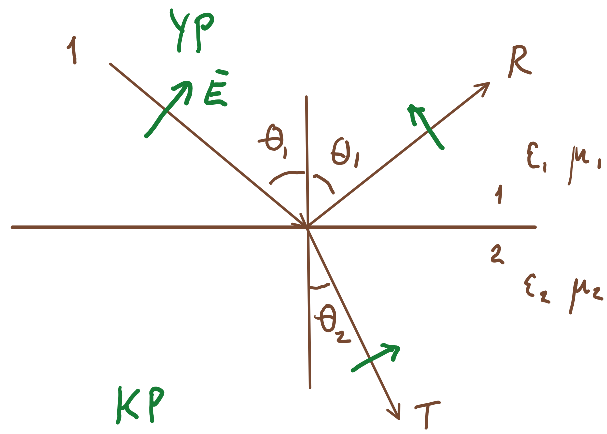
Kertausta heijastuksesta:



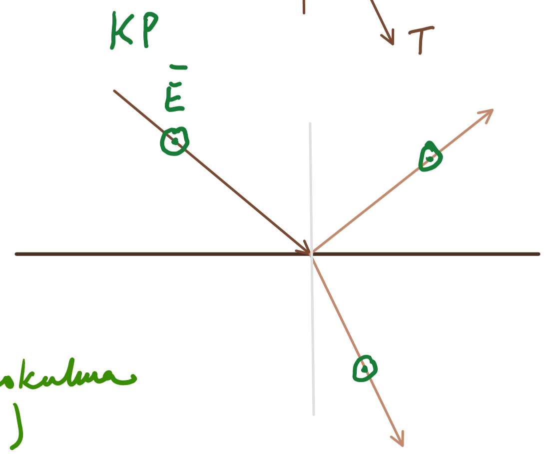
Snellin laki

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_1 = \sqrt{\mu_{r_1} \varepsilon_{r_1}} \qquad n_2 = \sqrt{\mu_{r_2} \varepsilon_{r_2}}$$

$$\eta_1 = \sqrt{\mu_1/\varepsilon_1} \qquad \eta_2 = \sqrt{\mu_2/\varepsilon_2}$$

$$R = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad (\text{kohtisuora tulo, } \theta_1 = \theta_2 = 0)$$



$$R = \frac{\eta_{2t} - \eta_{1t}}{\eta_{2t} + \eta_{1t}} \quad \leftarrow \text{vino tulokulma } (\theta_1 > 0)$$

$$\eta_{it} = \begin{cases} \eta_i \cos\theta_i & (YP) \\ \eta_i / \cos\theta_i & (KP) \end{cases} \qquad (i = 1, 2)$$

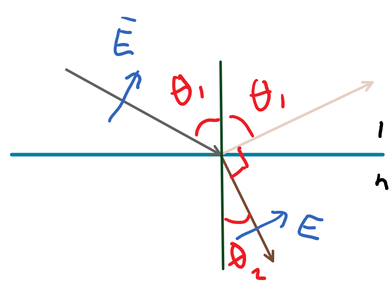
Dielektrinen rajapinta ja Brewsterin kulma

1 — ILMA : ε_0, μ_0 $\quad \eta_1 = \eta_0$

2 — DIEL. AINE $\mu_0,\ \varepsilon_r\varepsilon_0$ $\quad \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{\eta_0}{n}$

$$R_{YP} = \frac{\eta_2 \cos\theta_2 - \eta_1 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1}$$

$$= \frac{\cos\theta_2 - n\cos\theta_1}{\cos\theta_2 + n\cos\theta_1} \overset{?}{=} 0$$



$$n\cos\theta_1 = \cos\theta_2$$

$$\sin\theta_1 = n \sin\theta_2$$

$$\cancel{n} \sin\theta_1 \cos\theta_1 = \cancel{n} \sin\theta_2 \cos\theta_2$$

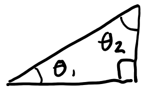
$$2 \qquad\qquad 2$$

$$\sin 2\theta_1 = \sin 2\theta_2 \qquad (2\theta_1 = 2\theta_2 \rightarrow \text{ei rajapintaa})$$

$$\sin 2\theta_1 = \sin 2\theta_2 \qquad (2\theta_1 = 2\theta_2 \rightarrow \text{ei rajapintaa})$$

$$2\theta_1 + 2\theta_2 = 180^\circ$$

$$\theta_1 + \theta_2 = 90^\circ$$



$$\sin\theta_1 = \cos\theta_2 \longrightarrow \sin\theta_1 = n\cos\theta_1$$

$$\Rightarrow \tan\theta_1 = n \quad \Rightarrow R_{Y_P} = 0$$

$\rightarrow$ Brewsterin Kulma: $\theta_{Br} = \arctan(n)$

Sähkömagneettinen säteily.

$$\nabla \times \bar{E} = -j\omega\mu\bar{H} \qquad (\rho = 0,\ \bar{J} = 0)$$

$$\nabla \times \bar{H} = j\omega\varepsilon\bar{E}$$

$$\underbrace{\nabla\times\nabla\times\bar{E}}_{\nabla(\nabla\cdot\bar{E}) - \nabla^2\bar{E}} = -j\omega\mu j\omega\varepsilon\bar{E} = \underbrace{\omega^2\mu\varepsilon}_{k^2}\bar{E}$$

$$\nabla^2\bar{E} + k^2\bar{E} = 0 \quad \rightarrow \quad \text{Tasoaalto: } E''(z) + k^2\bar{E}(z) = 0$$

$$e^{-jkz}$$

Palloaalto $\quad P(r)$

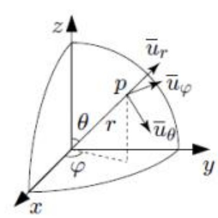
$$\nabla^2 P(r) + k^2 P(r) = 0$$

$$\nabla^2 P = \frac{1}{r^2}\left(r^2 P'\right)'$$

$$= \frac{1}{r^2}\left(2r P' + r^2 P''\right)$$

$$= P'' + \frac{2}{r}P'$$

Pallokoordinaatisto



$$\nabla f(\bar{r}) = \bar{u}_r\frac{\partial}{\partial r}f + \bar{u}_\theta\frac{1}{r}\frac{\partial}{\partial\theta}f + \bar{u}_\varphi\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}f$$

$$\nabla\times\bar{f} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \bar{u}_r & r\bar{u}_\theta & r\sin\theta\,\bar{u}_\varphi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi} \\ f_r & r f_\theta & r\sin\theta\, f_\varphi \end{vmatrix}$$

$$\nabla\cdot\bar{f} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 f_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta f_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}f_\varphi$$

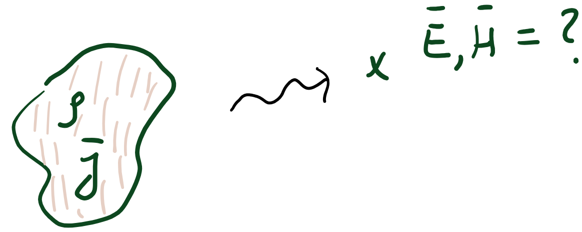
$$\underline{\nabla^2 f} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \cancel{\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right)} + \cancel{\frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}}$$

$$\frac{1}{r}(rP)'' = \frac{1}{r}(P + rP')' = \frac{1}{r}(P' + P' + rP'') = P'' + \frac{2}{r}P'$$

$$\frac{1}{r}(rP)'' + k^2 P = 0 \quad \Rightarrow \quad (rP)'' + k^2 (rP) = 0$$

$$rP(r) = e^{-jkr} \qquad (e^{+jkr})$$

$$A\,\frac{e^{-jkr}}{r} \qquad (\text{PALLOAALLON KENTTÄ})$$



$$\rho \neq 0, \quad \bar{J} \neq 0$$

1) $\nabla \cdot \bar{B} = 0 \Rightarrow \bar{B} = \nabla \times \bar{A}$

2) $\nabla \times \bar{E} = -j\omega \bar{B} = -j\omega \nabla \times \bar{A} \Rightarrow \nabla \times (\underbrace{\bar{E} + j\omega \bar{A}}_{-\nabla\phi}) = 0$

3) $\nabla \times \mu \bar{H} = \mu \bar{J} + \mu j\omega\varepsilon \bar{E}$

$$\nabla \times (\nabla \times \bar{A}) = \mu \bar{J} + j\omega\mu\varepsilon(-\nabla\phi - j\omega\bar{A})$$

$$\nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A} = \mu \bar{J} - j\omega\mu\varepsilon\nabla\phi + \underbrace{\omega^2\mu\varepsilon}_{k^2}\bar{A}$$

LORENZIN EHTO : $\nabla \cdot \bar{A} = -j\omega\mu\varepsilon\phi$

$$\nabla^2 \bar{A}(\bar{r}) + k^2 \bar{A}(\bar{r}) = -\mu \bar{J}$$

4) $\nabla \cdot \bar{D} = \nabla \cdot \varepsilon \bar{E} = \rho$

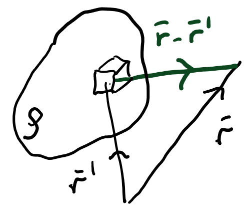
$$\Rightarrow \nabla \cdot (-\nabla\phi - j\omega\bar{A}) = \rho/\varepsilon$$

$$-\nabla^2\phi - j\omega \underbrace{\nabla\cdot\bar{A}}_{-j\omega\mu\varepsilon\phi} = \rho/\varepsilon$$

$$\nabla^2\phi + \underbrace{\omega^2\mu\varepsilon}_{k^2}\phi = -\rho/\varepsilon$$

SÄHKÖSTATIIKKA: $\omega = 0$

$$\nabla^2\phi(\bar{r}) = -\frac{\rho(\bar{r})}{\varepsilon}$$



$$\phi(\bar{r}) = \int \frac{\rho(\bar{r}')\,dv'}{4\pi\varepsilon|\bar{r}-\bar{r}'|}$$

DYNAMIIKKA $(k \neq 0)$

$$\phi(\bar{r}) = \int \frac{\rho(\bar{r}')e^{-jk|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|}dv'$$

$$\bar{A}(\bar{r}) = \int \frac{\mu\bar{J}(\bar{r}')e^{-jk|\bar{r}-\bar{r}'|}}{4\pi|\bar{r}-\bar{r}'|}dv'$$

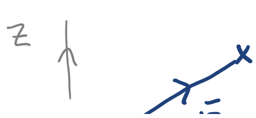
(vaikea)-

$$\rho, \bar{J} \longrightarrow \phi, \bar{A} \longrightarrow \bar{E}, \bar{H}$$

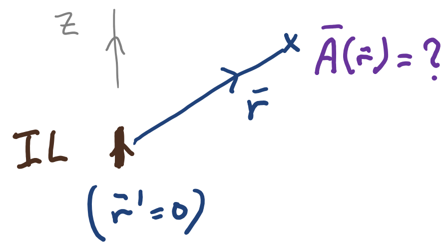
HERTZIN DIPOLI

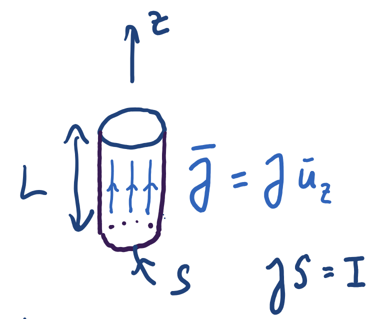
$L \ll \lambda$, VIRTAMOMENTTI IL

$$kL = \frac{2\pi L}{\lambda} \ll 1$$



$\bar{A}(\bar{r}) = ?$





$JS = I$

$dV' = SL$

$$\bar A(\bar r) = \frac{\mu\, e^{-jkr}}{4\pi r} \overbrace{\int \bar J \, dv'}^{IL}$$

$$= \mu I L \frac{e^{-jkr}}{4\pi r} \bar u_z$$

$\bar u_r \cos\theta - \bar u_\theta \sin\theta$

MAGNEETTIKENTTÄ

$$\bar B = \nabla \times \bar A \Rightarrow \bar H = \frac{\nabla \times \bar A}{\mu}$$

$$\bar A = \mu I L \frac{e^{-jkr}}{4\pi r}\left(\bar u_r \cos\theta - \bar u_\theta \sin\theta\right)$$

$$\bar H = \frac{\nabla \times \bar A}{\mu} = \frac{1}{\mu}\frac{1}{r^2 \sin\theta}\begin{vmatrix} \bar u_r & r\bar u_\theta & r\sin\theta\, \bar u_\varphi \\ \partial/\partial r & \partial/\partial\theta & \cancel{\partial/\partial\varphi}\ 0 \\ A_r & rA_\theta & 0 \end{vmatrix}$$

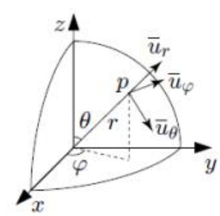
Pallokoordinaatisto

$$\nabla f(\bar r) = \bar u_r \frac{\partial}{\partial r} f + \bar u_\theta \frac{1}{r}\frac{\partial}{\partial \theta} f + \bar u_\varphi \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi} f$$

$$\nabla \times \bar f = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \bar u_r & r\bar u_\theta & r\sin\theta\,\bar u_\varphi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi} \\ f_r & rf_\theta & r\sin\theta\, f_\varphi \end{vmatrix}$$

$$\nabla \cdot \bar f = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 f_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta f_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi} f_\varphi$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}$$



$$= \frac{1}{\mu r^{\cancel{2}} \cancel{\sin\theta}} \cancel{r} \cancel{\sin\theta}\, \bar u_\varphi \left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial\theta} A_r\right)$$

$$-\mu I L \frac{e^{-jkr}}{4\pi}\sin\theta$$

$$+jkIL\mu \frac{e^{-jkr}}{4\pi}\sin\theta$$

$$\mu I L \frac{e^{-jkr}}{4\pi r}\cos\theta$$

$$\bar A = \mu I L \frac{e^{-jkr}}{4\pi r}\left(\bar u_r \cos\theta - \bar u_\theta \sin\theta\right)$$

$$\frac{\partial}{\partial r} e^{-jkr} = -jk\, e^{-jkr}$$

$$\frac{\partial}{\partial r}\frac{1}{jkr} = -\frac{1}{jkr^2}$$

$$\mu I L \frac{\ }{4\pi r}\cos\theta$$

$$\frac{\partial}{\partial r}\frac{1}{jkr} = -\frac{1}{jkr^2}$$

$$\frac{\partial}{\partial \theta}\sin\theta = \cos\theta$$

$$\frac{\partial}{\partial \theta}\cos\theta = -\sin\theta$$

$$\bar{H}(\bar{r}) = \frac{1}{\cancel{\mu} r}\cancel{\mu} I L \frac{1}{4\pi}\,\bar{u}_\varphi\left(+jk\sin\theta\, e^{-jkr} + \sin\theta\,\frac{e^{-jkr}}{jkr}jk\right)$$

$$= \bar{u}_\varphi\, jkIL\,\frac{e^{-jkr}}{4\pi r}\sin\theta\left(1+\frac{1}{jkr}\right)$$

SÄHKÖKENTTÄ

$$\nabla\times\bar{H} = j\omega\varepsilon\bar{E} \quad\Rightarrow\quad \bar{E}(\bar{r}) = \frac{\nabla\times\bar{H}}{j\omega\varepsilon}$$

$$\bar{H} = jkIL\frac{e^{-jkr}}{4\pi r}\sin\theta\,\bar{u}_\varphi\left(1+\frac{1}{jkr}\right)$$

$$\nabla\times\bar{H} = j\omega\varepsilon\bar{E}$$

Pallokoordinaatisto

$$\nabla f(\bar{r}) = \bar{u}_r\frac{\partial}{\partial r}f + \bar{u}_\theta\frac{1}{r}\frac{\partial}{\partial\theta}f + \bar{u}_\varphi\frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}f$$

$$\nabla\times\bar{f} = \frac{1}{r^2\sin\theta}\begin{vmatrix}\bar{u}_r & r\bar{u}_\theta & r\sin\theta\,\bar{u}_\varphi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\varphi}\\ f_r & rf_\theta & r\sin\theta f_\varphi\end{vmatrix}$$

$$\bar{E}(\bar{r}) = \frac{\nabla\times\bar{H}}{j\omega\varepsilon} = \frac{1}{j\omega\varepsilon r^2\sin\theta}\begin{vmatrix}\bar{u}_r & r\bar{u}_\theta & r\sin\theta\,\bar{u}_\varphi\\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\varphi\\ 0 & 0 & r\sin\theta H_\varphi\end{vmatrix} = \frac{1}{j\omega\varepsilon r^2\sin\theta}\left(\bar{u}_r\frac{\partial}{\partial\theta}(r\sin\theta\, H_\varphi) - r\bar{u}_\theta\frac{\partial}{\partial r}(r\sin\theta\, H_\varphi)\right)$$

$$= \bar{u}_r\boxed{E_r} + \bar{u}_\theta\boxed{E_\theta}$$

$$E_\theta = \frac{1}{j\omega\varepsilon r^{\cancel{2}}\cancel{\sin\theta}}\left[-\cancel{r}\frac{\partial}{\partial r}\left(\cancel{r}\cancel{\sin\theta}\, jkIL\frac{e^{-jkr}}{4\pi \cancel{r}}\sin\theta\left(1+\frac{1}{jkr}\right)\right)\right]$$

$$\frac{\partial}{\partial r}\left(e^{-jkr}\left(1+\frac{1}{jkr}\right)\right) = -jk\,e^{-jkr}\left(1+\frac{1}{jkr}\right) - \frac{1\cdot jk}{(jk)^2r^2}e^{-jkr}$$

$$= -jk\,e^{-jkr}\left(1+\frac{1}{jkr}+\frac{1}{(jkr)^2}\right)$$

$$E_\theta = \frac{(-jk)^2 IL}{j\omega\varepsilon r}\,\frac{e^{-jkr}}{4\pi}\sin\theta\left(1+\frac{1}{jkr}+\frac{1}{(jkr)^2}\right)$$

$-\omega^2\mu\varepsilon$

$$= j\omega\mu IL\frac{e^{-jkr}}{4\pi r}\sin\theta\left(1+\frac{1}{jkr}+\frac{1}{(jkr)^2}\right)$$

$$\frac{\partial}{\partial\theta}\left(\cancel{r}\sin\theta\, jkIL\frac{e^{-jkr}}{4\pi\cancel{r}}\sin\theta\left(1+\frac{1}{jkr}\right)\right) = jkIL\frac{e^{-jkr}}{4\pi}2\sin\theta\cos\theta\left(1+\frac{1}{jkr}\right)$$

$$\Rightarrow E_r = \frac{jkIL\;jk}{j\omega\varepsilon r^2\cancel{\sin\theta}\;jk}\frac{e^{-jkr}}{4\pi}2\cancel{\sin\theta}\cos\theta\left(1+\frac{1}{jkr}\right)$$

$$(k^2 = \omega^2\mu\varepsilon)$$

$$E_r = j\omega\mu IL\frac{e^{-jkr}}{4\pi r}2\cos\theta\left(\frac{1}{jkr}+\frac{1}{(jkr)^2}\right)$$

Siispä Hertzin dipolin täydelliset kentät:

$$\bar{E}(\bar{r}) = j\omega\mu IL\frac{e^{-jkr}}{4\pi r}\left[2\cos\theta\,\bar{u}_r\left(\frac{1}{jkr}+\frac{1}{(jkr)^2}\right) + \bar{u}_\theta\sin\theta\left(1+\frac{1}{jkr}+\frac{1}{(jkr)^2}\right)\right]$$

$$\bar{H}(\bar{r}) = jkIL\frac{e^{-jkr}}{4\pi r}\sin\theta\,\bar{u}_\varphi\left(1+\frac{1}{jkr}\right)$$

Ajasta riippuvat Hertzin dipolin kentät saa näistä vanhalla kaavalla: $\bar{E}(\bar{r},t) = Re\left\{\bar{E}(\bar{r})\, e^{j\omega t}\right\}$

Münchenin yliopiston fysiikan laitoksen animaatiosivusto:

https://www.en.didaktik.physik.uni-muenchen.de/multimedia/dipolstrahlung/index.html

HERTZIN DIPOLIN LÄHIKENTÄT

$$kr \ll 1 \quad \Rightarrow \quad e^{-jkr} \to 1$$

$$\bar{H}(\bar{r}) = \cancel{j}\cancel{k}\, IL \frac{1}{4\pi r} \sin\theta\ \bar{u}_\varphi \frac{1}{\cancel{jk}r} = \bar{u}_\varphi \frac{IL \sin\theta}{4\pi r^2}$$

SAMAT

BIOT-SAVART

$I d\bar{l} \uparrow \quad r \quad x \to \bar{u}_r$

$$\bar{H}(\bar{r}) = \frac{I\, d\bar{l} \times \bar{u}_r}{4\pi r^2}$$

$$\bar{E}(\bar{r}) = j\omega\mu IL \frac{1}{4\pi r}\left(2\cos\theta\ \bar{u}_r + \sin\theta\ \bar{u}_\theta\right)\frac{1}{(jkr)^2}$$

$$= \frac{-j\omega\mu IL}{\underbrace{k^2}_{\omega^2\mu\varepsilon}} \cdot \frac{1}{4\pi r^3}\left(\bar{u}_r\, 2\cos\theta + \bar{u}_\theta \sin\theta\right)$$

$$= \underbrace{\frac{-jIL}{\omega}}_{p} \cdot \frac{1}{4\pi\varepsilon r^3}\left(\bar{u}_r\, 2\cos\theta + \bar{u}_\theta \sin\theta\right)$$

$$-jIL = \omega p$$

$$IL = \underbrace{j\omega}_{\partial/\partial t} p$$

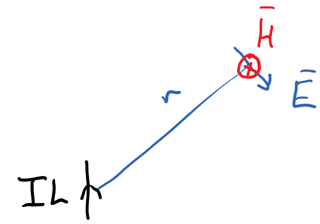
STAATTINEN DIPOLI $\quad \bar{p} = \bar{u}_z p$

$$\phi = \frac{p\cos\theta}{4\pi\varepsilon r^2}$$

$$\Rightarrow \bar{E} = \frac{p}{4\pi\varepsilon r^3}\left(2\cos\theta\ \bar{u}_r + \sin\theta\ \bar{u}_\theta\right)$$

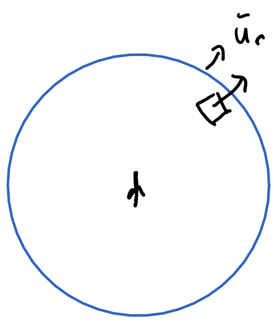
KAUKOKENTÄT $|jkR| >> 1$

$$\begin{cases} \bar E(\bar r) = j\omega\mu IL\,\frac{e^{-jkr}}{4\pi r}\sin\theta\;\bar u_\theta \\ \bar H(\bar r) = jkIL\,\frac{e^{-jkr}}{4\pi r}\sin\theta\;\bar u_\varphi \end{cases}$$



$$\frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\varepsilon}} = \eta \qquad \bar u_r \times \bar E = \eta \bar H$$

Hertzin dipolin säteilyteho P_s



$$\bar S = \frac12 \bar E\times\bar H^* = \frac12 \overbrace{\bar E\times\left(\frac{\bar u_r\times\bar E}{\eta}\right)^*}^{\bar u_r(\bar E\cdot\bar E^*) - \underbrace{(\bar u_r\cdot\bar E)}_{0}\bar E^*}$$

$$= \frac{|\bar E|^2}{2\eta}\,\bar u_r$$

$$P_s = \oiint \bar S\cdot d\bar a$$

$$P_s = \frac{1}{2\eta}\iint \frac{(\omega\mu IL)^2}{(4\pi \cancel{r})^2}\sin^2\theta\,\overbrace{\bar u_r\cdot\bar u_r}^{1}\,\cancel{r^2}\sin\theta\,d\theta\,d\varphi$$

$$= \frac{\omega^2\mu^2(IL)^2}{2\eta(4\pi)^2}\underbrace{\int_0^{2\pi}d\varphi}_{2\pi}\int_0^{\pi}\sin^3\theta\,d\theta$$

$$\frac{4}{3}$$

$$\sin\theta\cdot\sin^2\theta = \sin\theta(1-\cos^2\theta)$$

$$\begin{cases} \int_0^\pi \sin\theta\,d\theta = \Big/_0^\pi -\cos\theta = 2 \\ \int_0^\pi \sin\theta\cos^2\theta\,d\theta = \Big/^\pi -\frac{\cos^3\theta}{3} = \frac{2}{3} \end{cases}$$

$$\overbrace{\omega^2\mu^2}^{(k\eta)^2}(IL)^2$$

$$= \frac{\omega^2 \mu^2 (IL)^2}{\cancel{2} \eta \, 4\pi \, \cancel{4\pi}} \cancel{2\pi} \frac{\cancel{4}}{3}$$

$$\left(\int_0^{\pi} \sin\theta \cos^2\theta \, d\theta = \Big/_0 - \frac{\cos^3\theta}{3} = \frac{2}{3} \right.$$

$$P_s = \frac{\eta}{12\pi} (kIL)^2$$

Säteilyresistanssi R_s

$$P_s = \frac{1}{2} R_s I^2$$

$$\Rightarrow \quad R_s = \frac{2P_s}{I^2} = \frac{\eta}{6\pi} (kL)^2$$