Demo Problem 1: Introduction to R

a) Change your working directory. Try the commands `help(c)` and `help(matrix)`.

b) Calculate the affine transformation $\boldsymbol{y} = \boldsymbol{x}\boldsymbol{A}^{-1} + \boldsymbol{b}$, where

$$\boldsymbol{A} = \begin{pmatrix} 2 & 1 & 5 \\ -2 & 7 & 0 \\ 5 & -8 & -1 \end{pmatrix}, \qquad \boldsymbol{x}^T = \begin{pmatrix} 8 \\ -4 \\ 2 \end{pmatrix}, \qquad \boldsymbol{b}^T = \begin{pmatrix} 3 \\ 10 \\ -19 \end{pmatrix}.$$

c) Install the package `mvtnorm` and load the corresponding functions to your workspace. Set the seed to 123 using the command `set.seed(123)`. Generate 100 observations from a two dimensional normal distribution with expected value $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. Visualize the observations.

$$\boldsymbol{\mu} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Sigma} = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix}.$$

d) Use the data from part c) and calculate the sample mean $\bar{\boldsymbol{x}}$ and the sample covariance matrix $\boldsymbol{S}_x$. Calculate the eigenvalues and eigenvectors from the matrix $\boldsymbol{S}_x$. Verify from the data, that the following equations hold: $\operatorname{Tr}(\boldsymbol{S}_x) = \lambda_1 + \lambda_2 + \ldots + \lambda_p$ and $\operatorname{Det}(\boldsymbol{S}_x) = \lambda_1\lambda_2 \ldots \lambda_p$, where λ_i are the eigenvalues of $\boldsymbol{S}_x$.

e) Calculate the affine transformation $\boldsymbol{y}_i = \boldsymbol{A}\boldsymbol{x}_i + \boldsymbol{b}$, where

$$\boldsymbol{b} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad \text{and} \quad \boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix},$$

verify that $\bar{\boldsymbol{y}} = \boldsymbol{A}\bar{\boldsymbol{x}} + \boldsymbol{b}$ and $\boldsymbol{S}_y = \boldsymbol{A}\boldsymbol{S}_x\boldsymbol{A}^T$. What does affine equivariance mean in practice?

f) Upload the data from the file `data.txt` into your workspace. Create a function, that centers your data (removes the mean) and pairwise scatterplots the variables. Calculate the sample covariance and correlation matrices and the corresponding eigenvalues- and vectors.

Demo problem 2: The Eigenvalues of a Symmetric Matrix

Show that the eigenvalues of a real valued symmetric matrix are always real valued.

Homework Problem 1: Functions

In this exercise do not use the built-in functions `cov`, `cor`, `cov2cor` or any additional R packages.

a) Create an R function that takes a data matrix $\boldsymbol{X} \in \mathbb{R}^{n \times p}$, $n > p$, as an argument and returns the unbiased estimator of the covariance matrix.

b) Create an R function that takes a full-rank covariance matrix $\boldsymbol{A} \in \mathbb{R}^{p \times p}$ as an argument and returns the square root of the inverse matrix such that $\boldsymbol{A}^{-\frac{1}{2}}\boldsymbol{A}^{-\frac{1}{2}} = \boldsymbol{A}^{-\mathbf{1}}$.

c) Create an R function that takes a full-rank covariance matrix $\boldsymbol{A}$ as an argument and returns the corresponding correlation matrix.