} = (1+x)^{1/2} = 1 + \frac{1}{2} x - \frac{1}{8} x^2 \cdots$$

$$f'(x) = \frac{1}{2} (1+x)^{-1/2}$$

$$f''(x) = -\frac{1}{4} (1+x)^{-3/2}$$

Desibeli

$$10 \lg \frac{P}{P_0} \quad \underline{dB}$$

1000 - 30 dB

100 - 20 dB

0,01 - -20 dB

$$10 \lg \frac{E^2}{E_0^2} = \underbrace{10 \cdot 2}_{20} \lg \frac{E}{E_0}$$

dBm

Teho suhteutettuna milliwattiin

$$10 \lg \frac{P}{1\,mW}$$

$$1\,W = 30\,dBm$$

Hyvä johde: Helmholtzin yhtälö

$$\nabla \times \bar{E} = -j\omega\mu \bar{H}$$

$$\nabla \times \bar{H} = \bar{J} + \cancel{j\omega\varepsilon} \bar{E}, \quad \bar{J} = \sigma \bar{E}$$

$$\underbrace{\nabla \times (\nabla \times \bar{E})}_{\nabla(\underbrace{\nabla \cdot \bar{E}}_{=0}) - \nabla^2 \bar{E}} = -j\omega\mu\sigma \bar{E}$$

$$\nabla^2 \bar{E} - \underbrace{j\omega\mu\sigma}_{k^2} \bar{E} = 0$$

$$\bar{J} = \sigma \bar{E}$$

$\underbrace{\quad}_{=0}$ $\qquad\qquad$ $\underbrace{\qquad}_{k^2}$ $\qquad$ $\bar{J} = \sigma \bar{E}$

$$\nabla^2 \bar{J} = j\omega\mu\sigma\bar{J} \qquad \nabla^2 \bar{J}(\bar{r}) + k^2 \bar{J}(\bar{r}) = 0$$

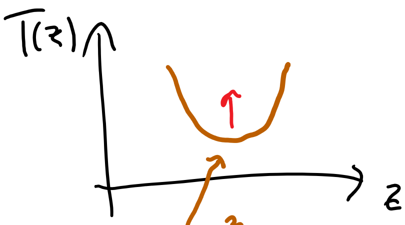
Lämpöyhtälö

aika-alue: $j\omega \hat{=} \frac{\partial}{\partial t}$

$$\Rightarrow \nabla^2 T(\bar{r},t) = a \frac{\partial}{\partial t} T(\bar{r},t)$$

(a: materiaalivakio)

$$\nabla^2 T \hat{=} \frac{\partial^2}{\partial z^2} T$$

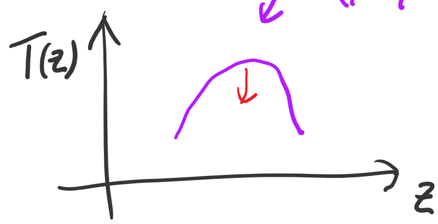


$\nabla^2 T > 0$

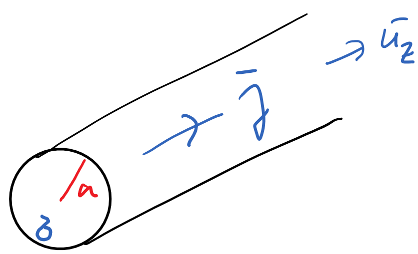
$\Rightarrow \frac{\partial}{\partial t} T > 0$

$\nabla^2 T < 0$

$\Rightarrow \frac{\partial}{\partial t} T < 0$



Pyöreä johdin ja virran ahtautuminen



ω

$$\bar{J}(\bar{r}) = \bar{u}_z K(\rho)$$

$$\nabla^2 K(\rho) + k^2 K(\rho) = 0$$

$$\frac{1}{\rho}(\rho K')' + k^2 K = 0$$

$$K''(\rho) + \frac{1}{\rho} K'(\rho) + k^2 K(\rho) = 0$$

$$K(\rho) = A\, J_0(k\rho)$$

$-j\omega\mu\sigma = \underline{1-j}$

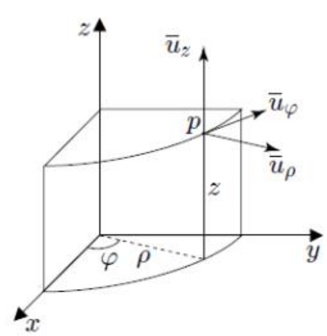
Sylinterikoordinaatisto

$$\nabla f(\bar{r}) = \bar{u}_\rho \frac{\partial}{\partial \rho} f + \bar{u}_\varphi \frac{1}{\rho}\frac{\partial}{\partial \varphi} f + \bar{u}_z \frac{\partial}{\partial z} f$$

$$\nabla \times \bar{f} = \frac{1}{\rho} \begin{vmatrix} \bar{u}_\rho & \rho\bar{u}_\varphi & \bar{u}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ f_\rho & \rho f_\varphi & f_z \end{vmatrix}$$

$$\nabla \cdot \bar{f} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho f_\rho) + \frac{1}{\rho}\frac{\partial}{\partial \varphi} f_\varphi + \frac{\partial}{\partial z} f_z$$

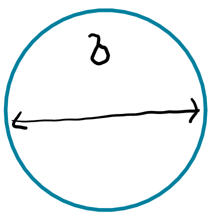
$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

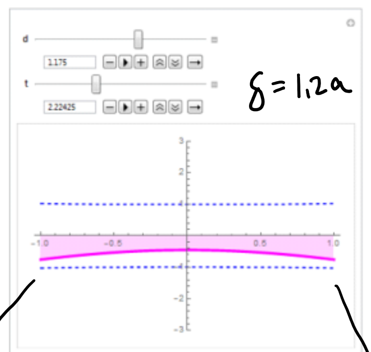


$$\uparrow \quad -j\omega\mu\sigma = \frac{1-j}{\delta}$$

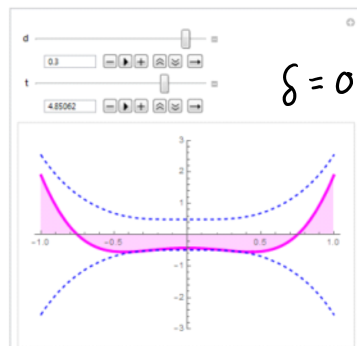
$$\bar{J}(\bar{r}) = \bar{u}_z I_0 \frac{k}{2\pi a} \frac{J_0(k\rho)}{J_1(ka)}$$

$$\mathbf{J}(\mathbf{r}, t) = \mathrm{Re}\left\{\mathbf{J}(\mathbf{r})\mathrm{e}^{\mathrm{j}\omega t}\right\}$$





$\delta = 1{,}2a$



$\delta = 0{,}3\,a$

reuna

reuna