ELEC-E8116 Model-based control systems
/exercises 11 (the last exercise)

Problem 1. Consider a SISO system in a two-degrees-of-freedom control configuration. Let the loop transfer function be $L(j\omega) = G(j\omega)F_y(j\omega)$, where the symbols are standard used in the course.

a. Define the *sensitivity* and *complementary sensitivity functions* and determine where in the complex plane it holds

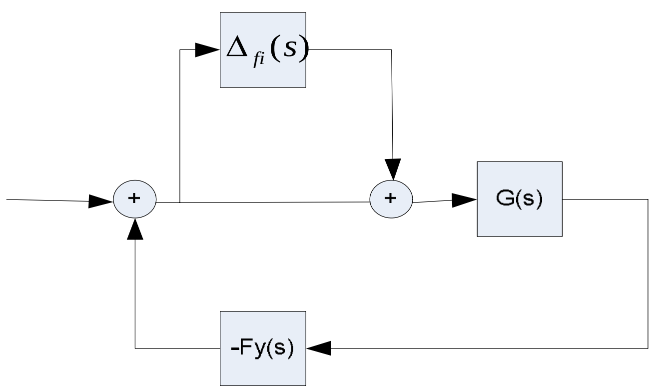
$$|S(j\omega)| < 1, \quad |S(j\omega| = 1, \quad |T(j\omega)| < 1 \text{ and } |T(j\omega)| = 1$$

b. Let the Nyquist diagram of the loop transfer function approach from below the point where $|S(j\omega_n| = 1$ and assume that it also holds then $|T(j\omega_n| = 1$. Assuming that there are no right hald poles of the open loop transfer function, what is the phase margin of the closed-loop system? Hint. In the complex plane (xy) let $L(j\omega) = x(\omega) + jy(\omega)$.

Problem 2. You are given the nominal plant

$$G(s) = \frac{10}{s^2 + 4}$$

with an input feedback uncertainty $\|\Delta_{fi}(s)\|_\infty \le 0.5$, and the controller $F_y(s) = \dfrac{4(s+2)}{s+8}$ (see Fig.) What can be said about robust stability of the closed-loop system?



Problem 3. Consider a SISO system and a state feedback control

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$u(t) = -Lx(t)$$

where L is chosen as a solution to the infinite time optimal (LQ) horizon problem.

a. Prove that the loop gain is $H(s) = L(sI - A)^{-1} B$

b. Prove that $|1 + H(i\omega)| \geq 1$

c. Show that for the LQ controller

- phase margin is at least 60 degrees
- gain margin is infinite
- the magnitude of the sensitivity function is less than 1
- the magnitude of the complementary sensitivity function is less than 2.

Problem 4. MPC control. Check the Matlab code in the file *mpcgain2.mat* to verify that it is correct. Ref: Wang's book. The code calculates in general MIMO case the augmented model and the related matrices, so that the MPC problem can easily be solved by writing a suitable software, which calls mpcgain2.

The code can be used in the last homework, if one wishes. No separate solution to problem 4 is provided.