1 Proof of Exercise 5 Demo

Let x be a p-variate continuous random variable. Show that $\text{Cov}\,[x]$ is positive semidefinite.

Recall the definition for positive semi-definiteness:

Definition 1.1 *A symmetric and real-valued $p \times p$ matrix A is said to be positive semidefinite if the scalar $a^\top Aa$ is non-negative for every real-valued column vector $a \in \mathbb{R}^p$.*

Now,

$$
\begin{aligned}
&a^\top \text{cov}[x]a = a^\top \mathbb{E}\left[(x - \mathbb{E}[x])\,(x - \mathbb{E}[x])^\top\right] a \\
&= \mathbb{E}\left[a^\top (x - \mathbb{E}[x])\,(x - \mathbb{E}[x])^\top a\right] && \Big|\quad y = a^\top (x - \mathbb{E}\,[x]) \in \mathbb{R} \\
&= \mathbb{E}\left[yy^\top\right] = \mathbb{E}\left[y^2\right] \geq 0.
\end{aligned}
$$