# Exercise 9 – Solutions

**15.11.2022**

## #1 Finding Pareto-optimal solutions with Excel

a) The MOO problem is

$$\underset{[x_1\ x_2]\in\mathbb{N}^2}{\text{v-max}} \begin{bmatrix} 2x_1 + x_2 \\ x_2 \end{bmatrix}$$

$$\text{s.t.}$$

$$2x_1 + 3x_2 \leq 20$$

b) See the Excel model solution on MyCourses site.

c) The overall value of an alternative (ticket buying strategy) is given by the function $V(y_1, y_2) = w_1 v_1^N(y_1) + w_2 v_2^N(y_2)$, where $v_1^N(y_1) = \frac{y_1}{20}$ and $v_2^N(y_2) = \frac{y_2}{6}$. The given preference statement implies $V(y_1 + 1, y_2) - V(y_1, y_2) = V(y_1, y_2 + 2) - V(y_1, y_2)$ from which it can be solved $\frac{w_1}{20} = \frac{2w_2}{6} \Rightarrow w_1 \approx 0.87, w_2 \approx 0.13.$
With these weights, the best ticket buying strategy is $(y_1, y_2) = (10,0)$ as it provides the largest overall value among the relevant decision alternatives (the PO solutions), see the Excel model solution for concrete comparisons.

d) With the weighted max-norm approach we minimize the value of $\Delta$. Thus $c^T = [0 \quad 0 \quad 1]$. Now the inequality constraint $Ax \leq b$ must include the additional constraints (considering the utopian vector and $\Delta$) and the budget constraint.

Notice $\lambda_1\big(f_1^* - f_1(x)\big) \leq \Delta \;\Leftrightarrow\; -\lambda_1 f_1(x) - \Delta \leq -\lambda_1 f_1^* \;\Leftrightarrow\; -\lambda_1(2x_1 + x_2) - \Delta \leq -\lambda_1 f_1^*$.

Similarly $\lambda_2\big(f_2^* - f_2(x)\big) \leq \Delta \Leftrightarrow -\lambda_2 x_2 - \Delta \leq -\lambda_2 f_2^*$

In one matrix, including budget constraint:

$$\begin{bmatrix} 2 & 3 & 0 \\ -2\lambda_1 & -\lambda_1 & -1 \\ 0 & -\lambda_2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \Delta \end{bmatrix} \leq \begin{bmatrix} 20 \\ -\lambda_1 f_1^* \\ -\lambda_2 f_2^* \end{bmatrix} \Leftrightarrow Ax \leq b$$

Notice, that $x_1, x_2 \in \mathbb{N}$