ELEC-E8116 Model-based control systems
Intermediate exam 2. 15. 12. 2021 / Solutions

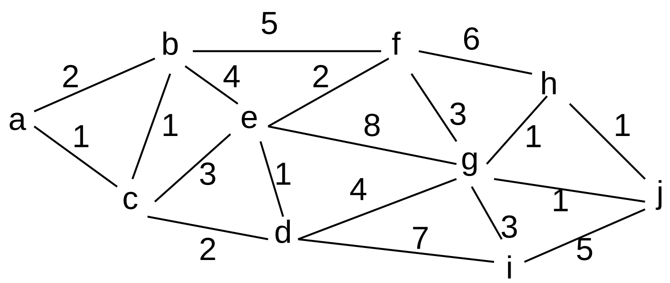
- *Write the name of the course, your name and student number to each answer sheet.*
- *There are three (3) problems and each one must be answered.*
- *Read the instructions in a separate file (Instructions), which is available in the Exam Assignment and which has also been published in advance.*
- *In problem 0 sign with your name (typesetting is enough if you use computer document) in which you assure that you follow the exam regulations.*

Each problem 1-3 gives the maximum of 5 points.

Note 1: Solutions obtained by computer are not presented and not accepted. Computer can be used in verification, but that is only for yourself to check the results, if needed.
Note 2: In many questions below you have to explain something (e.g. concepts). It is then not enough that you take some formulas, expressions or text from the course open book material; instead, you have to answer the questions in your own words such that the reviewer can see that you have really understood the issue. The answers do not have to be long, but they must be clear to the reader.

0. Write your signature confirming that you follow the exam rules.

1. **a**. Explain the concept "Principle of Optimality" (2p)
b.In the below figure the cost of moving from one node to another is given by the numbers; movement is allowed only from left to right.



Use *dynamic programming* to solve the following problem: Find the minimum cost path from node *a* to the desired final state *j*. (Copy the diagram on your paper and show clearly how you have solved the problem using dynamic programming. If you cannot present the figure in your answer sheet, then present anyway exactly each phase of your solution). (3p)

Solution

a. On the optimal route on any state, irrespective of how we have come there, we have to choose the optimum solution from there to the end state.

b. Use dynamic programming, start from the end state and go backwards in time to the beginning. Denote min_x, the minimum cost from state x to the end, and u_x=u_xy the first control from state x (which goes to state y).

min_h = 1, u_h=u_hj
min_i=5, u_i=u_ij
min_g=min(1+min_h, 1,3+min_i)=min(2,1,8)=1, u_g=u_gj
min_f=min(6+min_h,3+min_g)=min(7,4)=4, u_f=u_fg
min_d=min(4+min_g,7+min_i)=min(5,12)=5, u_d=u_dg
min_e=min(2+min_f,8+min_g,1+min_d)=min(6,9,6)=6, u_e=u_ef=u_ed
min_b=min(5+min_f,4+min_e)=min(9,10)=9, u_b=u_bf
min_c=min(1+min_b,3+min_e,2+min_d)=min(10,9,7)=7, u_c=u_cd
min_a=min(2+min_b,1+min_c)=min(11,8)=8, u_a=u_ac

Optimal cost from a to j is 8 and the optimal route is a-c-d-g-j.

Note that if the node e had been on the optimal route, there would have been two routes of equal costs from there on. But e was not on the optimal route.

2. **a**. Explain shortly the meaning of the concept *fundamental limitations in control*? (1p)
b. List and explain in your own words the main fundamental limitations in control. You can restrict the answer to SISO systems. (2p)
c. Show the validity of the following approximative inequalities and explain what they have to do with fundamental limitations in control

$$|S| < \varepsilon \Rightarrow |GF_y| > \frac{1}{\varepsilon}, \quad |T| < \varepsilon \Rightarrow |GF_y| < \varepsilon \qquad (2p)$$

Solution:

a. Fundamental limitations in control are characteristics which limit the control performance somehow and which can not be circumvented by any means or controllers.

b. For example: $S+T=I$, the waterbed formula giving limitations to S, the Bode integral giving limitations to the decrease rate of L, the limitations to bandwidth caused by process delay and/or RHP zeros, the minimum bandwidth required by RHP poles, the limitations in control caused by the minimum and maximum of the control variable. See Chapter 4 in lecture slides for details.

c. Epsilon is a small positive number. $L=GF_y$ is the loop transfer function. The first inequality shows that for those frequencies where S is small, the loop transfer function has to be high. The second inequality shows that when the complementary transfer function is small, the loop transfer function has to be small also.

$$|S| = \left|\frac{1}{1+L}\right| = \frac{1}{|1+L|} < \varepsilon$$

$$\Rightarrow |1+L| > \frac{1}{\varepsilon}$$

$$\Rightarrow \frac{1}{\varepsilon} < |1+L| < 1 + |L|$$

$$\Rightarrow |L| > \frac{1}{\varepsilon} - 1 \approx \frac{1}{\varepsilon}$$

Note that the triangle inequality was used in the second last row.

$$|T| = \left|\frac{L}{1+L}\right| = \left|\frac{1}{\frac{1}{L}+1}\right| < \varepsilon$$

$$\Rightarrow \left|\frac{1}{L}+1\right| > \frac{1}{\varepsilon}$$

$$\Rightarrow \frac{1}{\varepsilon} < \left|\frac{1}{L}+1\right| < \left|\frac{1}{L}\right| + 1$$

$$\Rightarrow \left|\frac{1}{L}\right| > \frac{1}{\varepsilon} - 1 \approx \frac{1}{\varepsilon}$$

$$\Rightarrow |L| < \varepsilon$$

3. Let us assume that you have to develop a controller for a linear multivariable process with *n* inputs and *n* outputs. Answer and explain the following, by using figures, formulas etc. when appropriate:

- Define Relative Gain Array (RGA) and explain how it is used in control. (2p)
- What is the difference between *decoupled* and *centralized* controllers? (1p)
- How would you design a Singular Value Decomposition (SVD)-based decoupled controller for the process? The decoupling is here considered necessary only at the zero frequency. Present the idea and also the necessary formulas. (2p)

Solution:

See lecture slides, Chapter 6, part Multivariable controllers.

The RGA is used to estimate the influence of input i to output j, taking into account the influence of other inputs to output j also. The idea is to try to *decouple* the multivariable process such that it could be controlled by SISO controllers, ignoring possible interconnections. The RGA is used then to choose which input is used to control which output.

The RGA was earlier used in zero frequency (s=0) only, but it is nowadays used more widely, especially in the neighborhood of the bandwidth frequency.

If the system is not diagonal enough with any couplings u_i - u_j, then it is possible to use decouplers, typically pre and post compensators, which make the process (approximately) diagonal. The decoupled controller is then used to control the augmented plant. Of course, in the end the decoupling elements are part of the controller. Singular value decomposition is one way to design the decouplers.

Centralized control means that the multivariable system is controlled as one unit, without any decouplings. For example, LQ controllers belong to this class.

Bonus problem: You can earn maximum 2 extra points by solving the below problem. (However, the maximum of the exam is 15 points).

Consider the pure delay term in a SISO transfer function. Derive the first order Padé approximation for it and explain how that can be used to explain one fundamental limitation in control.

Solution:

The delay term (usually a part of the process transfer function) is

$$e^{-Ts}$$

where T is the delay. A rational approximation (Padé approximation) can be obtained by

$$e^{-Ts} = \frac{e^{-(T/2)s}}{e^{(T/2)s}} \approx \frac{1 - \frac{T}{2}s}{1 + \frac{T}{2}s}$$

where the first order Taylor approximation for the exponential function has been used. The smaller T (or s), the better the approximation. Higher order Padé approximations can be obtained by including more terms in the Taylor approximation in the numerator and denominator.