ELEC-E8116 Model-based control systems
Intermediate exam 2. 8.12.2022

- *Write the name of the course, your name and student number to each answer sheet.*
- *There are three (3) problems and each one must be answered.*
- *No literature is allowed. A calculator can be used as a calculating aid. However, it must not be used for advanced calculations, e.g. matrix calculus, Laplace transformations, connection to the Internet etc.*
- *Your solutions must be presented so that it becomes clear how you have solved the problem.*

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1. Consider a SISO-process with the transfer function

$$G(s) = \frac{s + \dfrac{1}{T_1}}{\left(s + \dfrac{1}{T_2}\right)\left(s + \dfrac{1}{T_3}\right)}, \quad T_1 < 0, \quad T_2 < 0, \quad T_3 > 0$$

Explain, what kind of fundamental limitations on control performance can be stated for this system? (Present also some calculations; the formulas in the end of the problems can be of help.) (5 p)

2. Consider the first order process

$$\begin{aligned} \dot{x}(t) &= u(t), \quad x(0) = x_0 \\ y(t) &= x(t) \end{aligned}$$

so that the state is directly measurable . It is desired to find a control that minimizes the criterion

$$J = \frac{1}{2}\int_0^\infty \left(x^2 + Ru^2\right)dt \qquad (R > 0)$$

Determine the optimal control law and the closed loop state equation. What is the optimal state trajectory and optimal cost? (2+1+1+1 p)

3. Let us assume that you have to develop a controller for a linear multivariable process with n inputs and n outputs. Answer and explain the following:

- Define Relative Gain Array (RGA) and explain how it is used in control. (2 p)
- What is the difference between *decoupled* and *centralized* controllers? (1 p)

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- How would you design a Singular Value Decomposition (SVD)-based decoupled controller for the process? The decoupling is here considered necessary only at the zero frequency. Present the idea and also the necessary formulas. (2 p)

Some formulas that might be useful:

$$\dot{x} = Ax + Bu, \quad t \geq t_0$$

$$J(t_0) = \frac{1}{2} x^T(t_f) S(t_f) x(t_f) + \frac{1}{2} \int_{t_0}^{t_f} \left(x^T Q x + u^T R u \right) dt$$

$$S(t_f) \geq 0, \quad Q \geq 0, \quad R > 0$$

$$-\dot{S}(t) = A^T S + SA - SBR^{-1} B^T S + Q, \quad t \leq t_f, \quad \text{boundary condition } S(t_f)$$

$$K = R^{-1} B^T S$$

$$u = -Kx$$

$$J^*(t_0) = \frac{1}{2} x^T(t_0) S(t_0) x(t_0)$$

$$\int_0^{\infty} \log \left| S(i\omega) \right| d\omega = \pi \sum_{i=1}^{M} \mathrm{Re}\,(\mathrm{p_i})$$

$$\left| W_T(p_1) \right| \leq 1 \quad \Rightarrow \quad \omega_0 \geq \frac{p_1}{1 - 1/T_0}$$

$$\left| W_S(z) \right| \leq 1 \quad \Rightarrow \quad \omega_0 \leq \left(1 - 1/S_0 \right) z$$