ELEC-E8116 Model-based control systems

Intermediate exam 1. 27. 10. 2021

- *Write the name of the course, your name and student number to each answer sheet.*
- *There are three (3) problems and each one must be answered.*
- *Read the instructions in a separate file (Instructions), which is available in the Exam Assignment and which has also been published in advance.*
- *In problem 0 sign with your name (typesetting is enough if you use computer document) in which you assure that you follow the exam regulations.*

Each problem 1-3 gives the maximum of 5 points.

Note: Solutions obtained by computer are not presented and not accepted. Computer can be used in verification, but that is only for yourself to check the results, if needed.

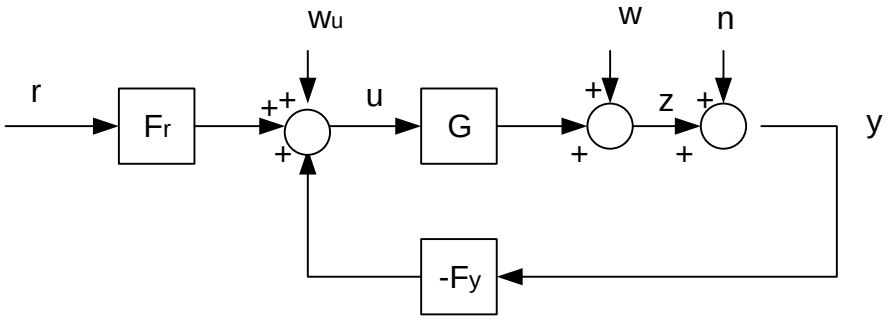
0. Write your signature.

1. Explain briefly the following concepts (max 1p. each)

- Singular value decomposition and singular values
- Input and output directions of a multivariable system
- Minimal realization of a multivariable transfer function
- Internal stability
- Robust stability

Note for problems 2 and 3: When writing equations with matrices remember that a square matrix can have inverse A^{-1} (if exists), but you cannot divide by a matrix ($1/A$ is illegal operation for a matrix). Also note that for matrices $AB \neq BA$ except for some rare special cases.

2. Consider a **multivariable** control configuration in the below figure, where signal y is m-dimensional and signal u n-dimensional (m and n are positive integers).



a. What are the dimensions of signals r, w_u, w, n and matrices G, F_y, F_r? (1 p.)

b. Give the condition by which the 2 DOF (two degrees-of-freedom) control configuration in the figure changes into a 1 DOF configuration. Draw a figure. (2 p.)

c. From the 1 DOF configuration identify the *loop transfer function*, *closed loop transfer function*, *sensitivity function* and *complementary sensitivity function*. Then answer: If the sensitivity function is known, can you calculate the loop transfer function? If the answer is yes, show the resulting formula for L. (2 p.)

3. **a**. Let G and F_y be matrices of dimensions $m \times n$ and $n \times m$ respectively (m and n are positive integers). Calculate and try to get as simple result as possible to

$$(I + GF_y)^{-1}GF_y - GF_y(I + GF_y)^{-1} = ?$$

where the inverse matrices are assumed to exist and I:s are identity matrices of appropriate dimensions. You may use a well-known matrix identity without proving it. (2 p.)

b. Explain in your own words the concept *bandwidth* from control viewpoint. Then explain what *loop shaping* in control means. (Relate the two concepts to each other in your answer). (3 p.)