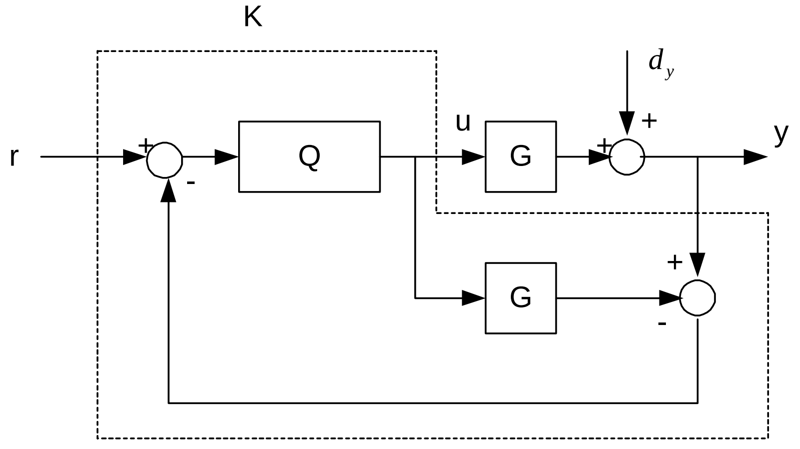
ELEC-E8116 Model-based control systems
/exercises with solutions 9

Problem 1: Consider the following IMC-control configuration, in which the process G is assumed stable.



a. Prove that to study the internal stability, the stability of the transfer functions

$$K(I+GK)^{-1} = Q$$
$$(I+GK)^{-1} = I - GQ$$
$$(I+KG)^{-1} = I - QG$$
$$G(I+KG)^{-1} = G(I-QG)$$

must be investigated. Prove that the system is internally unstable, if either Q or G is unstable.

b. Let a stable controller K be given. How can you characterize those processes, which can be stabilized with this controller? (Hint: Change the roles of the controller and process.)

Solution:

a. For the control it holds

$$u = Q[r-(y-Gu)] = Q(r-y) + QGu$$

from which it follows easily

$$u = (I-QG)^{-1}Q(r-y)$$

But this has the form

$$u = K(r - y)$$

where $K = (I - QG)^{-1}Q$

and

$$Q = K(I + GK)^{-1}$$

By this controller the configuration is equivalent to the ”one-degree-of-freedom”-structure. Based on lectures (Chapter 3, Internal stability of closed-loop systems) it is known that the system is internally stable, if the transfer functions

$$K(I + GK)^{-1} = Q$$
$$(I + GK)^{-1} = I - GQ$$
$$(I + KG)^{-1} = I - QG$$
$$G(I + KG)^{-1} = G(I - QG)$$

are stable (the ”right sides” follow easily from the choice of Q).

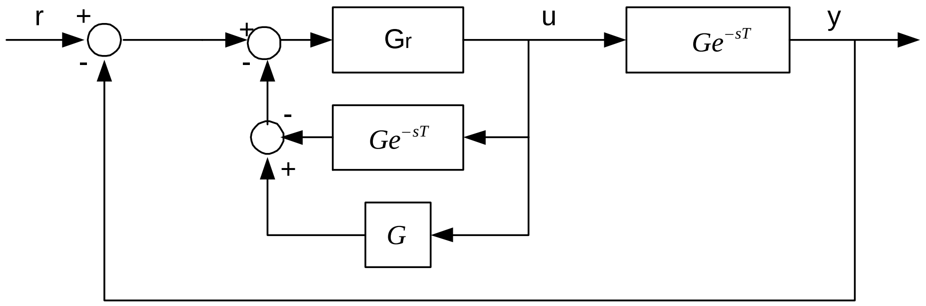
But the functions are clearly stable, if Q and G are stable. Correspondingly, if either one is unstable, the system is internally unstable.

b. These systems can be represented in the form (parameterization)

$$G = (I - QK)^{-1}Q = Q(I - KQ)^{-1}$$

where Q is any stable transfer function matrix.

Problem 2. Consider the control configuration shown in the figure (known as the *Smith-predictor*). Calculate the closed loop transfer function and verify the idea behind this controller. Compare to the *IMC*-controller and prove that the Smith predictor always leads to an internally unstable system, if the process is unstable.



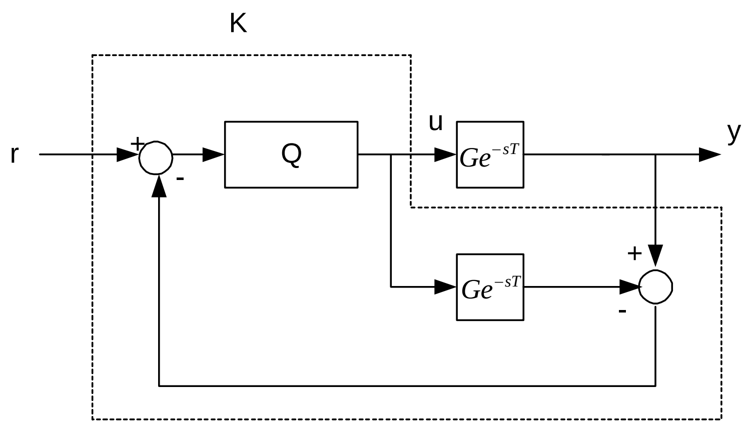
Solution: By using block diagram algebra the transfer function from *r* to *y* is easily calculated to be

$$Y(s)=\frac{G_r(s)G(s)}{1+G_r(s)G(s)}e^{-sT}R(s)$$

which reveals the idea behind this control configuration: the basic controller G_r can be designed to give a good closed loop response without paying any attention to the process delay. The real response is then the same but added with a pure delay *T*. The term e^{-sT} is not shown in the characteristic equation (which would happen, if G_r would directly control the process with delay). But note that in this ideal case the process is exactly known and the intermediate block in the controller generates the predicted value of the output. In reality an inaccurate process model has to be used for this purpose.

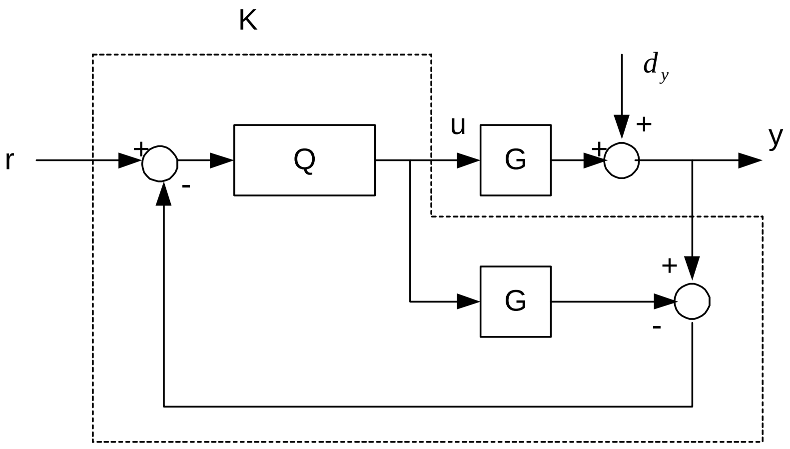
But: moving the block Ge^{-sT} in the figure a bit (without changing the control signal *u*, of course), the configuration below is obtained. That is directly the *IMC*-structure. There

$$Q=\frac{G_r}{1+G_rG}$$



But what if the process *G* is unstable? Look at the previous problem, in which it was shown that the closed loop system is internally unstable, if *Q* or *G* is unstable. Because the Smith predictor structure was above shown to be equivalent to the *IMC*-structure, the closed loop is inevitably (internally) unstable, if the process is unstable.

Problem 3. Consider the IMC control structure, which is used to control a stable and minimum phase SISO process *G*.



Note that in addition to the reference r a disturbance signal d_y is modelled to enter at the output of the process. By using the IMC design discussed in the lectures analyse the response to step inputs at r and d_y.

Solution:

The figure represents a two-degrees-of-freedom control configuration, where the inputs to the controller K are r and y. Again, it is easy to write

$$u = Q\left[r - (y - Gu\right] = Q(r-y) + QGu \Rightarrow u = (I - QG)^{-1}Q(r-y)$$

But that can be interpreted as a one-degree-of-freedom configuration with the controller

$$u = K_1(r-y), \quad K_1 = (I-QG)^{-1}Q = \frac{Q}{1-QG} \qquad \text{(SISO!)}$$

Using the design (see lecture slides)

$Q = \dfrac{1}{(\lambda s+1)^n}G^{-1}$ and writing equations from the topology in the figure

$$y = d_y + Gu = d_y + GK_1(r-y) \Rightarrow y = \frac{GK_1}{1+GK_1}r + \frac{1}{1+GK_1}d_y$$

Setting K_1 to this gives after simple calculations

$$y = \frac{\dfrac{GQ}{1-QG}}{1+\dfrac{GQ}{1-QG}}r + \frac{1}{1+\dfrac{GQ}{1-QG}}d_y = GQr + (1-QG)d_y = \frac{1}{(\lambda s+1)^n}r + \left[1 - \frac{1}{(\lambda s+1)^n}\right]d_y$$

Note that $GQ = QG$ for SISO systems. Also $y = GQr + (1-QG)d_y$ could have been obtained directly from the figure (careful!).

Setting $s = 0$ we find that the static gain from r to y is 1 and from d_y to y 0, so that the output follows the reference and mitigates the disturbance asymptotically. Note that internal stability was guaranteed by the fact that G was stable and minimum phase (G^{-1} stable) and Q stable.