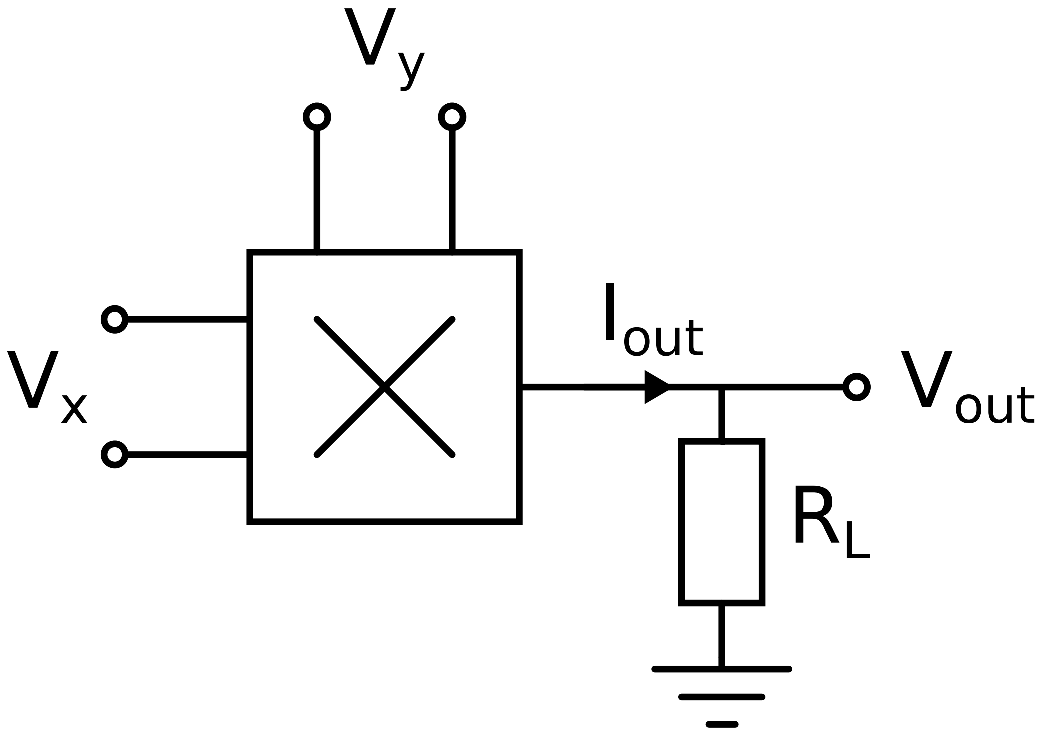
ELEC-E3510 Basics of IC Design

Lecture 9:
Multipliers

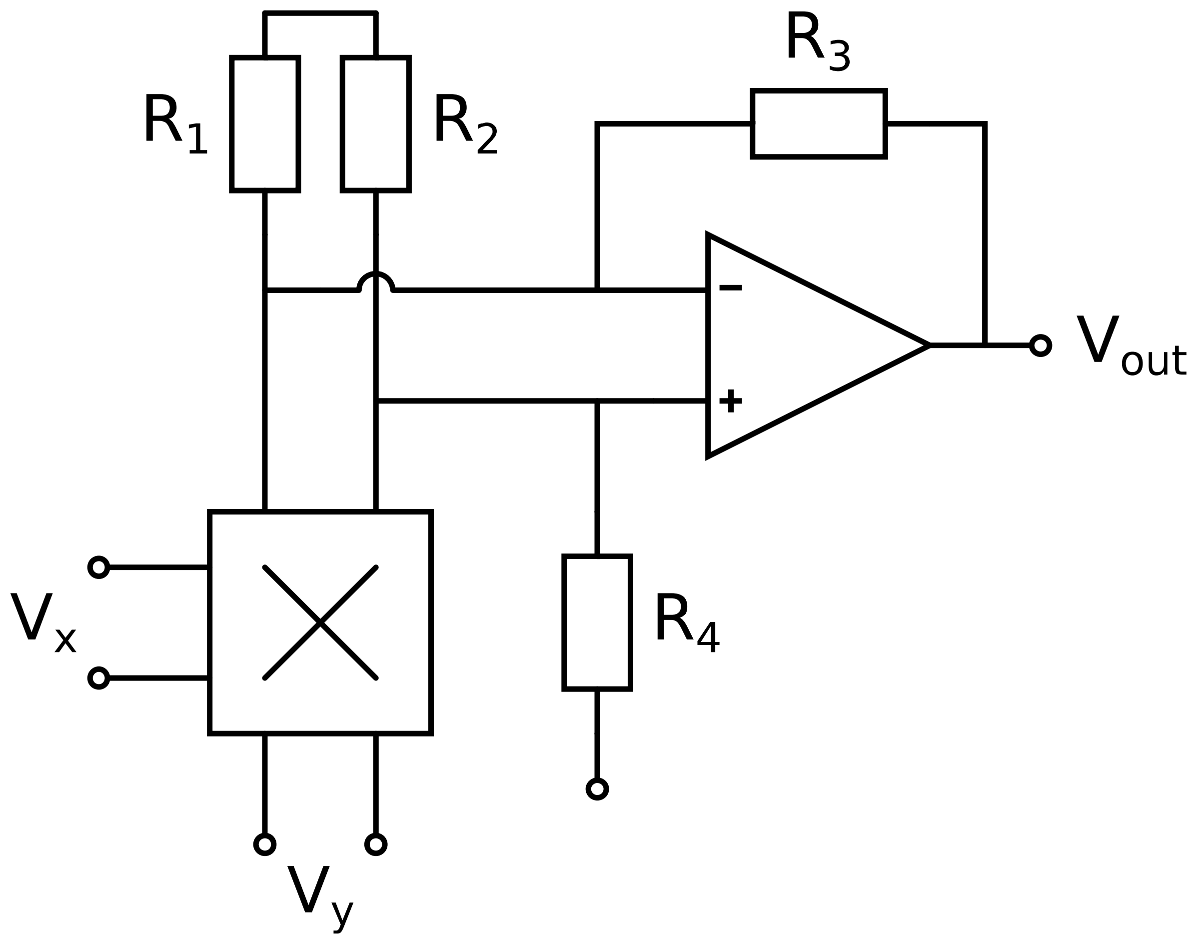
Analog multiplier



$I_{out} = kV_xV_y$

$V_{out} = I_{out} \cdot R_L = kR_LV_xV_y$

Single-ended output

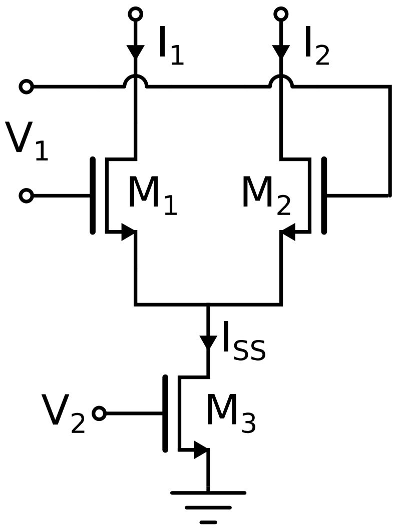


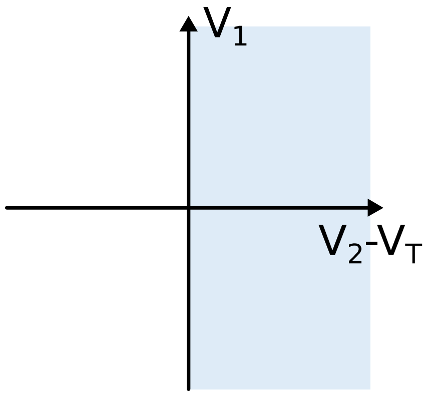
Assume $R_3 = R_4 = R \Rightarrow$

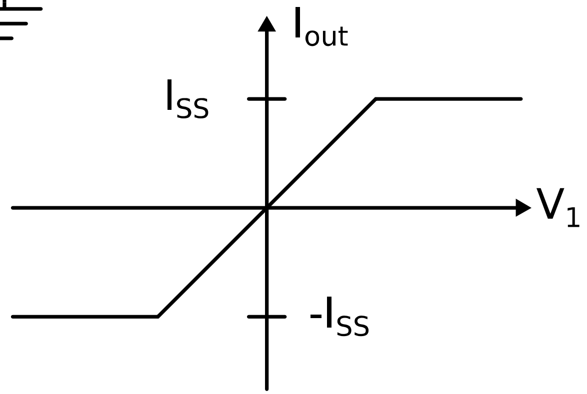
$V_{out} = RI_{out} + V_{REF}$

$V_{out} = \sqrt{2}k\, R_L V_x V_y + V_{REF}$

2-quadrant multiplier







For soure coupled pair

$$I_{out} = I_1 - I_2 = k_1 V_1 \sqrt{\frac{2I_{SS}}{k_1} - V_1^2}$$

Assume $V_1^2 << \frac{2I_{SS}}{k}$; $k_1 = \frac{1}{2}\mu C_{OX}$ $\qquad V_i << \sqrt{\frac{I_{SS}}{k}}$

$$I_{out} = I_1 - I_2 = g_{m1} V_1$$

$$= k_1 V_1 \sqrt{\frac{2I_{SS}}{k_1}}$$

M_3 in saturation

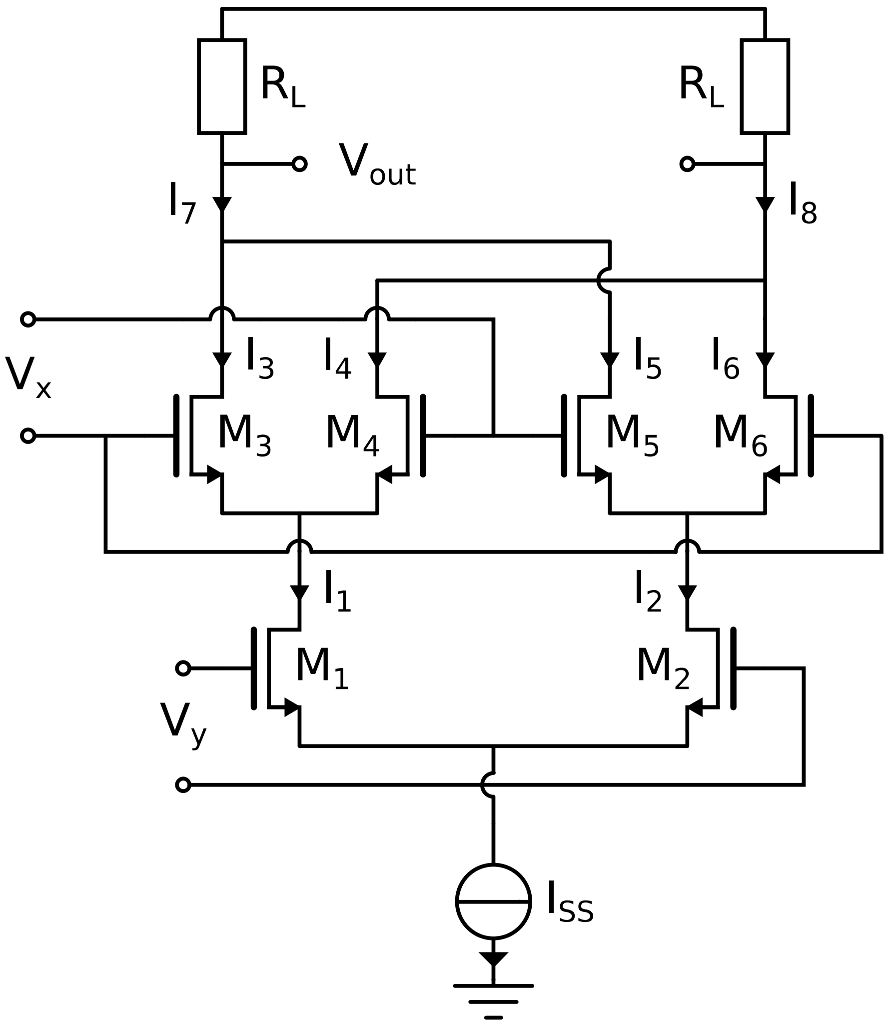
$$I_{SS} = k_3 (V_2 - V_T)^2$$

$$\Rightarrow I_{out} = \sqrt{2k_1} V_1 \sqrt{k_3} (V_2 - V_T)$$

$$= \sqrt{2k_1 k_3} (V_1 V_2 - V_1 V_T)$$

$$= \sqrt{2k_1 k_3} (V_2 - V_T) \cdot V_1$$

4-quadrant multiplier (Gilbert)



Assume all transistors are equal.
Gilbert cell output current difference is

$$I_{OUT} = I_7 - I_8 = (I_3 + I_5) - (I_4 + I_6)$$
$$= (I_3 - I_4) - (I_6 - I_5)$$

M_3 and M_4 as well as M_5 and M_6 form source coupled pairs ⇒

$$I_3 - I_4 = kV_x\sqrt{\frac{2I_1}{k} - V_x^2}$$

$$I_6 - I_5 = kV_x\sqrt{\frac{2I_2}{k} - V_x^2}$$

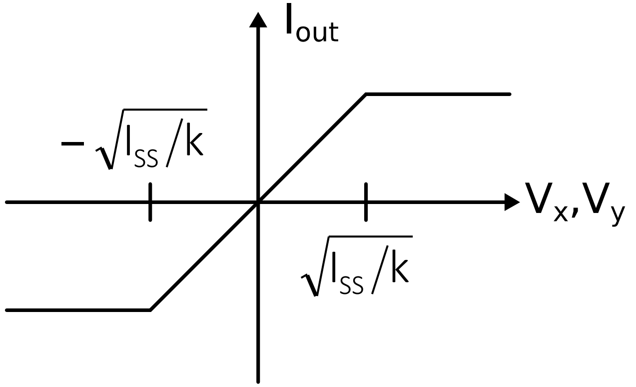
$$\Rightarrow I_{OUT} = kV_x\left(\sqrt{\frac{2I_1}{k} - V_x^2} - \sqrt{\frac{2I_2}{k} - V_x^2}\right)$$

Also M_1 and M_2 form a source coupled pair:

$$I_1 = \frac{k}{2}\left(\sqrt{\frac{I_{SS}}{k} - \frac{V_y^2}{2}} + \frac{V_y}{\sqrt{2}}\right)^2$$

$$I_2 = \frac{k}{2}\left(\sqrt{\frac{I_{SS}}{k} - \frac{V_y^2}{2}} - \frac{V_y}{\sqrt{2}}\right)^2$$

$$\Rightarrow \quad I_{out} = kV_x\left[\sqrt{\left(\sqrt{\frac{I_{SS}}{k} - \frac{V_y^2}{2}} + \frac{V_y}{\sqrt{2}}\right)^2 - V_x^2} - \sqrt{\left(\sqrt{\frac{I_{SS}}{k} - \frac{V_y^2}{2}} - \frac{V_y}{\sqrt{2}}\right)^2 - V_x^2}\right]$$



With small signal amplitudes i.e.

$$V_x, V_y << \sqrt{\frac{I_{SS}}{k}}$$

$$\Rightarrow \quad I_{out} \approx kV_x\left[\sqrt{\left(\sqrt{\frac{I_{SS}}{k} - \frac{V_y^2}{2}} + \frac{V_y}{\sqrt{2}}\right)^2} - \sqrt{\left(\sqrt{\frac{I_{SS}}{k} - \frac{V_y^2}{2}} - \frac{V_y}{\sqrt{2}}\right)^2}\right]$$

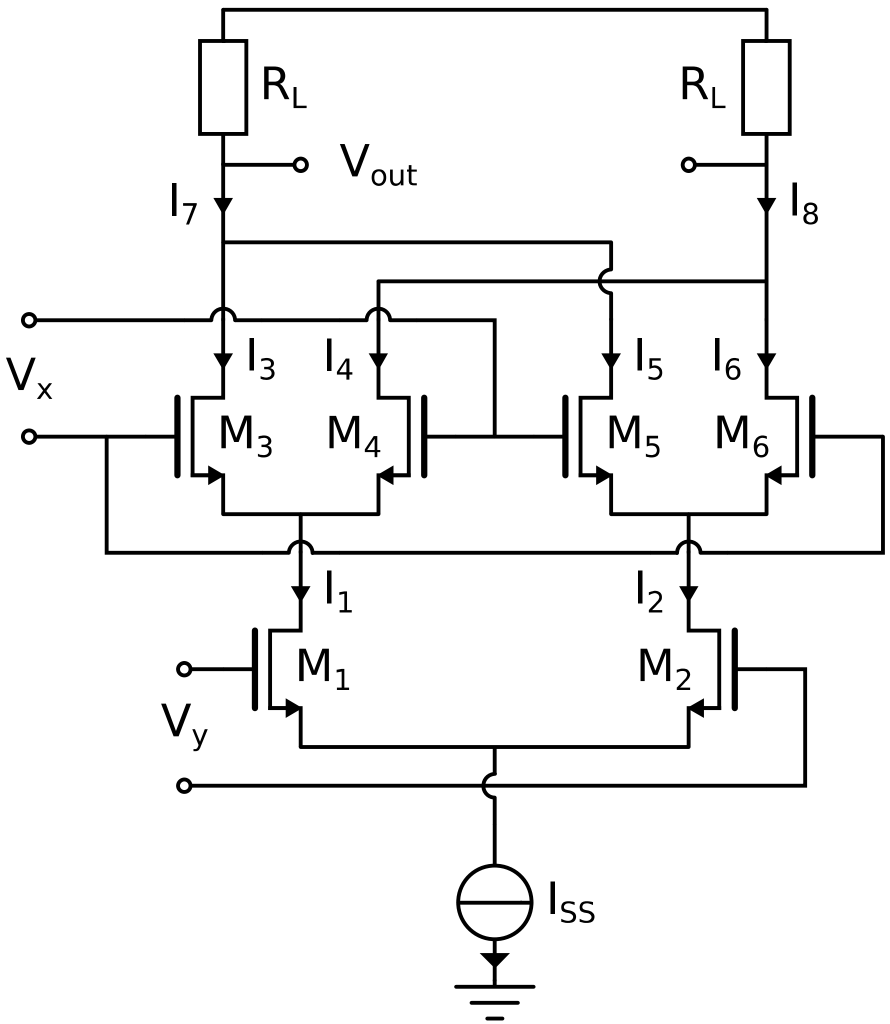
$$\approx kV_x\left[\sqrt{\frac{I_{SS}}{k} - \frac{V_y^2}{2}} + \frac{V_y}{\sqrt{2}} - \sqrt{\frac{I_{SS}}{k} - \frac{V_y^2}{2}} + \frac{V_y}{\sqrt{2}}\right]$$

$$\Rightarrow \quad \boxed{I_{out} \approx \sqrt{2}kV_xV_y}$$

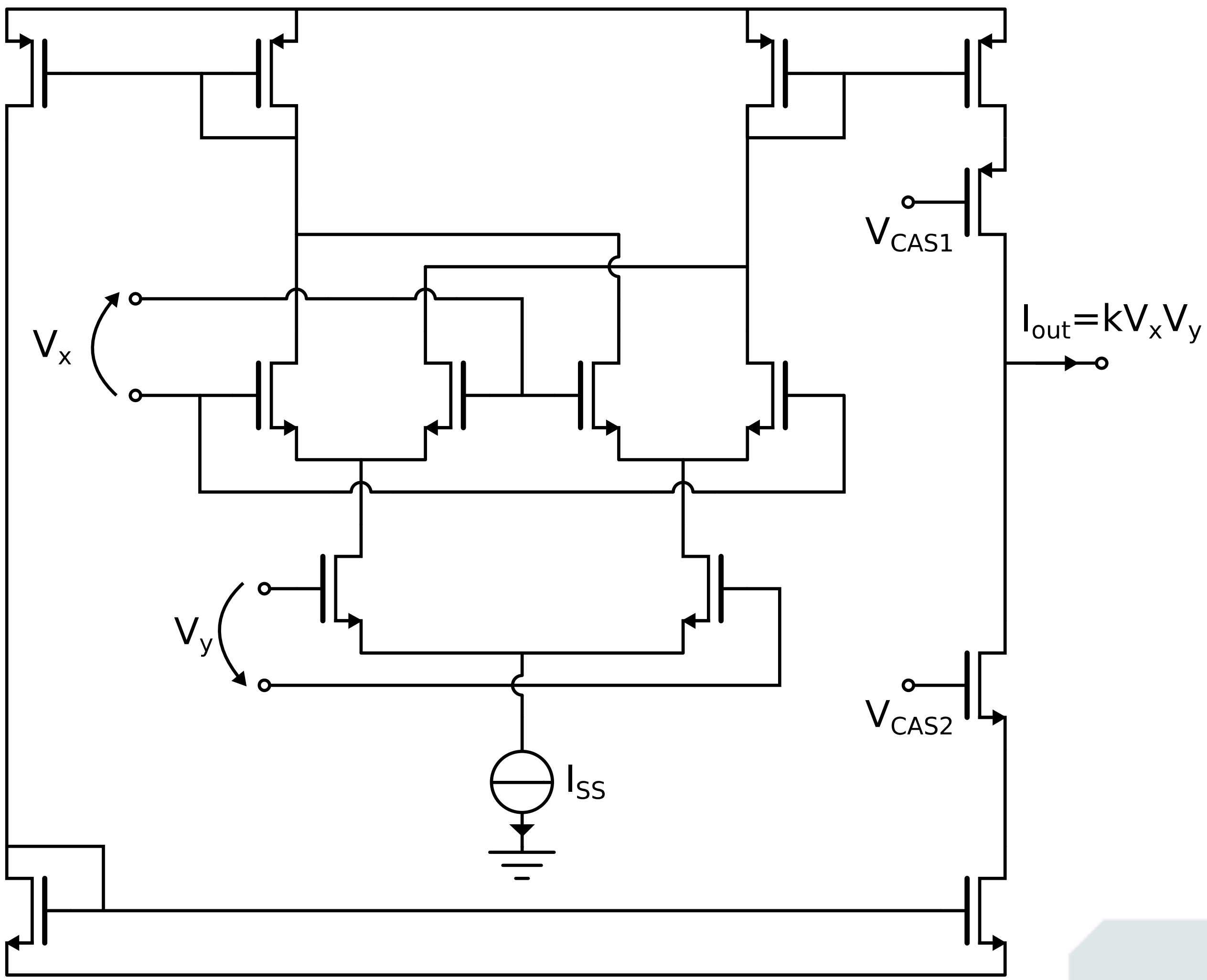
Linear region can be extended by:

1. Increasing I_{SS}
2. Decreasing k (big L, small W)
3. Use small input signal V_i $\quad V_i << \sqrt{\frac{I_{SS}}{k}}$
   (attenuate input signal)
   $\Rightarrow$ (source coupled pair) $\quad \Delta I \approx V_i\sqrt{2kI_{SS}}$
   $\Rightarrow$ 10 V supply voltage needed!

Differential to single-ended conversion



- Symmetrical load for Gilbert cell



Linearized source coupled pair

Source coupled pair:

$$I_1 = \frac{k}{2}\left(\sqrt{\frac{I_{SS}}{k} - \frac{V_1^2}{2}} + \frac{V_1}{\sqrt{2}}\right)^2$$

$$I_2 = \frac{k}{2}\left(\sqrt{\frac{I_{SS}}{k} - \frac{V_1^2}{2}} + \frac{V_1}{\sqrt{2}}\right)^2$$

Assume: $I_{SS}' = I_{SS} + \frac{1}{2}kV_i^2$

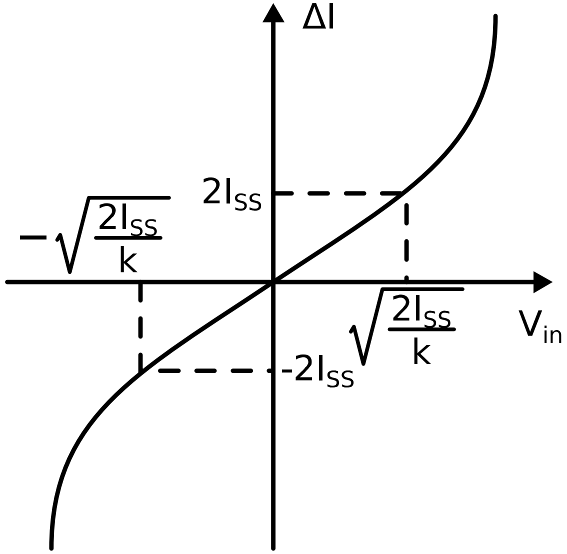
$$\Rightarrow \begin{cases} I_1 = \frac{k}{2}\left(\sqrt{\frac{I_{SS}}{k}} + \frac{V_i}{\sqrt{2}}\right)^2 \\ I_1 = \frac{k}{2}\left(\sqrt{\frac{I_{SS}}{k}} - \frac{V_i}{\sqrt{2}}\right)^2 \end{cases}$$

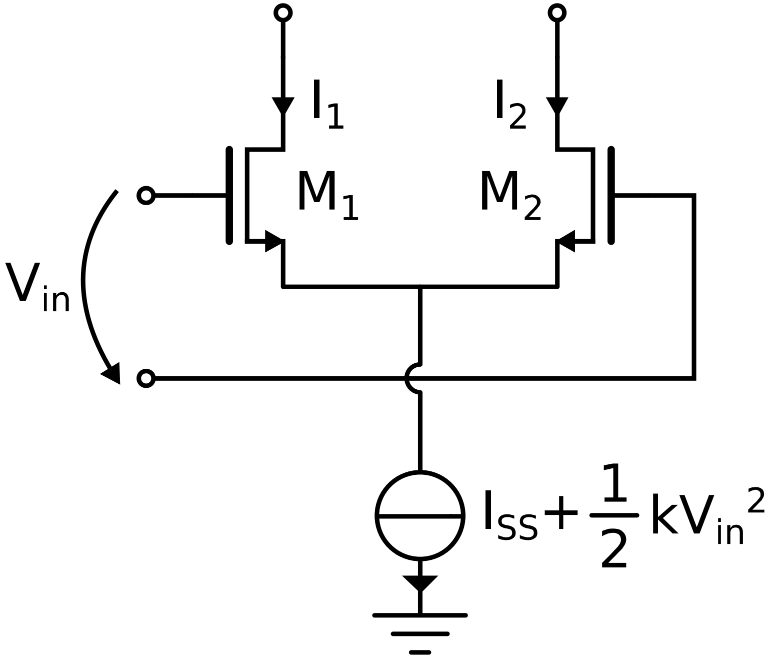
Thus output current is

$$I_{OUT} = I_1 - I_2 = \frac{k}{2}\left[\left(\sqrt{\frac{I_{SS}}{k}} + \frac{V_i}{\sqrt{2}}\right)^2 - \left(\sqrt{\frac{I_{SS}}{k}} - \frac{V_i}{\sqrt{2}}\right)^2\right]$$

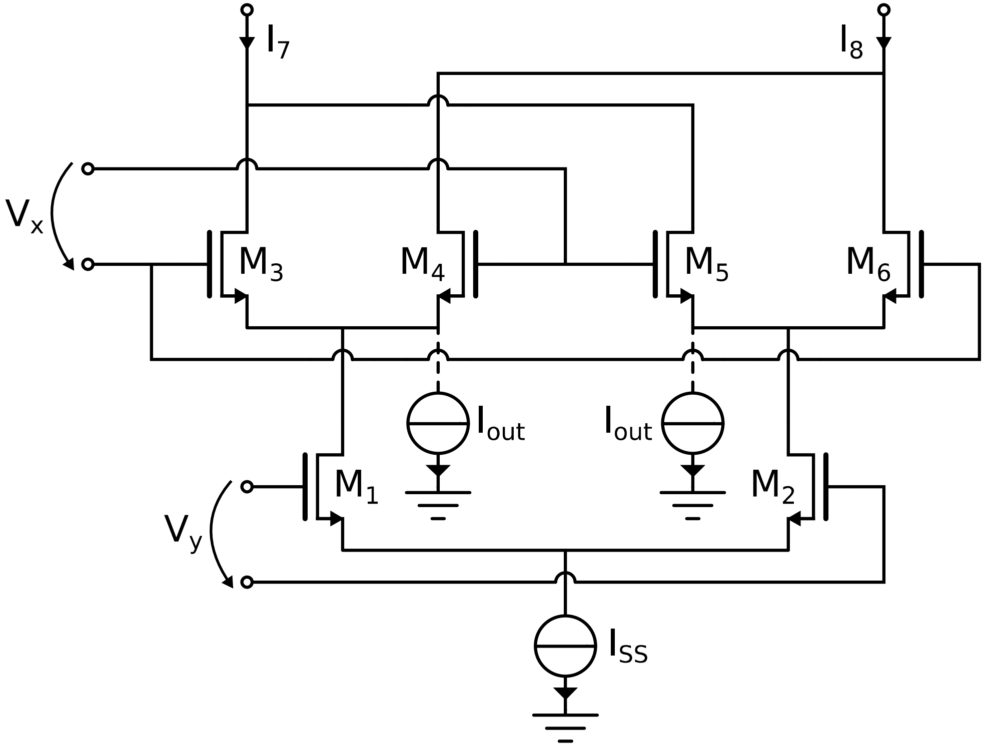
Which gives

$$I_{OUT} = V_i\sqrt{2kI_{SS}} \quad \text{(exactly!) when } V_x, V_y \le \sqrt{\frac{I_{SS}}{k}}$$





Linearized Gilbert cell



Current equations:

$$I_3 + I_4 = I_1 + I_0$$

$$I_6 + I_5 = I_2 + I_0$$

Assume: $I_0 = \frac{k}{2} V_x^2$

Source coupled pair:

$$I_3 - I_4 = kV_x \sqrt{\frac{2I_1}{k} - V_x^2}$$

$$I_6 - I_5 = kV_x \sqrt{\frac{2I_2}{k} - V_x^2}$$

$$\Rightarrow \quad I_3 - I_4 = \frac{k}{2} V_x \sqrt{\frac{2I_1}{k} + \frac{2I_0}{k} - V_x^2} = kV_x \sqrt{\frac{2I_1}{k}}$$

$$I_6 - I_5 = \frac{k}{2} V_x \sqrt{\frac{2I_2}{k} + \frac{2I_0}{k} - V_x^2} = kV_x \sqrt{\frac{2I_2}{k}}$$

$$\Rightarrow \quad I_{OUT} = kV_x \left(\sqrt{\frac{2I_1}{k}} - \sqrt{\frac{2I_2}{k}} \right)$$

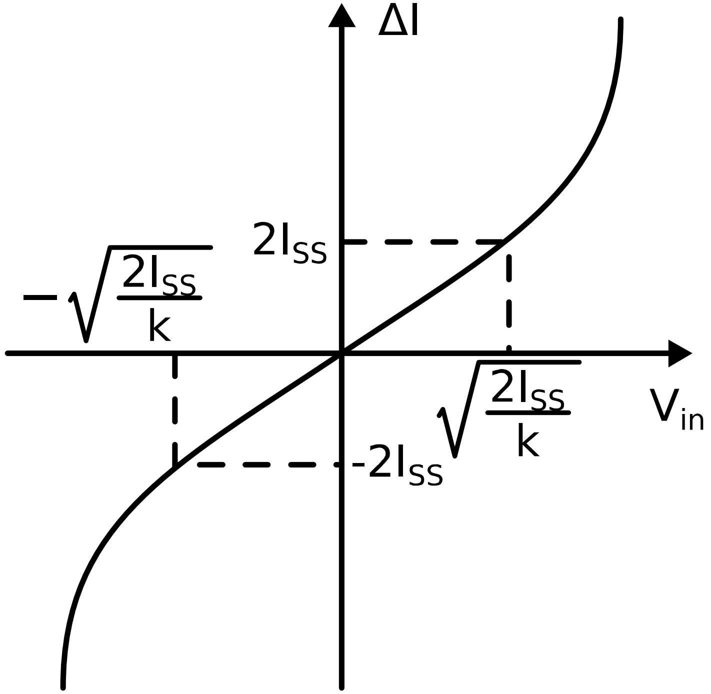
$$I_1 = \frac{k}{2} \left(\sqrt{\frac{I_{SS}}{k} - \frac{V_y^2}{2}} + \frac{V_y}{\sqrt{2}} \right)^2$$

$$I_2 = \frac{k}{2} \left(\sqrt{\frac{I_{SS}}{k} - \frac{V_y^2}{2}} - \frac{V_y}{\sqrt{2}} \right)^2$$

Calculate I_{out}:

$$I_{OUT} = kV_x\left[\sqrt{\left(\sqrt{\frac{I_{SS}}{k}-\frac{V_y^2}{2}}+\frac{V_y}{\sqrt{2}}\right)^2}-\sqrt{\left(\sqrt{\frac{I_{SS}}{k}-\frac{V_y^2}{2}}-\frac{V_y}{\sqrt{2}}\right)^2}\right]$$

$$I_{OUT} = \sqrt{2}kV_xV_y \quad \text{(exactly)} \quad \text{when} \quad V_x, V_y \le \sqrt{\frac{I_{SS}}{k}}$$

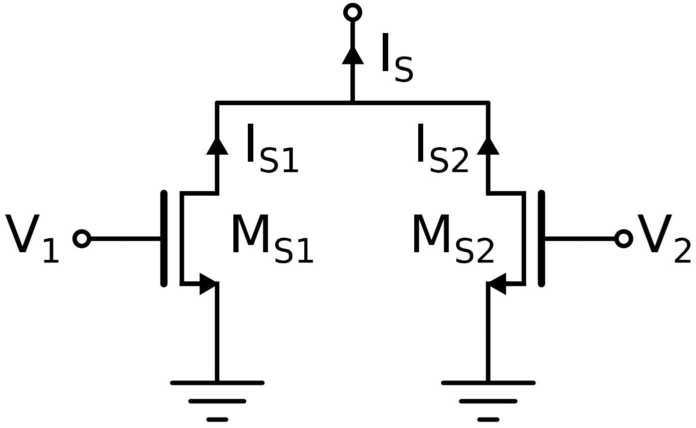


It is adequate to linearize x-input only.
Also y-input becomes linearized.

However, $V_x, V_y \le \sqrt{\frac{I_{SS}}{k}}$ therefore

⇒ To maximize the linear region, maximize $V_{D1,SAT}$.

Realization of current source $I_{SS} + I_0$



Input signal:

$$V_i = V_{CX} + V_X = V_{DC} + V_{AC}$$

Differential:

$$V_X = V_1 - V_2$$

Common-mode:

$$V_{CX} = \frac{1}{2}(V_1 + V_2)$$

$$\Rightarrow V_1 = V_{CX} + \frac{V_X}{2}$$

$$V_2 = V_{CX} - \frac{V_X}{2}$$

Current equation:

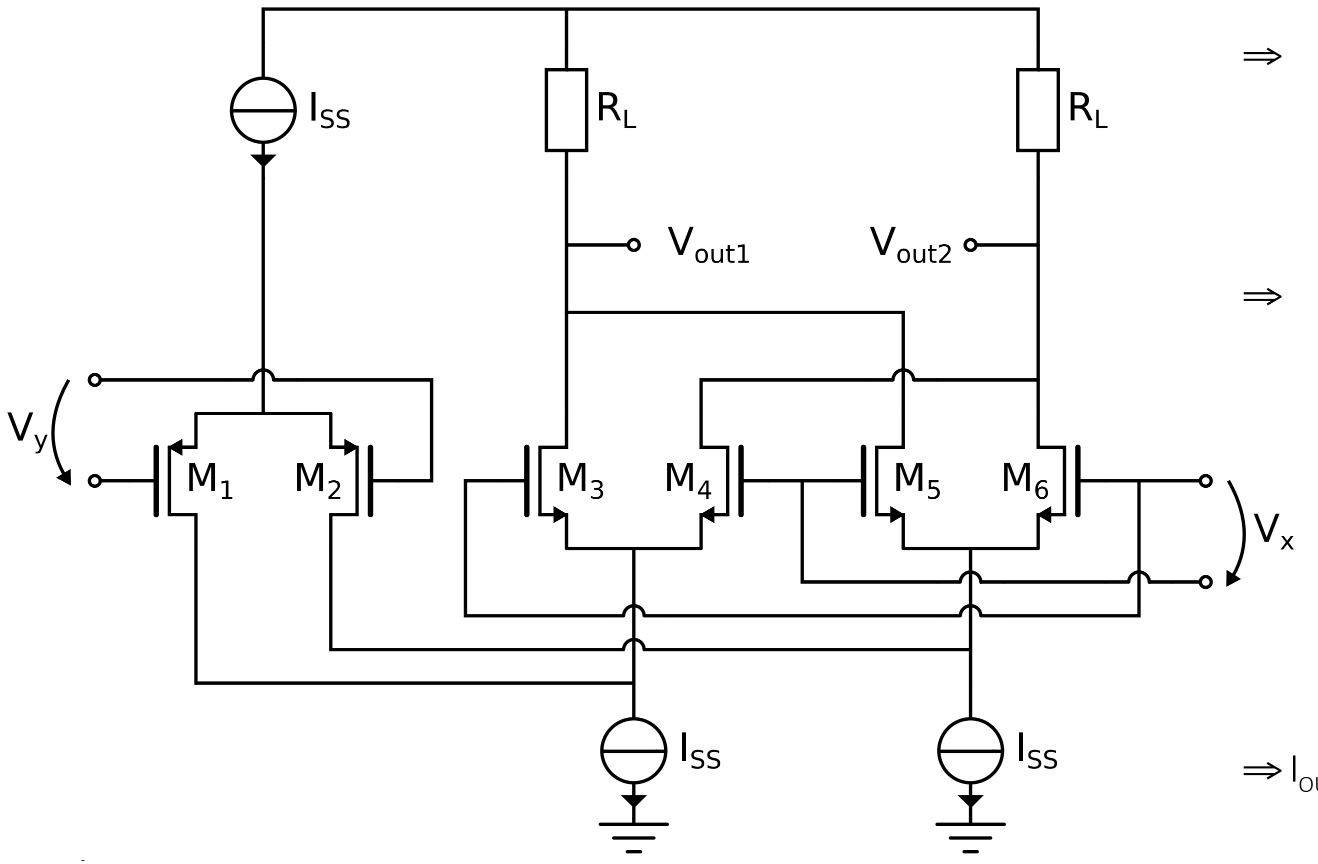
- Assume transistors in saturation

$$I_S = I_{S1} + I_{S2}$$

$$= k_n\left(V_{cx} + \frac{V_x}{2} - V_T\right)^2 + k_n\left(V_{cx} - \frac{V_x}{2} - V_T\right)^2$$

$$= I_{SS} + I_o$$

Folded Gilbert cell



$$I_3 + I_4 = I_{SS} - I_1 = I_2 \qquad ; I_1 + I_2 = I_{SS}$$

$$I_6 + I_5 = I_{SS} - I_2 = I_1$$

$$\Rightarrow \quad I_3 - I_4 = k_n V_x \sqrt{\frac{2I_2}{k_n} - V_x^2}$$

$$I_6 - I_5 = k_n V_x \sqrt{\frac{2I_1}{k_n} - V_x^2}$$

$$\Rightarrow \quad I_{OUT} = k_n V_x \left[\sqrt{\frac{2I_2}{k_n} - V_x^2} - \frac{2I_1}{k_n} - V_x^2 \right]$$

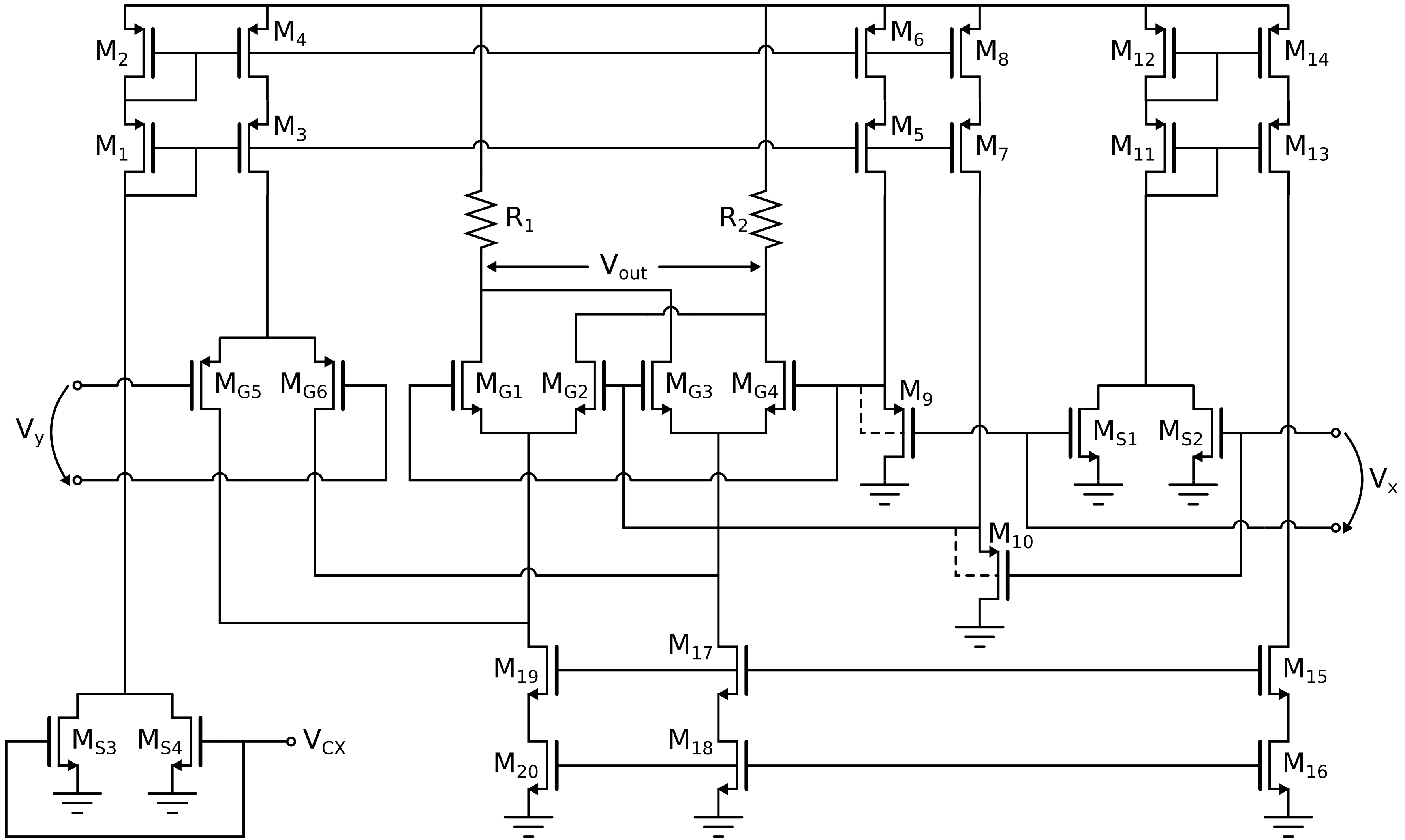
$$I_1 = \frac{k_p}{2} \left(\sqrt{\frac{I_{SS}}{k_p} - \frac{V_y^2}{2}} - \frac{V_y}{\sqrt{2}} \right)^2$$

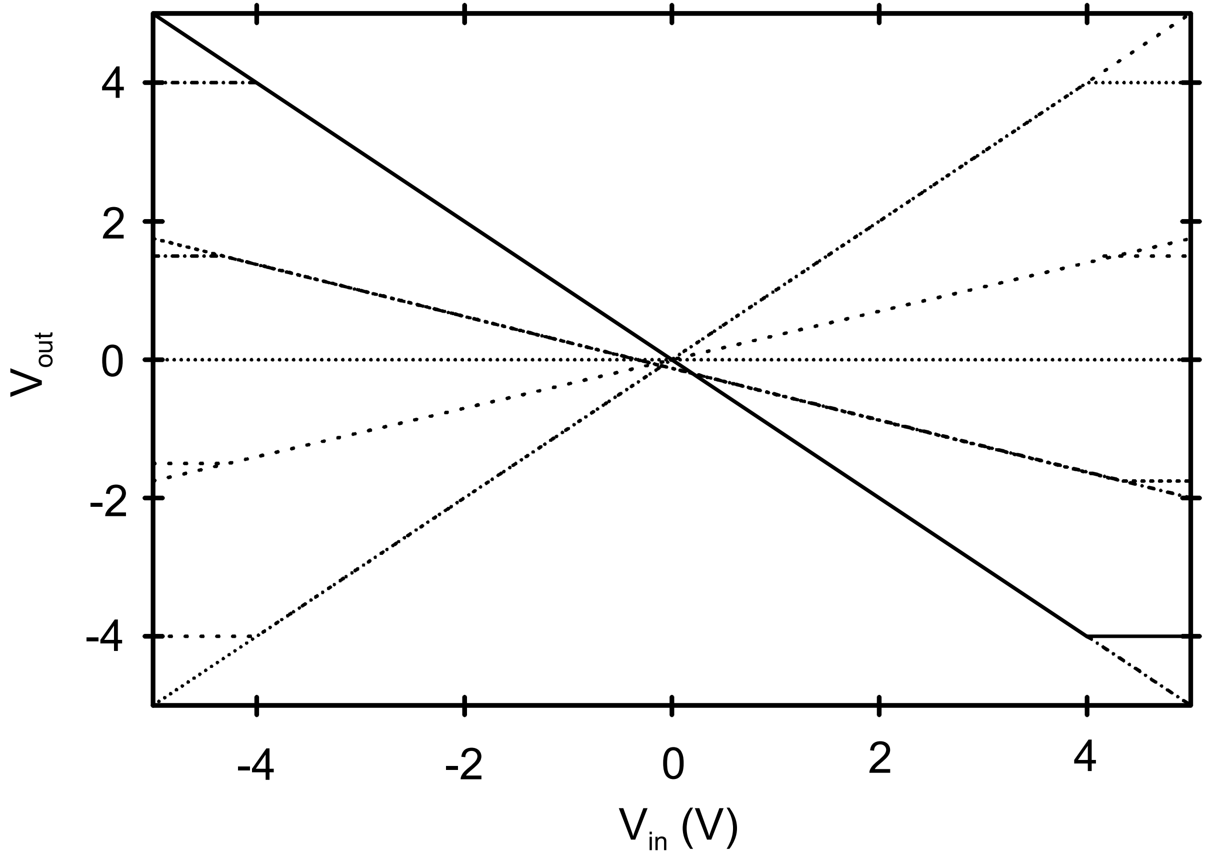
$$I_2 = \frac{k_p}{2} \left(\sqrt{\frac{I_{SS}}{k_p} - \frac{V_y^2}{2}} + \frac{V_y}{\sqrt{2}} \right)^2$$

$$\Rightarrow I_{OUT} = k_n V_x \left[\sqrt{\frac{k_p}{k_h} \left(\sqrt{\frac{I_{SS}}{k_p} - \frac{V_y^2}{2}} + \frac{V_y}{\sqrt{2}} \right)^2 - V_x^2} - \sqrt{\frac{k_p}{k_h} \left(\sqrt{\frac{I_{SS}}{k_p} - \frac{V_y^2}{2}} - \frac{V_y}{\sqrt{2}} \right)^2 - V_x^2} \right]$$

Linearized Folded Gilbert cell $\quad I_{OUT} = \sqrt{2k_n k_p} V_x V_y$

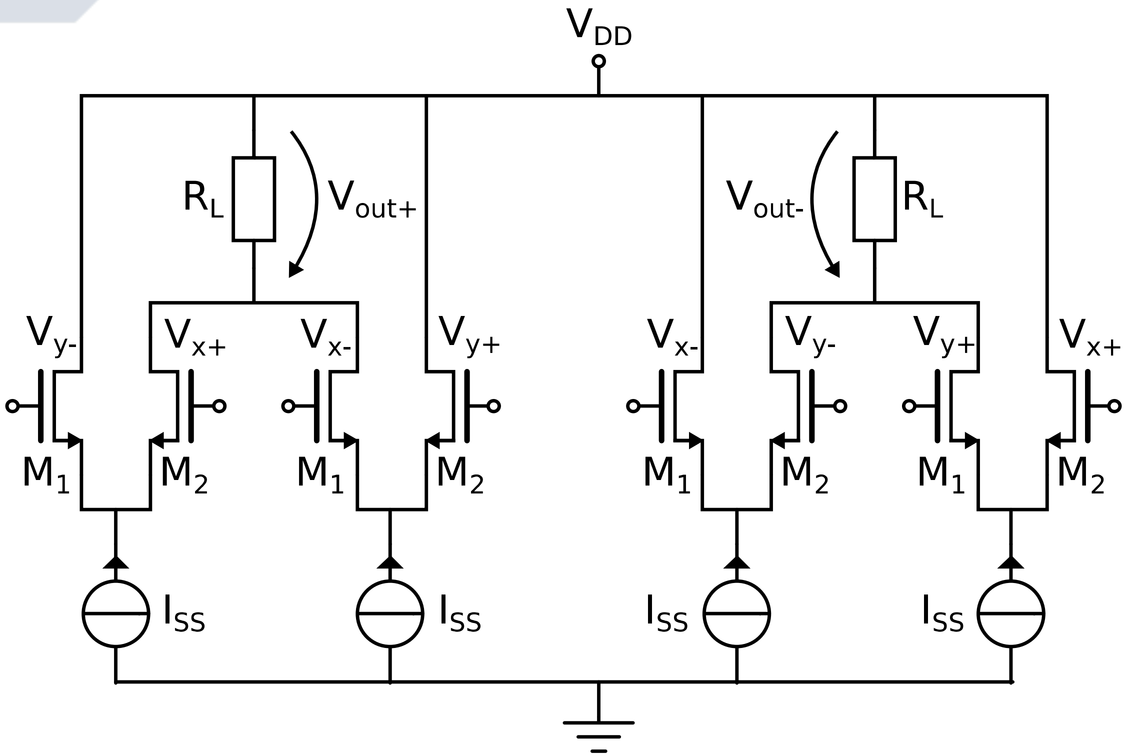
Linearized folded Gilbert multiplier





4
2
0
-2
-4
V_{out}
-4
-2
0
2
4
V_{in} (V)

Sum of squares multiplier



Assume $\left(\frac{W}{L}\right)_2 >> \left(\frac{W}{L}\right)_1$

$\Rightarrow$ M_2 is a source follower

$\Rightarrow$ $I_{D1} = k_1(V_{G1} - V_{S1} - V_T)^2 = k_1(V_{G1} - V_{G2} - V_T)^2$ (stronger than M_1)

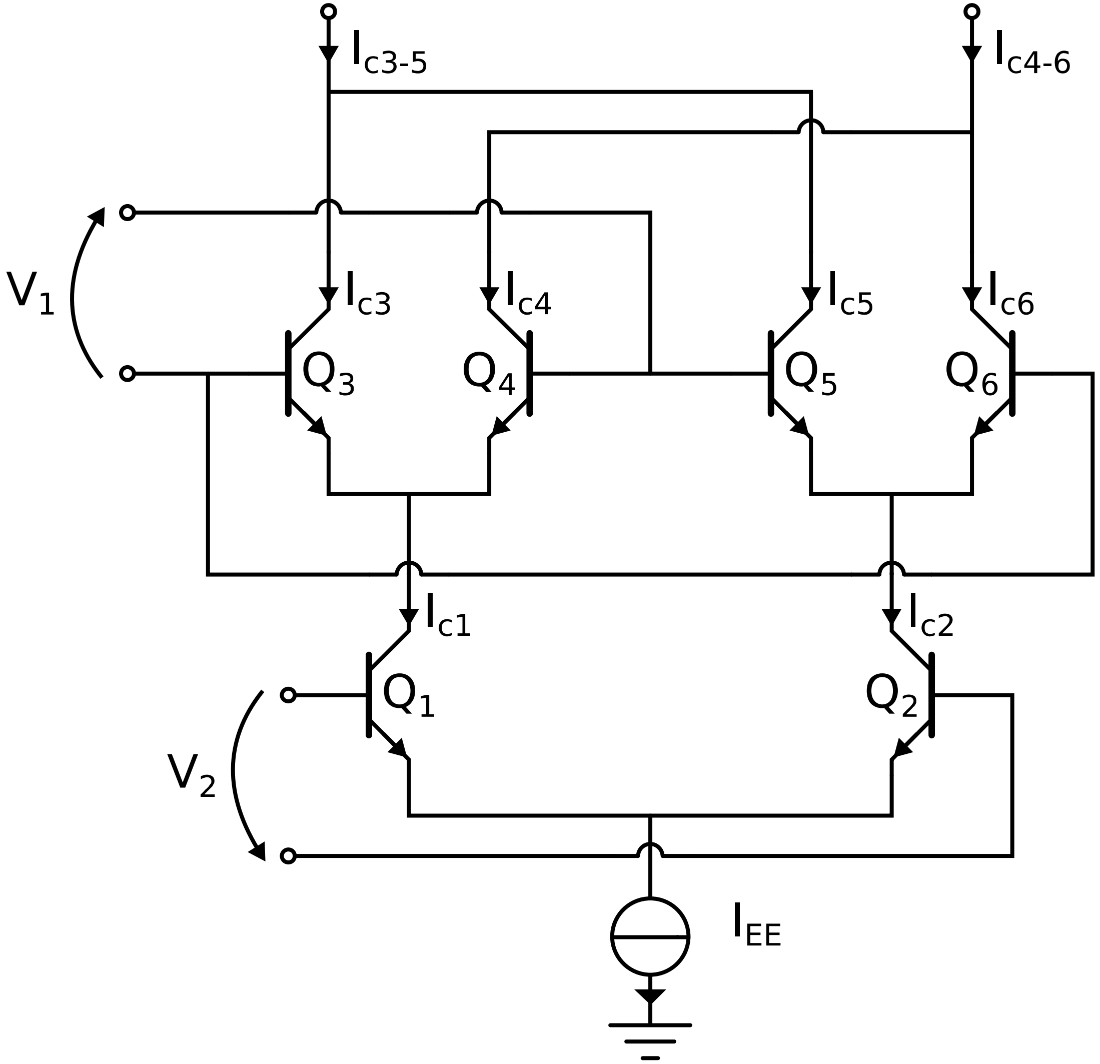
$I_1 = k\,(V_1 - V_2 - V_T)^2$
$I_2 = k\,(V_1 + V_2 - V_T)^2$
$I_3 = k\,(-V_1 + V_2 - V_T)^2$
$I_4 = k\,(-V_1 - V_2 - V_T)^2$

$$I_1 - I_2 = k\,(V_1 - V_T)^2 - 2k\,(V_1 - V_T) \cdot V_2 + kV_2^2$$
$$- k\,(V_1 - V_T)^2 - 2k\,(V_1 - V_T)V_2 + kV_2^2 - kV_2^2$$
$$= -4k(V_1 - V_T)\,V_2 = -4kV_1V_2 + 4kV_TV_2$$

$$-(I_3 - I_4) = 4k(-V_1 - V_T) \cdot (-V_2) = 4kV_1V_2 + 4kV_TV_2$$

$$I_1 - I_2 - (-(I_3 - I_4)) = -8kV_1V_2$$

Gilbert multiplier



Current equations:

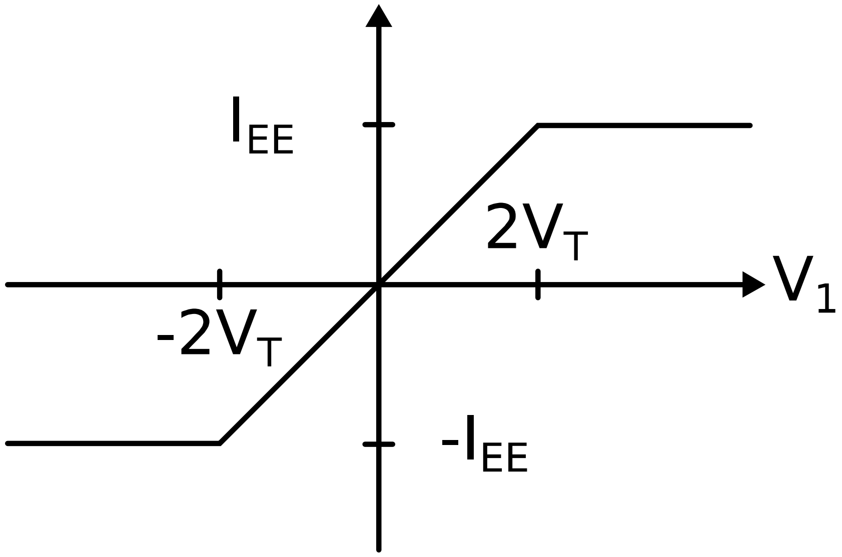
$$I_{OUT} = (I_{C3} + I_{C5}) - (I_{C4} + I_{C6})$$
$$= (I_{C3} - I_{C6}) - (I_{C4} - I_{C5})$$

Emitter coupled pair:

$$I_{C3} = \frac{I_{C1}}{1 + \exp\left(\frac{-V_1}{V_T}\right)} \qquad I_{C4} = \frac{I_{C1}}{1 + \exp\left(\frac{V_1}{V_T}\right)}$$

$$I_{C5} = \frac{I_{C2}}{1 + \exp\left(\frac{V_1}{V_T}\right)} \qquad I_{C6} = \frac{I_{C2}}{1 + \exp\left(\frac{-V_1}{V_T}\right)}$$

$$I_{C1} = \frac{I_{EE}}{1 + \exp\left(\frac{-V_2}{V_T}\right)} \qquad I_{C2} = \frac{I_{EE}}{1 + \exp\left(\frac{V_2}{V_T}\right)}$$



$$I_{C3} = \frac{I_{EE}}{\left[1+\exp\left(\frac{-V_1}{V_T}\right)\right]\left[1+\exp\left(\frac{-V_2}{V_T}\right)\right]}$$

$$I_{C4} = \frac{I_{EE}}{\left[1+\exp\left(\frac{-V_2}{V_T}\right)\right]\left[1+\exp\left(\frac{V_1}{V_T}\right)\right]}$$

$$I_{C5} = \frac{I_{EE}}{\left[1+\exp\left(\frac{V_1}{V_T}\right)\right]\left[1+\exp\left(\frac{V_2}{V_T}\right)\right]}$$

$$I_{C6} = \frac{I_{EE}}{\left[1+\exp\left(\frac{V_2}{V_T}\right)\right]\left[1+\exp\left(\frac{-V_1}{V_T}\right)\right]}$$

$$\Rightarrow I_{OUT} = I_{EE}\left[\tanh\left(\frac{V_1}{2V_T}\right)\right]\left[\tanh\left(\frac{V_2}{2V_T}\right)\right]$$

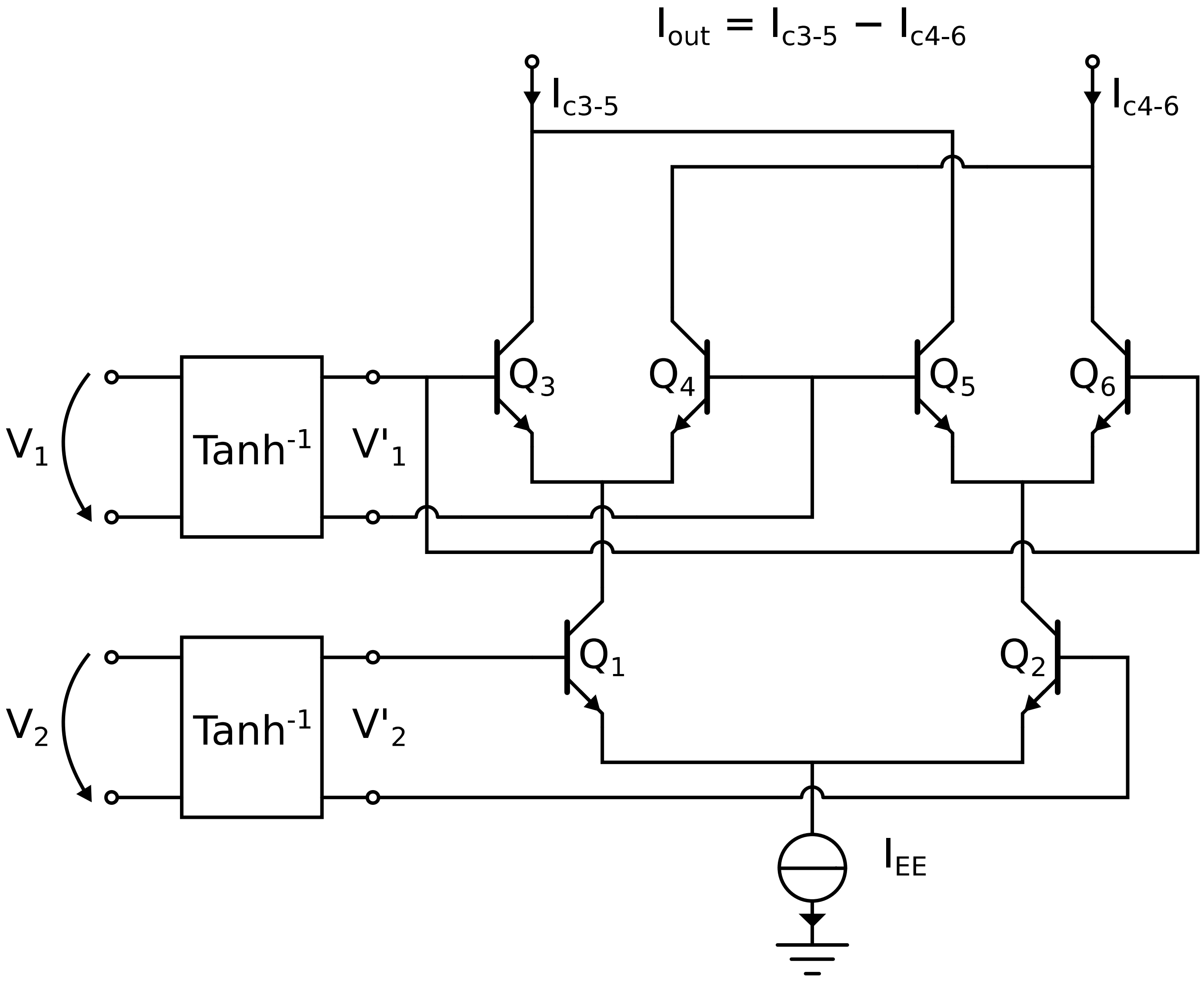
when $V_1 << 2V_T$

$V_2 << 2V_T$

$$\Rightarrow \tanh\frac{V_1}{2V_T} \approx \frac{V_1}{2V_T}$$

$$\Rightarrow I_{OUT} \approx I_{EE}\left(\frac{V_1}{2V_T}\right)\left(\frac{V_2}{2V_T}\right)$$

Linearized Gilbert multiplier



Linearized Gilbert multiplier

Voltage to current converter:

$I_1 = I_{01} + k_1V_1$

$I_2 = I_{01} - k_1V_1$

Output voltage:

$$\Delta V = V_{BE1} - V_{BE2}$$

$$= V_T \ln\left(\frac{I_{01} + k_1V_1}{I_S}\right) - V_T \ln\left(\frac{I_{01} - k_1V_1}{I_S}\right)$$

$$= V_T \ln\left(\frac{I_{01} + k_1V_1}{I_{01} - k_1V_1}\right)$$

$$\Rightarrow \quad \Delta V = 2V_T \tanh^{-1}\left(\frac{k_1V_1}{I_{01}}\right) \quad ; \tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

When applied for Gilbert multiplier input $\tanh^{-1}$ compensates for tanh-function

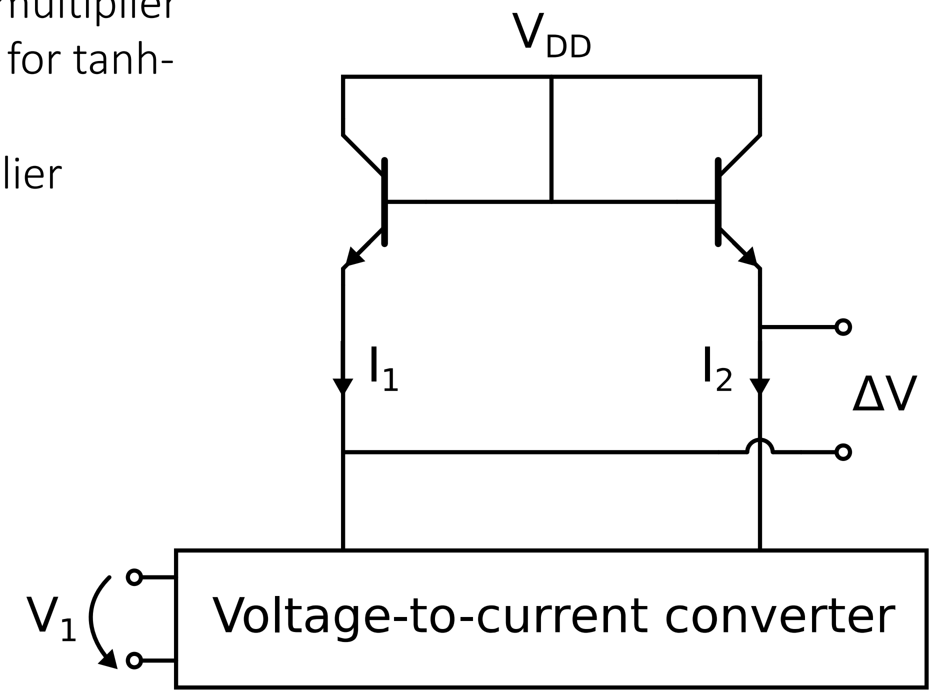
$\Rightarrow$ We obtain ideal multiplier

$$\Rightarrow \boxed{\Delta I = I_{EE}\left(\frac{k_1V_1}{I_{01}}\right)\left(\frac{k_2V_2}{I_{02}}\right)}$$

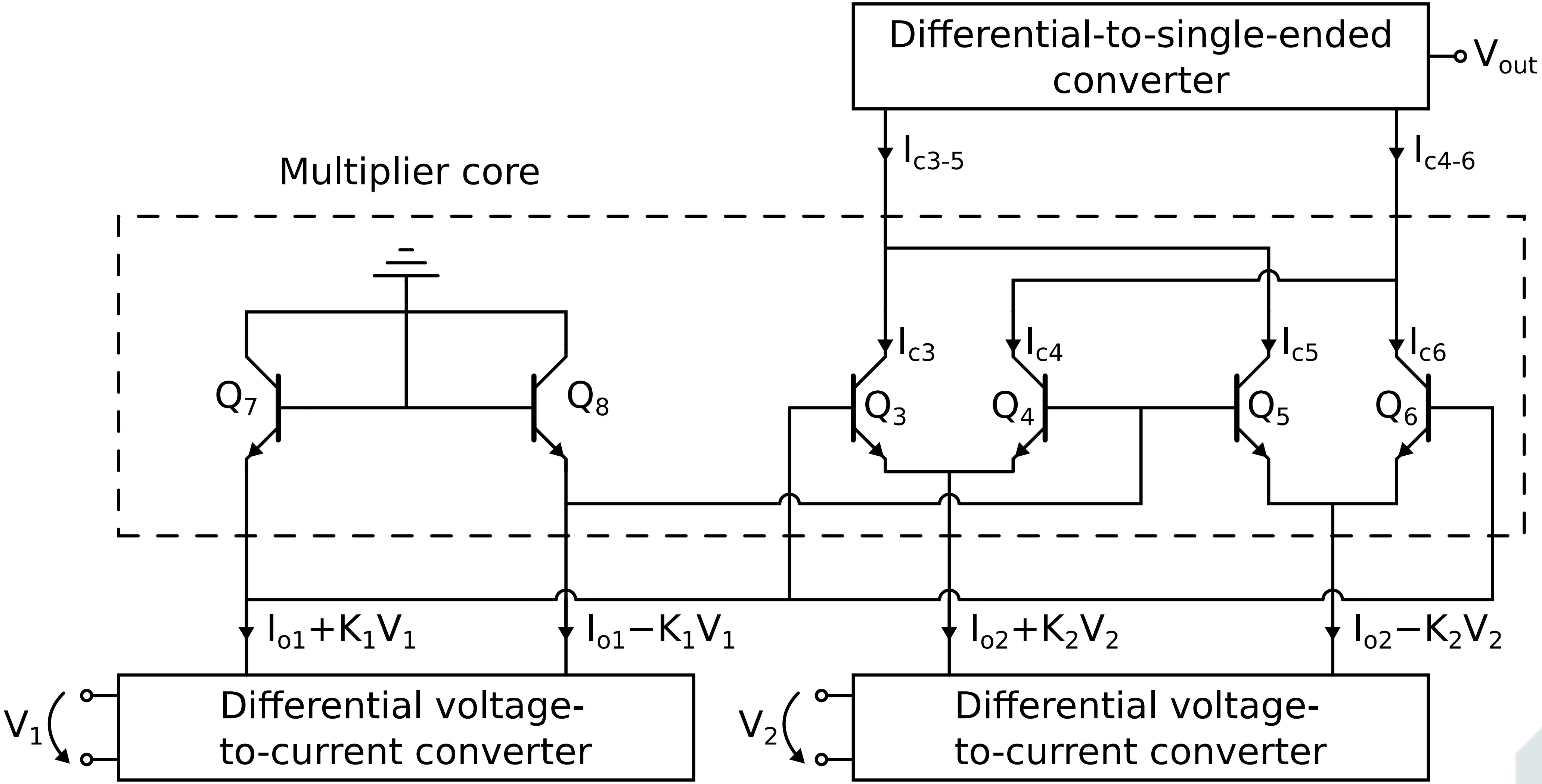
Linear region:

$$-\frac{I_{01}}{k_1} < V_1 < \frac{I_{01}}{k_1}$$

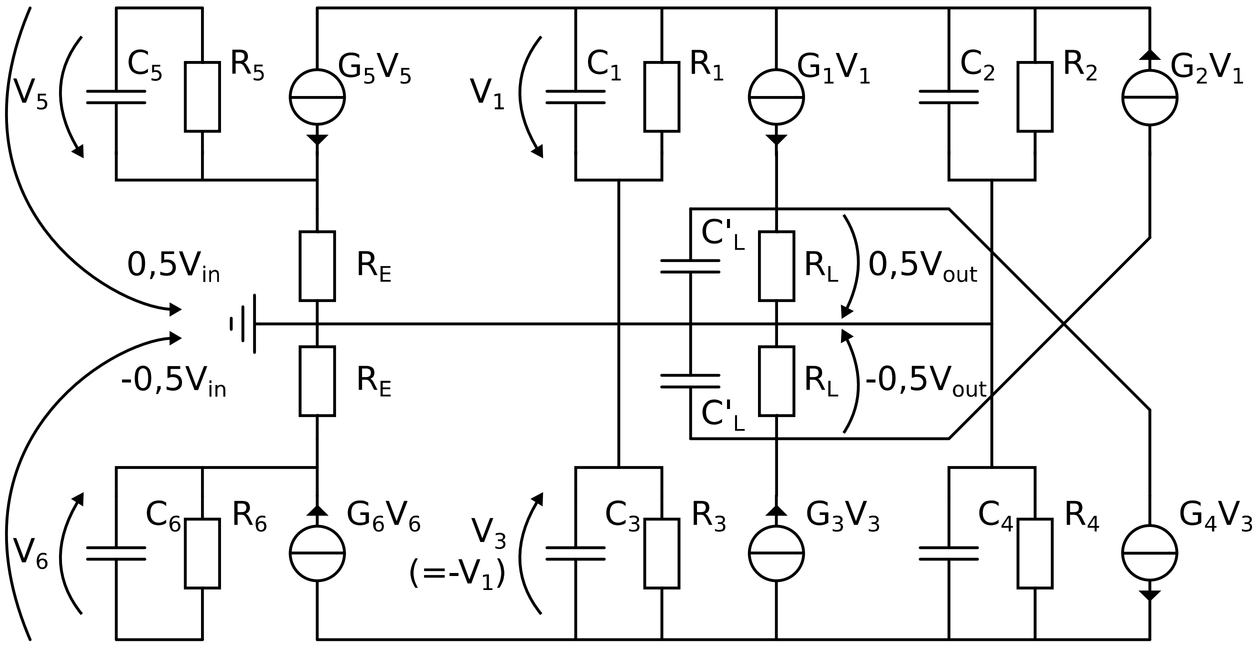
$$-\frac{I_{02}}{k_2} < V_2 < \frac{I_{02}}{k_2}$$



Linearized Gilbert multiplier



The small signal equivalent circuit of the Gilbert multiplier:



The multiplier has three poles:

- The pole associated with the base node of the input transistors:

$$p_1 = -\frac{R_{b5} + R_5 + R_{E1}}{R_5 \left(R_{b5} + R_{E1}\right) C_5} \approx -\frac{1}{R_E g_{m5}} \cdot \omega_{Tmax}$$

- The pole associated with the load:

$$p_2 = -\frac{1}{R_L \left(C_R + C_\mu^1 + C_\mu^3 + C_{js}^1 + C_{js}^3 + C_L\right)} = -\frac{1}{R_L C_L}$$

- The pole associated with th econtrol transistors (cascodes):

$$p_3 = -\frac{g_1 + g_2}{C_1 + C_2} \approx -\omega_T$$

CMOS Bridge-type multiplier

Assume transistors $M_{1\ldots4}$ in linear region.

$$I_{DS} = \beta_1(V_{GS} - V_T - \frac{1}{2}V_{DS})V_{DS}$$

Current equations of transistors $M_{1\ldots4}$:

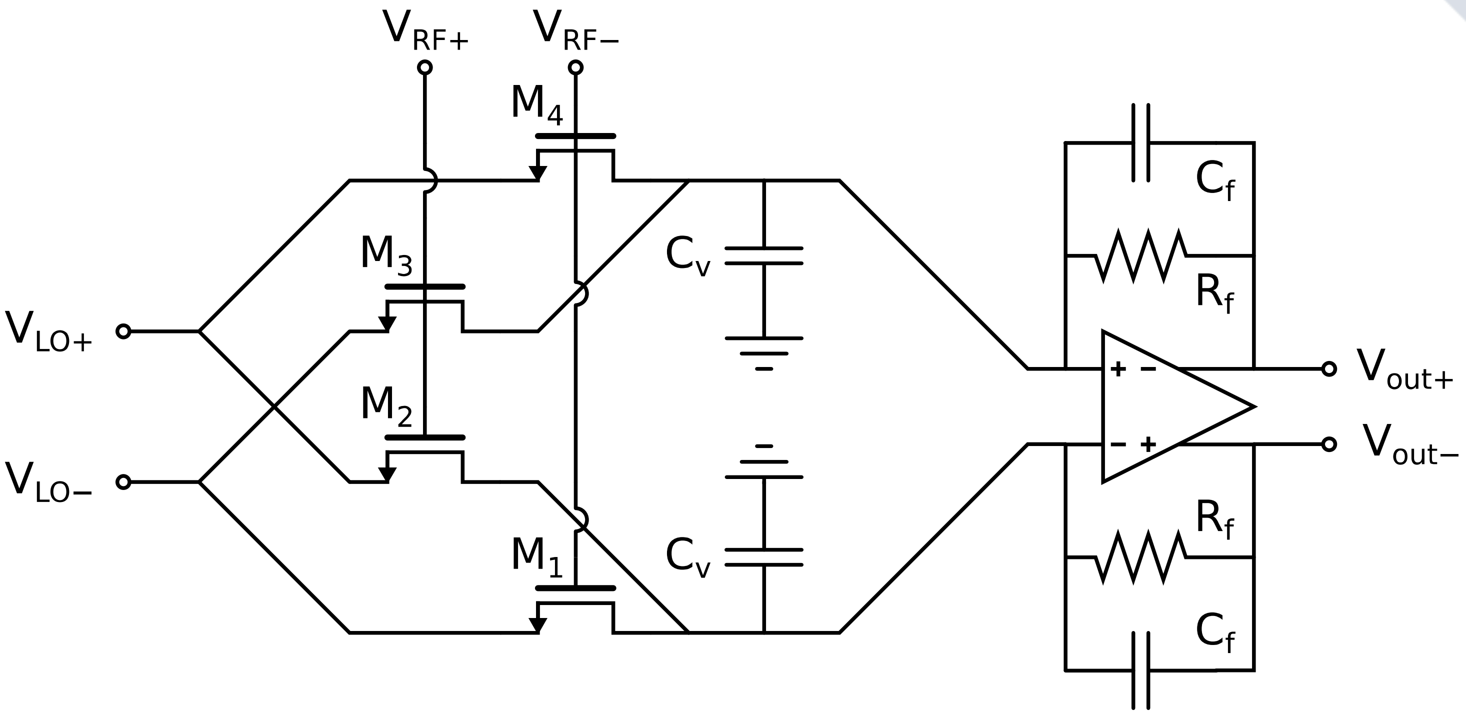
$$I_{DS,1} = \beta_1(V_{RF}^{+} - V_{CM} - V_{Tn,1} - \frac{V_{LO}^{+} - V_{CM}}{2})\cdot(V_{LO}^{+} - V_{CM})$$

$$I_{DS,2} = \beta_2(V_{-RF} - V_{CM} - V_{Tn,2} - \frac{V_{LO}^{-} - V_{CM}}{2})\cdot(V_{LO}^{-} - V_{CM})$$

$$I_{DS,3} = \beta_3(V_{RF}^{+} - V_{CM} - V_{Tn,3} - \frac{V_{LO}^{-} - V_{CM}}{2})\cdot(V_{LO}^{-} - V_{CM})$$

$$I_{DS,4} = \beta_4(V_{RF}^{-} - V_{CM} - V_{Tn,4} - \frac{V_{LO}^{+} - V_{CM}}{2})\cdot(V_{LO}^{+} - V_{CM})$$

V_{RF} and V_{LO} have the same common-mode level V_{CM-}.

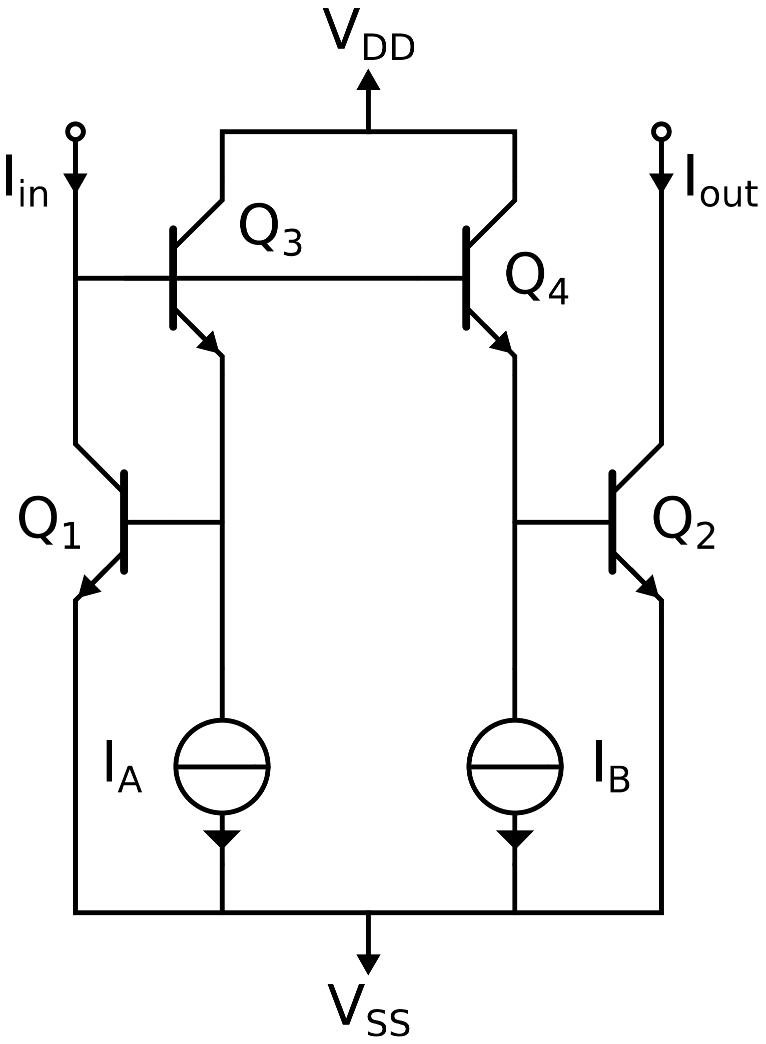


Differential output voltage:

$$V_{OUT}^{+} - V_{OUT}^{-} = -(R_f(I_1 + I_2) - R_f(I_3 + I_4))$$
$$= R_f((I_1 - I_4) - (I_3 - I_2))$$
$$= \beta * R_f(V_{RF}^{+} - V_{RF}^{-})\cdot(V_{LO}^{+} - V_{LO}^{-})$$

Ideal multiplier!

The current multiplier (B. Gilbert)



Voltage equation:

$$V_{BE1} + V_{BE3} = V_{BE2} + V_{BE4}$$

Applying the collector current equation of BJT:

$$V_T \ln\left(\frac{I_{C1}}{I_S}\right) + V_T \ln\left(\frac{I_{C3}}{I_S}\right) = V_T \ln\left(\frac{I_{C2}}{I_S}\right) + V_T \ln\left(\frac{I_{C4}}{I_S}\right)$$

$$\Rightarrow \ln\left(\frac{I_{C1}}{I_B}\right) = \ln\left(\frac{I_{C2}}{I_A}\right)$$

Assuming $I_{in} = I_{C1}$ and $I_{out} = I_{C2}$

we obtain $I_{out} = \frac{I_A}{I_B} I_{in}$