

## Exercise 11 – Solutions

29.11.2022

## #1

a) The local priority vectors are the normalized eigenvectors of the pairwise comparison matrices corresponding to the largest eigenvalues:

- Price: (0.57, 0.14, 0.29)
- Features: (0.11, 0.70, 0.19)
- Memory: (0.09, 0.74, 0.18)
- Criteria: (0.65, 0.23, 0.12)

b) Consistency indices are computed through  $CI = \frac{\lambda_{max}-3}{2}$ .

- Price:  $\lambda_{max} = 3 \Rightarrow CI = 0$
- Features:  $\lambda_{max} = 3.0092 \Rightarrow CI = 0.0046$
- Memory:  $\lambda_{max} = 3.0015 \Rightarrow CI = 0.00077$
- Criteria:  $\lambda_{max} = 3.0037 \Rightarrow CI = 0.0018$

The consistency of the matrices can be checked by comparing CI to RI=0.58:

- Price:  $CR = 0$
- Features:  $CR = 0.0079$
- Memory:  $CR = 0.0013$
- Criteria:  $CR = 0.0032$ .

All consistency ratios CR are below 0.10, whereby the pairwise comparisons matrices are sufficiently consistent. The only truly consistent matrix, however, is the one corresponding to the price criterion.

c)

	0.65	0.23	0.12	
	Price	Features	Memory	w
A	0.57	0.11	0.09	<b>0.41</b>
B	0.14	0.70	0.74	0.34
C	0.29	0.19	0.17	0.25

Phone A has the highest priority, and hence it should be selected.

d) When a replica of A is added to the group of alternatives as the fourth option, the local priority vectors for Price, Feature, and Memory become:

- Price: (0.36, 0.09, 0.18, 0.36)
- Features: (0.10, 0.63, 0.18, 0.10)
- Memory: (0.08, 0.68, 0.16, 0.08)

The consistency ratios are calculated through  $CI = \frac{\lambda_{max}-4}{3}$  for Price, Features, and Memory:

- Price:  $\lambda_{max} = 4 \Rightarrow CI = 0$
- Features:  $\lambda_{max} = 4.0104 \Rightarrow CI = 0.0035$

- Memory:  $\lambda_{max} = 4.0017 \Rightarrow CI = 0.00058$

The consistency of the matrices can be checked by comparing CI to RI=0.90 for Price, Features, and Memory:

- Price:  $CR = 0$
- Features:  $CR = 0.0038$
- Memory:  $CR = 0.00064$

Again, all consistency ratios CR are below 0.10, whereby the pairwise comparisons matrices are sufficiently consistent. The only truly consistent matrix, however, is the one corresponding to the price criterion.

	0.65	0.23	0.12	
	Price	Features	Memory	w
A	0.36	0.10	0.08	0.27
B	0.09	0.63	0.68	<b>0.29</b>
C	0.18	0.18	0.16	0.18
A replica	0.36	0.10	0.08	0.27

Now, phone B has the highest priority, and hence it should be selected. Thus the introduction of a replica of A to the group of alternatives caused a rank reversal to occur.