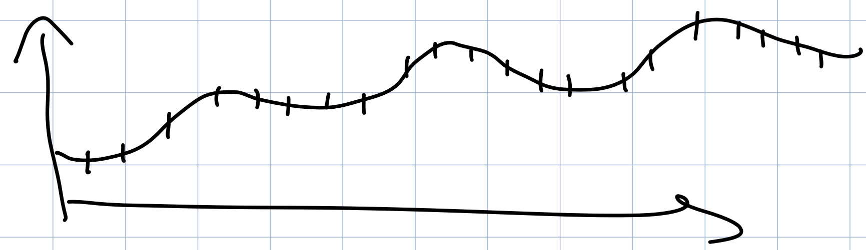


$$\frac{dx(t)}{dt} = a \cdot x(t) + b \cdot u(t)$$

$$\frac{dx}{dt} - ax = b \cdot u \qquad \Big| \cdot e^{-at}$$

$$e^{-at}\frac{dx}{dt} - e^{-at} a \cdot x = e^{-at} b\, u$$

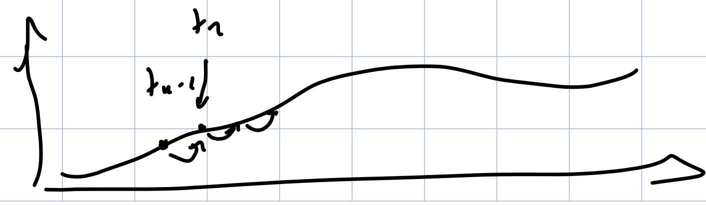
$$\frac{d}{dt}\left[e^{-at} x\right] = e^{-at} \cdot b\, u \qquad \Big| \int_{t_{n-1}}^{t_n}$$

$$e^{-at_n} x(t_n) - e^{-at_{n-1}} x(t_{n-1}) = \int_{t_{n-1}}^{t_n} e^{-at} \cdot b\, u(t)\, dt \qquad \Big| \cdot e^{at_n}$$

$$x(t_n) = e^{a(t_n - t_{n-1})} \cdot x(t_{n-1}) + \underline{\int e^{a(t_n - t)} \cdot b\, u(t)\, dt}$$

define $\Delta t = t_n - t_{n-1}$

$$x(t_n) = \underbrace{e^{a \cdot \Delta t}}_{f} x(t_{n-1}) + \underbrace{\int_{t_{n-1}}^{t_{n-1}+\Delta t} e^{a(t_{n-1} + \Delta t - t)} b \cdot u(t)\, dt}_{\tilde{u}_n}$$



$$x(t_n) = f \cdot x(t_{n-1}) + \tilde{v}_n$$

$$\frac{d\vec{x}(t)}{dt} = A\vec{x}(t) + B_v \vec{u}(t)$$

$$\frac{d\vec{x}}{dt} - A\vec{x}(t) = B_v \cdot \vec{u}(t) \qquad \Big| \, e^{-At}$$

...

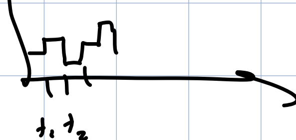
$$\frac{d}{dt}\left[e^{-At}\vec{x}\right] = e^{-At} \cdot B_v \vec{u}(t)$$

...

$$\vec{x}(t_n) = e^{A\cdot(t_n - t_{n-1})} x(t_{n-1}) + \int e^{A\cdot(t_n - t)} \cdot B_v \cdot \vec{u}(t)\, dt$$

$u(t_{n-1})$

$$x(t_n) = \underbrace{e^{a(t_n - t_{n-1})}}_{\text{number}} \cdot x(t_{n-1}) + \underline{\int_{t_{n-1}}^{t_n} e^{a(t_n - t)} \cdot b\, u(t)\, dt}$$




$$\dots = \int_{t_{n-1}}^{t_n} e^{a(t_n - t)}\, dt \cdot b\, u(t_{n-1})$$

$$= \underbrace{\frac{1}{a}\left[1 - e^{a(t_n - t_{n-1})}\right]}_{\text{check!}} \cdot b\, \underline{\underline{u(t_{n-1})}}$$

$$\Rightarrow \quad x(t_n) = \underset{\substack{\uparrow \\ e^{a(t_n - t_{n-1})}}}{f} \cdot x(t_{n-1}) + \underset{\substack{\uparrow \\ \text{a number}}}{\Gamma} \cdot u(t_{n-1})$$

---

$$\frac{d^2 p}{dt^2} = u \qquad \vec{x} = \begin{pmatrix} p \\ dp/dt \end{pmatrix} = \begin{pmatrix} p \\ v \end{pmatrix}$$

$$\vdots$$

$$\frac{d\vec{x}}{dt} = \underbrace{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}_{A} \vec{x} + \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{B_u} u$$

$$F = \exp(A \cdot \Delta t) = \sum_{i=0}^{\infty} \frac{1}{i!} (A \cdot \Delta t)^i$$

$$= \sum_{i=0}^{\infty} \frac{1}{i!} \underbrace{A^i}_{} \cdot \Delta t^i = \underbrace{A^0 \cdot \Delta t^0}_{I} + \underbrace{A^1 \cdot \Delta t^1}_{\begin{pmatrix} 0 & \Delta t \\ 0 & 0 \end{pmatrix}} = \begin{pmatrix} 1 & \Delta t \\ 0 & 1 \end{pmatrix}$$

$$A^0 = I$$

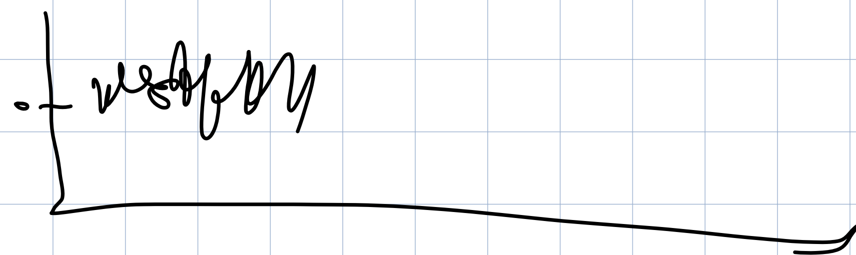
$$A^1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$A^3 = A \cdot A^2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

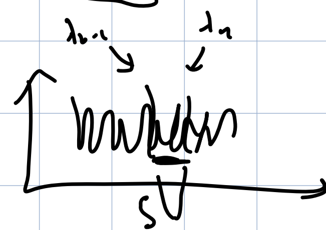
$$L = \int_{t_{n-1}}^{t_n} e^{A(t_n - t)} B_w \, dt = \int_{t_{n-1}}^{t_n} \begin{pmatrix} 1 & t_n - t \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} dt$$

$$= \int_{t_{n-1}}^{t_n} \begin{pmatrix} t_n - t \\ 1 \end{pmatrix} dt = \begin{pmatrix} \frac{1}{2}(t_n - t_{n-1})^2 \\ t_n - t_{n-1} \end{pmatrix}$$



1D:

$$x(t_n) = \overbrace{e^{a \cdot (t_n - t_{n-1})}}^{f} x(t_{n-1}) + \underbrace{\int_{t_{n-1}}^{t_n} e^{a \cdot (t_n - t)} \cdot b\, w(t)\, dt}_{\text{Gaussian}}$$




$$\frac{d\vec{x}}{dt} = \underbrace{\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}}_{A} \vec{x} + \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{B_w} w$$

$$e^{A \cdot \Delta t} = \begin{pmatrix} 1 & \Delta t \\ 0 & 1 \end{pmatrix}$$

$$Q_n = \int_{t_{n-1}}^{t_n} e^{A(t_n-\tau)} B_w \Sigma_w B_w^T e^{A^T(t_n-\tau)} \, d\tau$$

$$= \int_{t_{n-1}}^{t_n} \begin{pmatrix} 1 & t_n-\tau \\ 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix} \sigma_w^2 \begin{pmatrix} 0 \\ 1 \end{pmatrix}^T}_{\begin{pmatrix} 0 & 0 \\ 0 & \sigma_w^2 \end{pmatrix}} \begin{pmatrix} 1 & t_n-\tau \\ 0 & 1 \end{pmatrix}^T d\tau$$

$$= \int_{t_{n-1}}^{t_n} \begin{pmatrix} 0 & \sigma_w^2(t_n-\tau) \\ 0 & \sigma_w^2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ t_n-\tau & 1 \end{pmatrix} d\tau$$

$$= \int_{t_{n-1}}^{t_n} \begin{pmatrix} \sigma_w^2(t_n-\tau)^2 & \sigma_w^2(t_n-\tau) \\ \sigma_w^2(t_n-\tau) & \sigma_w^2 \end{pmatrix} d\tau$$

$$= \begin{pmatrix} \frac{1}{3}\sigma_w^2(t_n-t_{n-1})^3 & \frac{1}{2}\sigma_w^2(t_n-t_{n-1})^2 \\ \frac{1}{2}\sigma_w^2\underbrace{(t_n-t_{n-1})}_{\Delta t}{}^2 & \sigma_w^2(t_n-t_{n-1}) \end{pmatrix}$$

$$= Q$$