ELEC-E8107 - Stochastic models, estimation and control

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Exercises Session 5

Exercise 1

A nonlinear system dynamic model of a robot moving on the plane is given by the following equation.

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ \theta_{k+1} \\ v_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \Delta t cos(\theta_k) \\ 0 & 1 & 0 & \Delta t sin(\theta_k) \\ 0 & 0 & 1 & \frac{\Delta t}{L} tan(\phi) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \\ \theta_k \\ v_k \end{bmatrix} + q_k \tag{1}$$

Where v is the speed of the vehicle, θ is the heading and q_k is the process noise vector with covariance matrix Q_k. This covariance matrix can be assumed to be a diagonal matrix. The parameter ϕ is the steering angle and is considered a known input to the system. The constant parameter L is the distance between the front and back wheel of the robot. Here we assume $L = 15cm$.

Only the positions x and y of the robot are measured. The measurement noise is assumed Gaussian with zero mean and has 0.5 meters standard deviation. The measurement noise of x-axis and y-axis are assumed independent.

1. Write the measurement equation for the system.
2. Implement the bootstrap particle filter to estimate the state of the system.

Exercise 2

The process dynamic of a system is given as;

$$\begin{aligned} x(k+1) &= 0.8x(k) + 0.3u(k) + v(k) \\ y_1(k) &= x(k) + w_1(k) \\ y_2(k) &= x(k-1) + w_2(k) \end{aligned}$$

Where,

$$\begin{aligned} E[v(k)v(k)^T] &= 0.01 \\ E[w_1(k)w_1(k)^T] &= 0.1 \\ E[w_2(k)w_2(k)^T] &= 0.01 \end{aligned}$$

Following measurements are made from the system;

k	1	2	3
y	10	-10	0
y_1	0.0	3.2	-0.8
y_2	-	0.0	3.0

The task is to devise a Matlab routine that calculates estimates for $x(k)$ using the data available at time k.