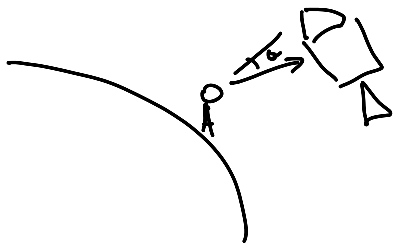


$$r = \sqrt{(x-x_r)^2 + (y-y_r)^2 + (z-z_r)^2}$$

$$\theta = \text{non-linear}(x, y, z)$$

$$\hat{x}_{n|n-1} = E[x_n \mid y_{1:n-1}]$$

lin: $[x_n = F x_{n-1} + q_n]$

$$= E[F x_{n-1} + q_n \mid y_{1:n-1}]$$

$$= F \underbrace{E[x_{n-1} \mid y_{1:n-1}]}_{\hat{x}_{n-1|n-1}}$$

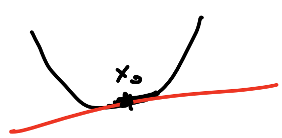
non: $x_{n|n-1} = E[x_n \mid y_{1:n-1}]$

$$= E[f(x_{n-1}) + q_n \mid y_{1:n-1}]$$

$$= E[f(x_{n-1}) \mid y_{1:n-1}]$$

$$\neq f(E[x_{n-1} \mid y_{1:n-1}])$$

taylor:



$$f(x) = f(x_0) + f'(x_0)(x - x_0)$$
$$+ \frac{1}{2!} f''(x_0)(x-x_0)^2 + \frac{1}{3!} f^{(3)}(x_0)(x-x_0)^3 + \dots$$
$$\approx f(x_0) + f'(x_0)(x - x_0)$$

$$
\begin{aligned}
\hat{x}_{n|n-1} &= E\{x_n \mid y_{1:n-1}\} \\
&= E\{ f(x_{n-1}) + q_n \mid y_{1:n-1}\} \\
&\approx E\{ f(\hat{x}_{n-1|n-1}) + F_x(\hat{x}_{n-1|n-1})(x_{n-1} - \hat{x}_{n-1|n-1}) \cancel{+q_n} \mid y_{1:n-1}\} \\
&= f(\hat{x}_{n-1|n-1}) - F_x(\hat{x}_{n-1|n-1})\hat{x}_{n-1|n-1} \\
&\quad + F_x(\hat{x}_{n-1|n-1}) \underbrace{E\{x_{n-1} \mid y_{1:n-1}\}}_{\hat{x}_{n-1|n-1}} \\
&= f(\hat{x}_{n-1|n-1})
\end{aligned}
$$

$$
\begin{aligned}
P_{n|n-1} &= E\{ (x_n - \hat{x}_{n|n-1})(\dots)^T \mid y_{1:n-1}\} \\
&= E\{ (f(x_{n-1}) + q_n - \hat{x}_{n|n-1})(\dots)^T \mid y_{1:n-1}\} \\
&\approx E\{ (\cancel{f(\hat{x}_{n-1|n-1})} + F_x(\hat{x}_{n-1|n-1})(x_{n-1} - \hat{x}_{n-1|n-1}) \\
&\qquad + q_n - \cancel{f(\hat{x}_{n-1|n-1})})(\dots)^T \mid y_{1:n-1}\} \\
&= E\{ (F_x \cdot (x_{n-1} - \hat{x}_{n-1|n-1}) + q_n)(F_x \cdot (x_{n-1} - \hat{x}_{n-1|n-1}) \\
&\qquad + q_n)^T \mid y_{1:n-1}\} \\
&= E\{ F_x \cdot (x_{n-1} - \hat{x}_{n-1|n-1})(x_{n-1} - \hat{x}_{n-1|n-1})^T \cdot F_x^T \\
&\qquad \cancel{+ F_x (x_{n-1} - \hat{x}_{n-1|n-1}) q_n^T} \\
&\qquad \cancel{+ q_n (x_{n-1} - \hat{x}_{n-1})^T F_x^T} + q_n q_n^T \mid y_{1:n-1}\} \\
&= F_x \cdot E\{ (x_{n-1} - \hat{x}_{n-1|n-1})(x_{n-1} - \hat{x}_{n-1})^T \} F_x^T \\
&\quad + E\{ q_n q_n^T \mid y_{1:n-1}\} \\
&= F_x \cdot P_{n-1|n-1} \cdot F_x^T + Q_n
\end{aligned}
$$

$$g = g(\hat{x}_{n|n-1}), \quad G = G_x(\hat{x}_{n|n-1})$$

$$J(x) = (y - g - G(x - \hat{x}))^T R^{-1} (y - g - G(x - \hat{x})) + (x - \hat{x})^T P^{-1} (x - \hat{x})$$

$$\frac{\partial J}{\partial x} = -2 G^T R^{-1} (y - g - G(x - \hat{x})) + 2 P^{-1} (x - \hat{x}) = 0$$

$$- G^T R^{-1} y + G^T R^{-1} g + G^T R^{-1} G x - G^T R^{-1} G \hat{x} + P^{-1} x - P^{-1} \hat{x} = 0$$

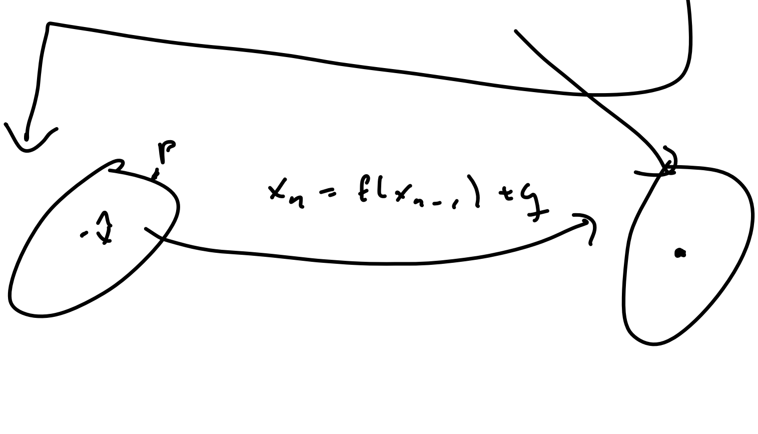
$$x = \underbrace{\left[G^T R^{-1} G + P^{-1} \right]^{-1}} \cdot \left[G^T R^{-1} y - G^T R^{-1} g + G^T R^{-1} G \hat{x} + P^{-1} \hat{x} \right]$$

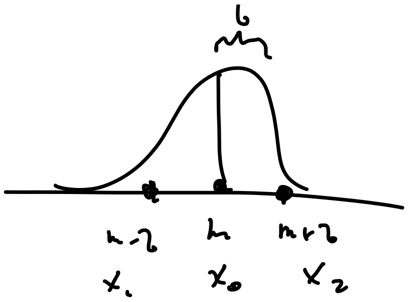
$$= \text{Woodbury} = \dots$$

$\hat{x}_{n-1|n-1}, \; P_{n-1|n-1}$

$E[x_n \mid y_{1:n-1}]$

$Cov[x_n \mid y_{1:n-1}]$

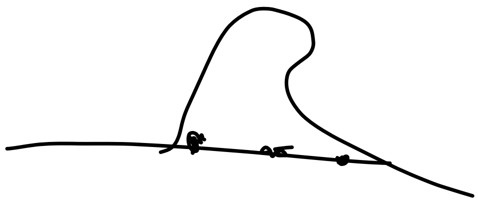




$$m = (x_0 + x_1 + x_2)/3$$

$$\sigma^2 = \left(\frac{x_2 - x_0}{2}\right)^2 + \left(\frac{x_1 - x_0}{2}\right)^2$$

$$z = h(x) \quad , \quad z_0 = h(x_0) \quad z_1 = h(x_1) \quad z_2 = h(x_2)$$



$$E[z] = \int h(x)\, p(x)\, dx$$

$$\approx \sum w^i h(x^i)$$

$$P = C C^T$$

$$P = \sqrt{P}\,\sqrt{P}^T$$