Exercise No 08 with solution.

Question 1:

A square wave inverter has a DC source of 100 V, and the output frequency is 50 Hz. The RL series load of 10 ohm and 30 mH is connected to the output. Find,

a) Expression of the load current.
b) RMS load current.
c) Average source current.

a) Expression of the Load Current:

As we know that the expression for load current is given as,

$$i_o(t) = \frac{V_{in}}{R} + \left(I_{min} - \frac{V_{in}}{R}\right)e^{-\frac{t}{\tau}} \qquad for\ 0 \le t \le \frac{T}{2}$$

$$i_o(t) = \frac{-V_{in}}{R} + \left(I_{max} + \frac{V_{in}}{R}\right)e^{\frac{-(t-T/2)}{\tau}} \qquad for \frac{T}{2} \le t \le T$$

Where,

$$T = \frac{1}{f} = \frac{1}{50} = 0.02\ sec$$

$$\tau = \frac{L}{R} = \frac{30x10^{-3}}{10} = 0.003$$

$$\frac{T}{2\tau} = \frac{0.02}{2x0.003} = 3.33$$

$$I_{max} = -I_{min} = \frac{V_{in}}{R}\left(\frac{1-e^{-\frac{T}{2\tau}}}{1+e^{-\frac{T}{2\tau}}}\right) = \frac{100}{10}\left(\frac{1-e^{-3.33}}{1+e^{-3.33}}\right) = 9.30A$$

Now,

$$i_o(t) = \frac{V_{in}}{R} + \left(I_{min} - \frac{V_{in}}{R}\right)e^{-\frac{t}{\tau}}$$

$$i_o(t) = \frac{100}{10} + \left(-9.30 - \frac{100}{10}\right)e^{-\frac{t}{0.003}}$$

$$i_o(t) = 10 - 19.3e^{-\frac{t}{0.003}} \qquad for\ 0 \le t \le \frac{1}{100}$$

and,

$$i_o(t) = \frac{-V_{in}}{R} + \left(I_{max} + \frac{V_{in}}{R}\right)e^{\frac{-(t-T/2)}{\tau}}$$

$$i_o(t) = \frac{-100}{10} + \left(9.30 + \frac{100}{10}\right)e^{\frac{-(t-0.02/2)}{0.003}}$$

$$i_o(t) = -10 + 19.3\,e^{\frac{-(t-0.01)}{0.003}} \qquad for\ \frac{1}{100} \le t \le \frac{1}{50}$$

$$I_{max} = -I_{min} = \frac{V_{in}}{R}\left(\frac{1-e^{-\frac{T}{2\tau}}}{1+e^{-\frac{T}{2\tau}}}\right) = \frac{100}{10}\left(\frac{1-e^{-3.33}}{1+e^{-3.33}}\right) = 9.30A$$

Now,

$$i_o(t) = \frac{V_{in}}{R} + \left(I_{min} - \frac{V_{in}}{R}\right)e^{-\frac{t}{\tau}}$$

$$i_o(t) = \frac{100}{10} + \left(-9.30 - \frac{100}{10}\right)e^{-\frac{t}{0.003}}$$

$$i_o(t) = 10 - 19.3e^{-\frac{t}{0.003}} \qquad for\ 0 \le t \le \frac{1}{100}$$

and,

$$i_o(t) = \frac{-V_{in}}{R} + \left(I_{max} + \frac{V_{in}}{R}\right)e^{\frac{-(t-T/2)}{\tau}}$$

$$i_o(t) = \frac{-100}{10} + \left(9.30 + \frac{100}{10}\right)e^{\frac{-(t-0.02/2)}{0.003}}$$

$$i_o(t) = -10 + 19.3\,e^{\frac{-(t-0.01)}{0.003}} \qquad for\ \frac{1}{100} \le t \le \frac{1}{50}$$

b) RMS Load Current:

For RMS value, we have the formula,

$$I_{RMS} = \sqrt{\frac{1}{T}(\int_0^T [i_o(t)]^2 dt}$$

$$I_{RMS} = \sqrt{\frac{2}{T}(\int_0^{T/2} \left[\frac{V_{in}}{R} + \left(I_{min} - \frac{V_{in}}{R}\right) e^{-\frac{t}{\tau}}\right]^2 dt}$$

$$I_{RMS} = \sqrt{\frac{2}{0.02}(\int_0^{0.01} \left[10 - 19.3e^{-\frac{t}{0.003}}\right]^2 dt}$$

$$I_{RMS} = \sqrt{100x0.44133}$$

$$I_{RMS} = 6.643\ A$$

c) Average Source Current:

As we know that,

$$P = VI$$

$$I_{in} = \frac{P_{in}}{V_{in}}$$

Where,

$$P_{in} = I_{rms}^2\ x\ R$$

$$P_{in} = (6.643)^2\ x\ 10$$

$$P_{in} = 441.33\ W$$

and,

$$I_{in} = \frac{441.33}{100} = 4.41\ A$$

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Question 2:
A square-wave inverter has a dc input of 150 V and supplies a series RL load with R=20Ω and L=40 mH. The output frequency is 60 Hz.

a) Determine an expression for steady state load current.
b) Determine the peak current in each switch component.
c) What is the maximum voltage across each switch?
Assume ideal components

Solution:

$$T = \frac{1}{f} = \frac{1}{60} = 0.0167\ s$$

$$\tau = \frac{L}{R} = \frac{0.040}{20} = 0.002$$

$$\frac{T}{2\tau} = \frac{0.0167}{2 * 0.002} = 4.175$$

Part (a):

$$I_{max} = -I_{min} = \frac{V_{in}}{R}\left(\frac{1 - e^{-T/2\tau}}{1 + e^{-T/2\tau}}\right) = \frac{150}{20}\left(\frac{1 - e^{-4.175}}{1 + e^{-4.175}}\right) = 7.27\ A$$

$$i_o(t) = \frac{V_{in}}{R} + \left(I_{min} - \frac{V_{in}}{R}\right)e^{-t/\tau} = 7.5 + (-7.27 - 7.5)e^{-t/0.002}$$

$$= 7.5 - 14.77e^{-t/0.002} \qquad 0 \le t \le \frac{1}{120}$$

$$i_o(t) = \frac{-V_{in}}{R} + \left(I_{max} + \frac{V_{in}}{R}\right)e^{-(t-\frac{T}{2})/\tau} = -7.5 + (7.27 + 7.5)e^{-(t-0.0084)/0.002}$$

$$= -7.5 + 14.77e^{-(t-0.0084)/0.002} \qquad \frac{1}{120} \le t \le \frac{1}{60}$$

Part (c): the peak current in each switch component is equal to the peak output current,

$$I_{max} = \frac{V_{in}}{R}\left(\frac{1 - e^{-T/2\tau}}{1 + e^{-T/2\tau}}\right) = \frac{150}{20}\left(\frac{1 - e^{-4.175}}{1 + e^{-4.175}}\right) = 7.27\ A$$

Part (d):

The maximum voltage across each switch is $V_{in} = 150\ V$