Mass balances

WAT-E2120 Physical and Chemical Treatment of Water and Waste

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General mass balance

- Also called material balances
- ”A mass balance is a tool to keep track of how much substance is in a given region of space at a given time”
- Conservation of mass, or rate at which substance i enters, exit, reacts and accumulates

$$\begin{pmatrix}\text{Rate of change}\\ \text{of mass of } i\\ \text{stored in the}\\ \text{system}\end{pmatrix} = \begin{pmatrix}\text{Rate at which}\\ i \text{ enters the}\\ \text{system from}\\ \text{outside}\end{pmatrix} - \begin{pmatrix}\text{Rate at which}\\ i \text{ exits the}\\ \text{system to}\\ \text{the outside}\end{pmatrix} + \begin{pmatrix}\text{Rate at which}\\ i \text{ is generated}\\ \text{inside the}\\ \text{reactor}\end{pmatrix} - \begin{pmatrix}\text{Rate at which}\\ i \text{ is destroyed}\\ \text{inside the}\\ \text{reactor}\end{pmatrix}$$

- Rate: mass/time, volume/time, moles/tıme (... ...)

Short form of the mass balance

Accumulation in system	**=**	**Input to system**	**–**	**Output from system**	**+**	**Generation in system**	**–**	**Consumption in system**

Even shorter:

ACC = IN – OUT + GEN – CON

Mass balance system boundaries

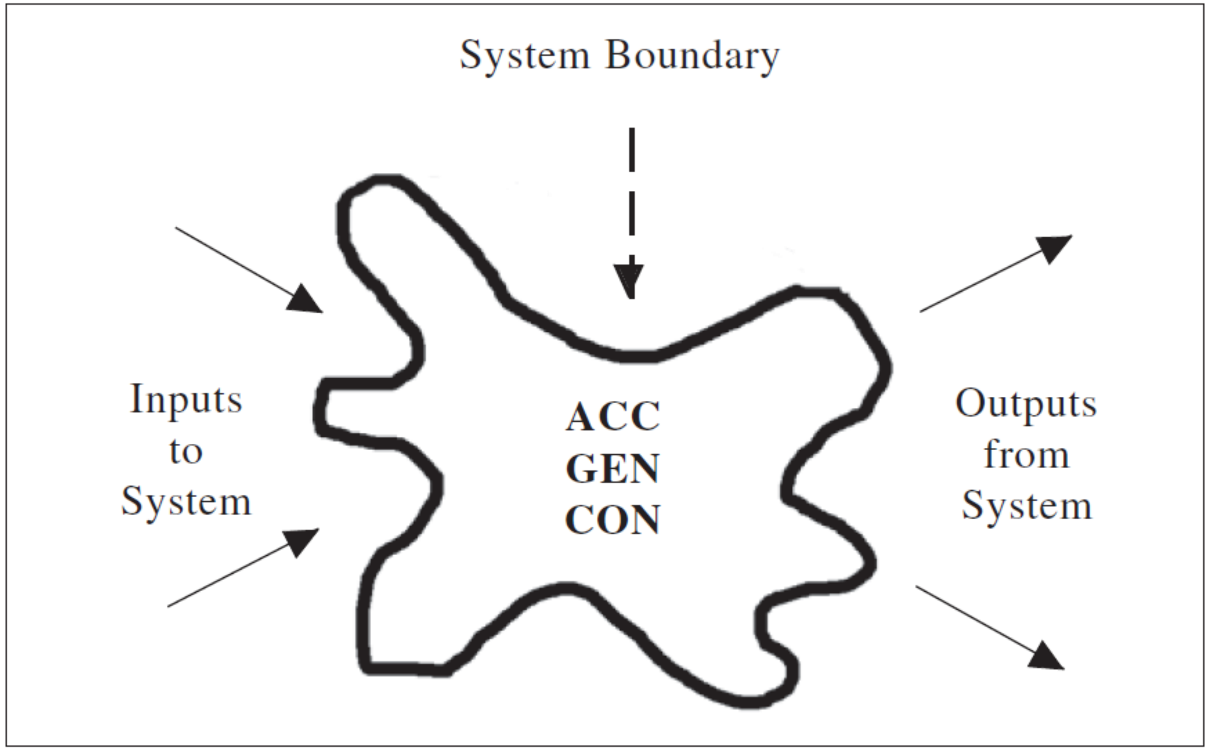
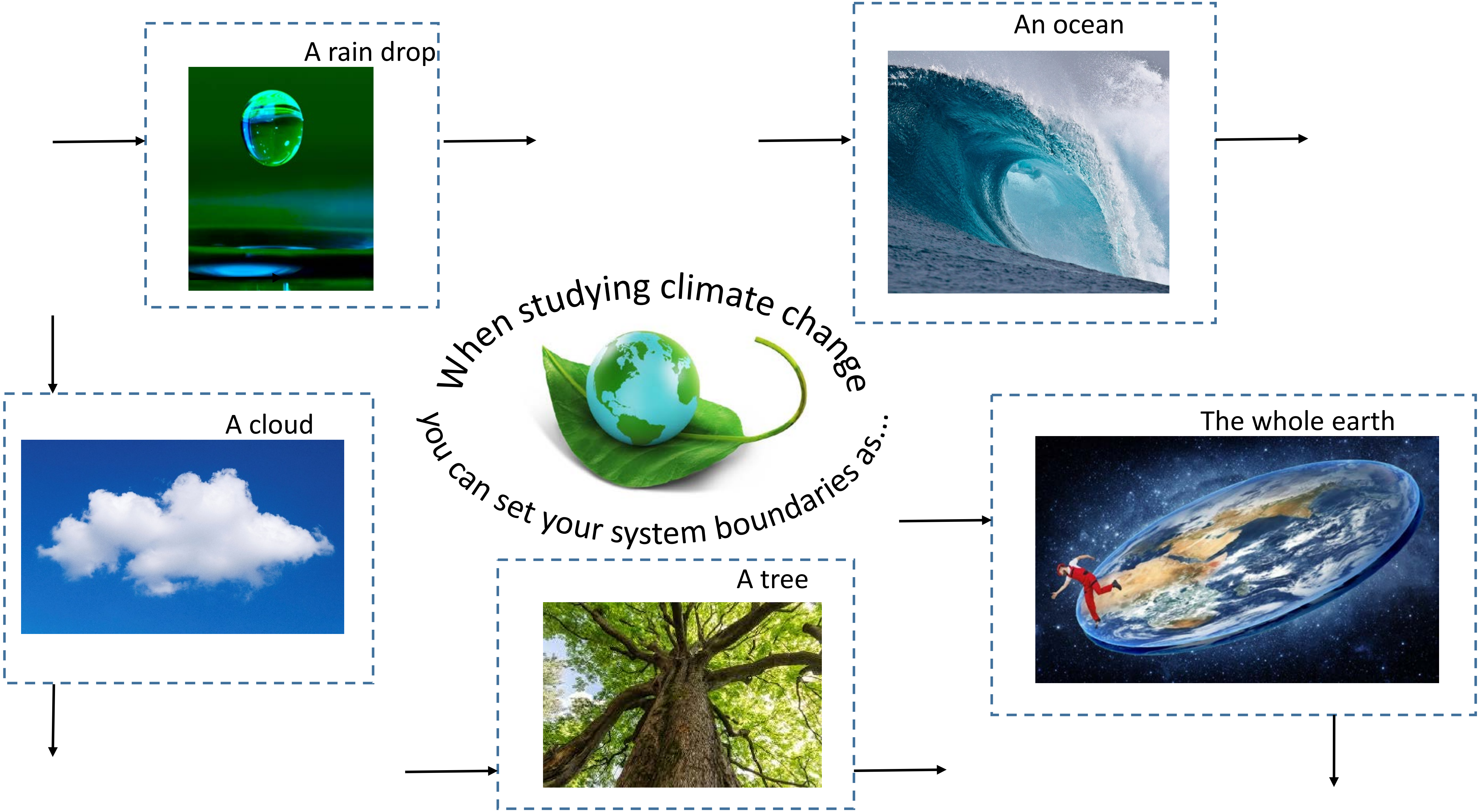


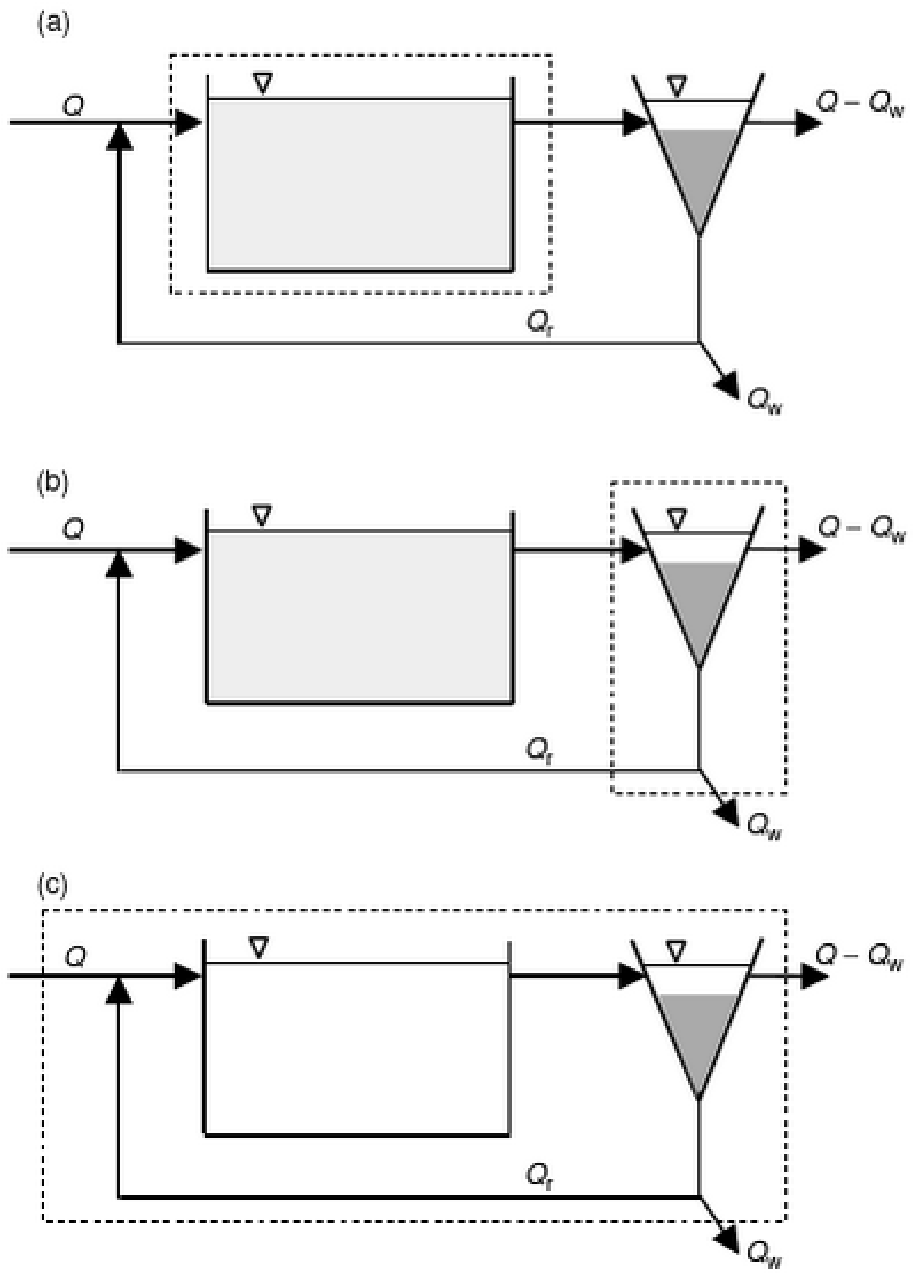
Figure 1.01. Conceptual diagram of a system.



A rain drop
An ocean
When studying climate change
you can set your system boundaries as...
A cloud
The whole earth
A tree

Mass balances

- System (boundaries) = Control volume = CV
- You choose the boundaries
  - For each purpose

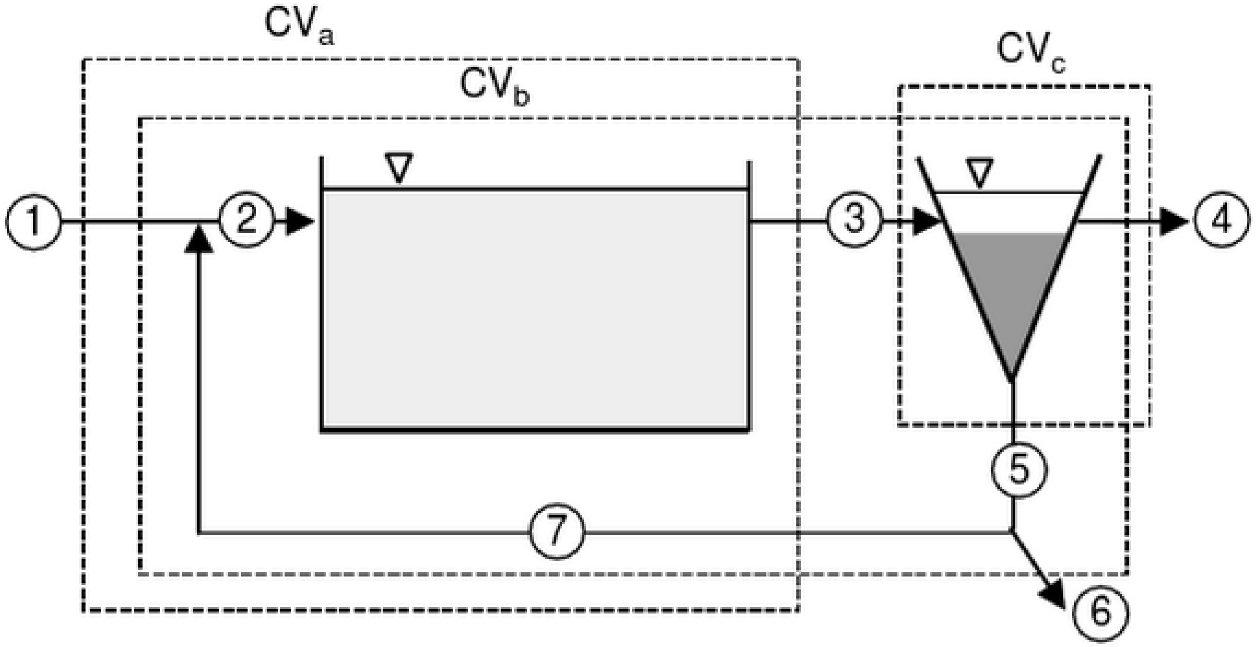


To choose the control volumes

- An open reactor and a settling basin
- With recycle of settled solids to the reactor influent
- Waste stream from the recycle stream
- Balance boundaries shown with broken line
- Q = Influent flow rate
- Q_r = Recycle stream flow rate
- Q_w = waste stream flow rate

In the picture the mass balances have been already utilized to calculate flow rates

Marking control volumes

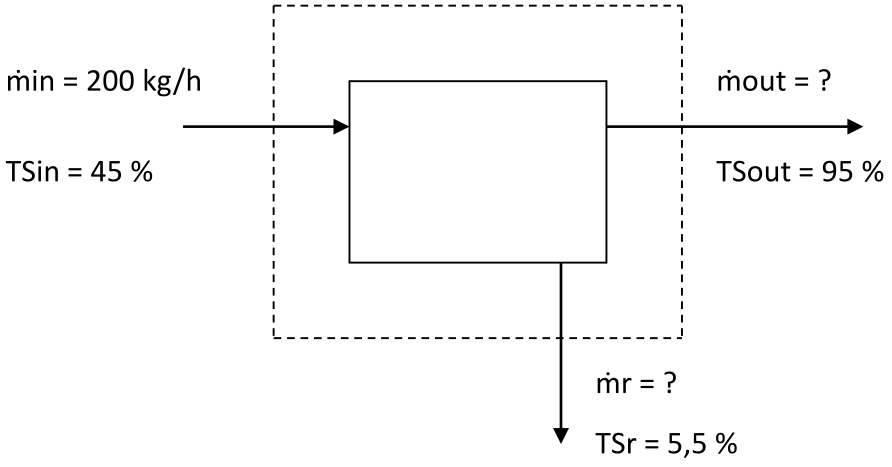


Various control volumes for mass balances that are useful for solving the example problem.

Mark down the flow rates and control volumes

Separation of solids

A capillary sludge drying system is fed 200 kg/h sludge, with suspended solids of 45% (55% water). In the dried sludge there is 95% of suspended solids and in the reject water stream leaving the drying system contains suspended solids 55 g/l. What is the flow of dried sludge (kg/h) and the reject water (kg/h)?



$\dot{m}in = \dot{m}out + \dot{m}r$ (1)

$\dot{m}in*TSin = \dot{m}out*TSout + \dot{m}r*TSr$ (2)

(1) => $\dot{m}out = \dot{m}in - \dot{m}r$

Insert in (2)

=> $\dot{m}in*TSin = (\dot{m}in-\dot{m}r)*TSout + \dot{m}r*TSr$

$= \dot{m}in*TSo - \dot{m}r*TSout + \dot{m}r*TSr$

$= \dot{m}in*TSout + (TSr-TSout)*\dot{m}r$

<=> $(TSr-TSout)*\dot{m}r = \dot{m}in*TSin - \dot{m}in*TSout$

<=> $\dot{m}r = (\dot{m}in*TSin-\dot{m}in*TSout) / (TSr-TSout)$

$= (200\ kg/h*0.45-200\ kg/h*0.95) / (0.055 - 0.95)$

$= (90\ kg/h - 190\ kg/h) / (-0.895)$

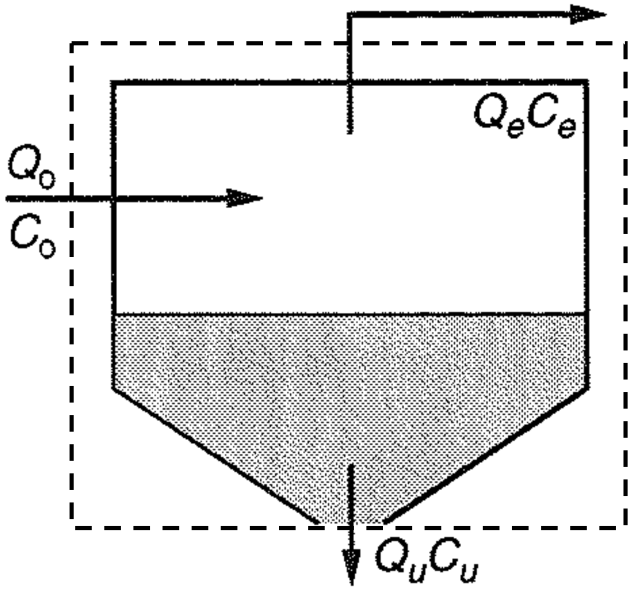
$= (-100\ kg/h) / (-0.895) = 112\ kg/h =$ <u>110 kg/h</u>

(1) : $\dot{m}out = \dot{m}in - \dot{m}r = 200\ kg/h - 110\ kg/h =$ <u>90 kg/h</u>

=> $\dot{m}r = 110\ kg/h$

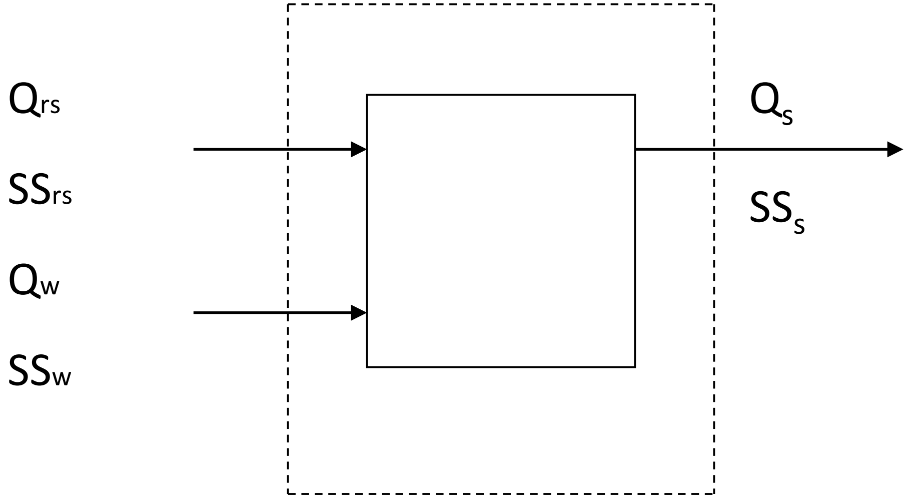
ja $\dot{m}out = 90\ kg/h$

Thickener



- $Q_0 = Q_u + Q_e$
- $Q_0C_0 = Q_uC_u + Q_eC_e$
- Sedimentation (settling) tank is similar to thickener

Mixing of solids



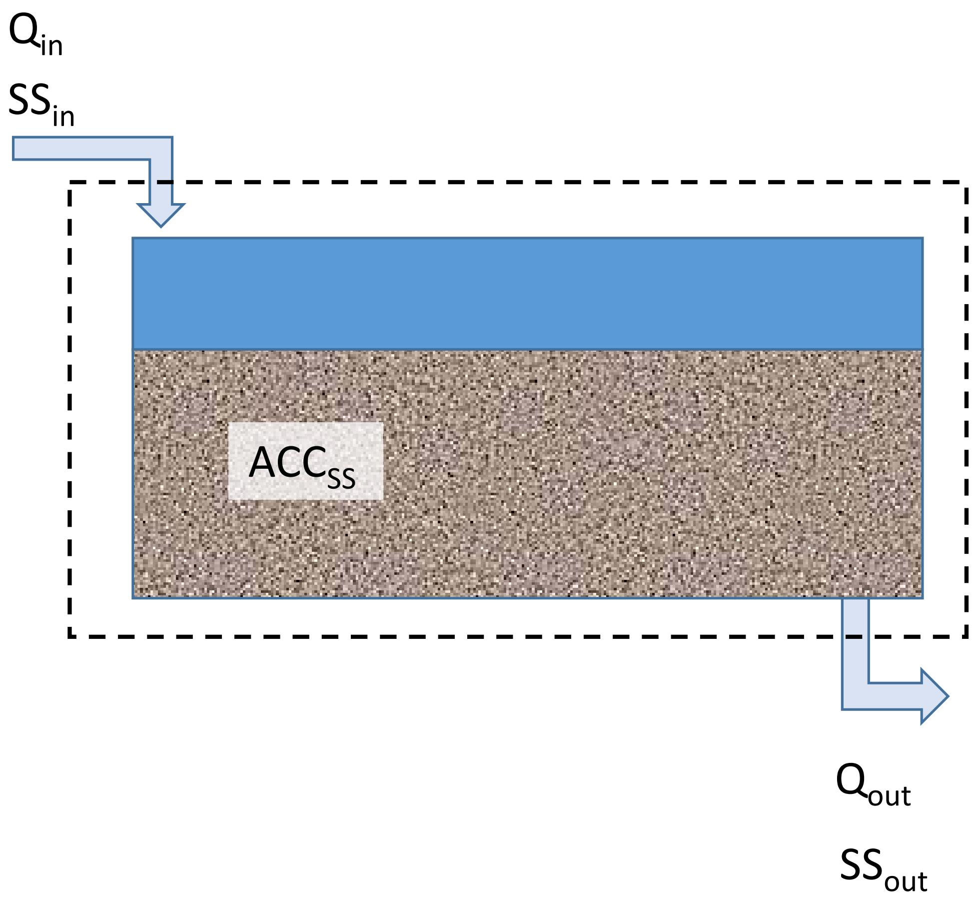
Exercise 4.

Sand filtration

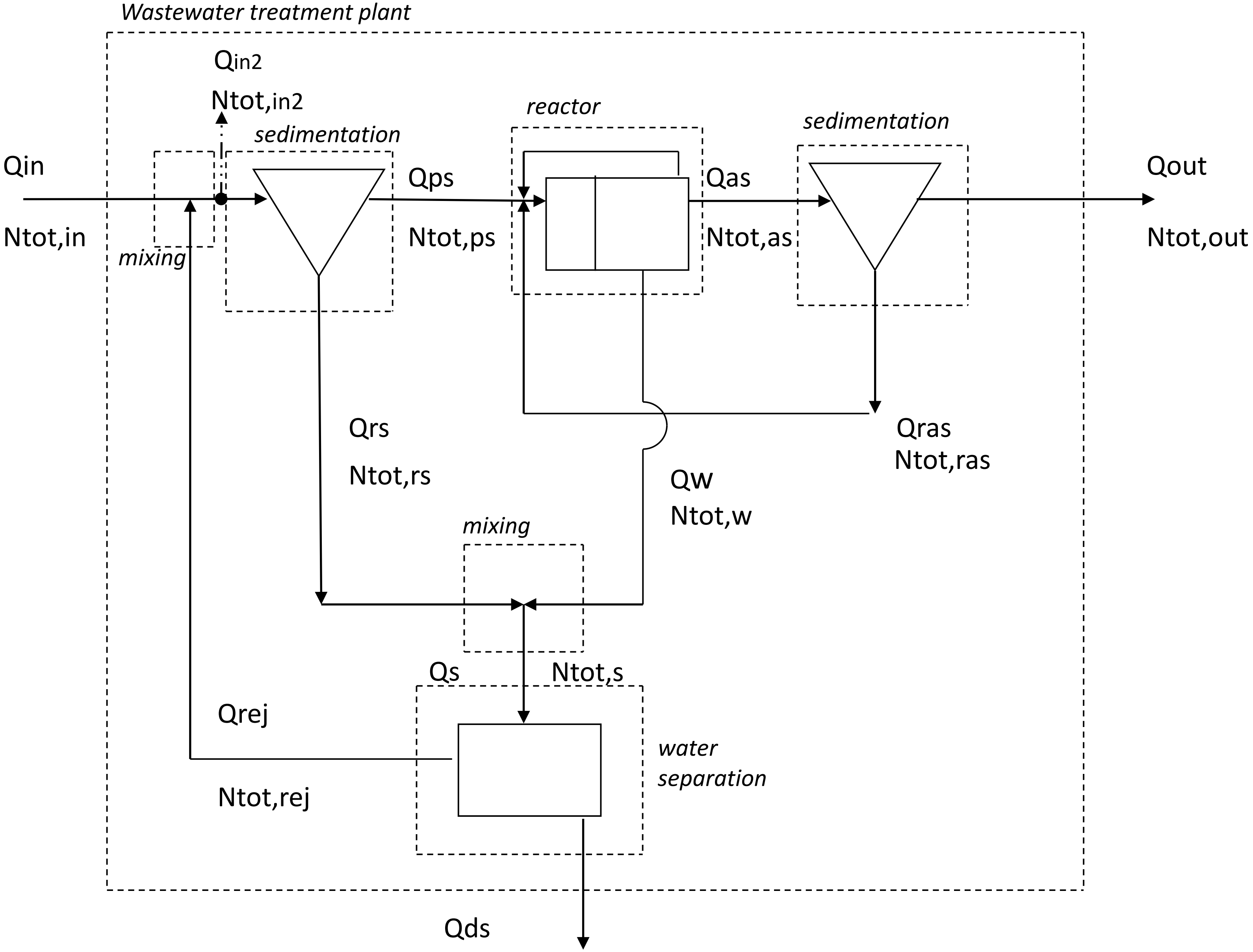
- Water treatment plant receives raw water that is treated with sand filtration to remove suspended solids (SS)
- SS is accumulated in the filter bed and removed by washing regularly
  - Q_{in}, Q_{out}
  - SS_{in}, SS_{out}
  - ACC_{SS}
- Material balances:

$$Q_{in} = Q_{out}$$

$$ACC_{SS} = Q_{out} * SS_{out} - Q_{in} * SS_{in}$$



Wastewater treatment plant balance as a block diagram



- The diagram shows both flow and nitrogen

ps = primary sedimentation
as = activated sludge
ras = return activated sludge
w = waste sludge
rs = raw sludge from promary sedimentation
rej = reject water from sludge drying
s = sludge
ds = dried sludge

Note.Ex. 4

Systems and processes

Practical problems are classified according to the type of system and the nature of the process occurring in the system, as follows:

Closed system	Zero material* is transferred in or out of the system [during the time period of interest].		
[Controlled mass]	i.e. in the material balance equation:	IN = OUT = 0	
	A process occurring in a closed system is called a BATCH process.		
Open system	Material is transferred in and/or out of the system.		
[Controlled volume]	i.e. in the material balance equation:	IN $\neq$ 0 and/or OUT $\neq$ 0	
	A process occurring in an open system is called a CONTINUOUS process.		
Steady-state process	A process in which all conditions are invariant with time.		
	i.e. at steady-state:	Rate ACC = 0	for all quantities.
Unsteady-state process	A process in which one or more conditions vary with time [these are *transient* conditions], i.e. at unsteady-state:	Rate ACC $\neq$ 0	for one or more quantities.

* *Energy can be transferred in and/or out of both closed and open systems.*

”How to” of mass balances

1. Draw a diagram and balance boundary/ies
2. Write down all known quantities
3. Identify and assign symbols to all unknown quantities
4. Determine the appropiate set of equations to solve the unknowns
5. Solve the unknowns