Exercise 10

22.11.2022

#1 Finding Pareto-optimal solutions with MATLAB

Consider a company with an R&D budget of 80M€. A team of managers is currently considering 40 R&D project proposals with costs c_j , $j = 1, \dots, 40$. Each proposal is presented by one of the company's three research divisions. Let $r_{kj} = 1$, if the j^{th} project is proposed by the k^{th} division, and $r_{kj} = 0$ otherwise. The projects can only be carried out by the proposing division.

The team wants you, the company's decision analyst, to present them the Pareto-optimal project portfolios with regard to the objectives of minimizing risk (variance of profit) and maximization of expected profit of the portfolio. The profits of the projects are assumed independent and normally distributed, with expected profits μ_j and standard deviations σ_j. The team requires that at least 20 % of the total R&D budget is allocated to each division (in terms of funding proposals).

a) Formulate a multi-objective integer linear programming model of the form (here natural numbers include 0)

$$\text{v-}\max_{x \in \mathbb{N}^m} \{Cx | Ax \leq B\}, \mathrm{C} \in \mathbb{R}^{\mathrm{n} \times \mathrm{m}}, \mathrm{A} \in \mathbb{R}^{\mathrm{q} \times \mathrm{m}}, B \in \mathbb{R}^{\mathrm{q}}$$

b) Assume you have a mixed integer linear programming (MILP) solver that solves problems of the form

$$\max_{\substack{x \in \mathbb{N}^m \\ y \in \mathbb{R}^{m'}}} \{c \begin{bmatrix} x \\ y \end{bmatrix} | A \begin{bmatrix} x \\ y \end{bmatrix} \leq B\}, \mathrm{c} \in \mathbb{R}^{1 \times (\mathrm{m}+\mathrm{m}')}, \mathrm{A} \in \mathbb{R}^{\mathrm{q} \times (\mathrm{m}+\mathrm{m}')}, B \in \mathbb{R}^{\mathrm{q}}$$

Formulate i) weighted sum and ii) weighted max-norm algorithms that use this solver to find Pareto-optimal portfolios.

c) The function "intlinprog" included in the Optimization Toolbox of Matlab is a MILP solver of the kind described in b).

1. Install the Optimization Toolbox to your computer (if you do not have it yet). See the installation help guide from the exercise folder if needed.
2. Write "help intlinprog" to Matlab's Command Window and familiarize yourself with the syntax of the function.
3. Open Ex_10_Template.m. Complete the missing parts of the code (6 instances) and use it to find Pareto-optimal solutions to the problem.
    i. How many Pareto optimal portfolios can you find?
    ii. Which projects are included in all Pareto-optimal portfolios?