PHYS-E055102, Low Temperature Physics, Exercise 11 DL 30.11.2022

1. Consider a non-interacting one-sided cavity with Hamiltonian
$$\hat{H}_{\text{sys}} = \hbar\omega_c \hat{a}^\dagger \hat{a}. \tag{1}$$

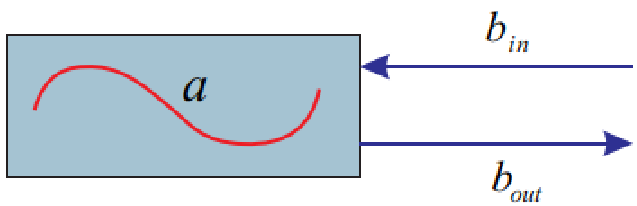

Figure 1: Schematic of the non-interacting one-sided cavity and the relevant fields.

   (a) Use the input/output theory to calculate input and output fields $\hat{b}_{\text{in}}(\omega)$ and $\hat{b}_{\text{out}}(\omega)$.
   Hint: Use Fourier transform to transform the equation of motion ($\omega \leftrightarrow t$).

   (b) Consider the limits of $\omega = \omega_c$ and ω very far from ω_c for the input and output field.

2. Derive the noise for a phase preserving linear amplifier in the limit of $G \gg 1$
$$(\Delta\hat{b})^2 = G\left((\Delta\hat{a})^2 + \frac{1}{2}\right). \tag{2}$$
Start by showing that the amplified mode $\hat{b} = \sqrt{G}\hat{a} + \hat{F}$ has an idler field $\hat{F} = \sqrt{G-1}\hat{c}^\dagger$ (for a single idler) to satisfy the commutation relation $[\hat{b}, \hat{b}^\dagger] = 1$.

3. You design a microwave experiment ($50\,\Omega$ system) in which you wish to have high bandwidth and your sensitivity tolerates $10\,\text{dB}$ loss due to an in-band reflection Γ. How large bandwidth you may get using Bode-Fano criterion compared with the perfect matching bandwidth (source made of R and C). How much does the band depend whether the reflection arises with R above or below $50\,\Omega$?

4. A simple example of a non-phase preserving degenerate parametric amplifier is a cavity where the different modes couple to each other. Here the pump

photons are split into two signal photons $\omega_s = \omega_p/2$. Such a system has the Hamiltonian

$$\hat{H}_{\text{sys}} = \hbar \left(\omega_p \hat{a}_p^\dagger \hat{a}_p + \omega_s \hat{a}_s^\dagger \hat{a}_s \right) + i\hbar\eta \left(\hat{a}_s^\dagger \hat{a}_s^\dagger \hat{a}_p - \hat{a}_s \hat{a}_s \hat{a}_p^\dagger \right), \tag{3}$$

where η is the real, positive non-linear susceptibility. Calculate the quadratures of the signal modes and explain their behaviour in dependence of gain.