Marine Environments and Its Impact on the Design of Ships and Marine Structures

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ABSTRACT

This paper presents a state-of-the-art summary of information on environmental conditions (wind, waves and currents) relevant to the design and safe operation of ships and marine structures. With increasing application of stochastic prediction techniques in evaluating responses of marine systems for design, results of the author's recent study on stochastic properties and prediction of severe environment pertinent to design application are presented. Included also are presently existing useful information and methodologies for providing a uniform source of information.

1. INTRODUCTION

The four decades following the introduction of the stochastic prediction approach in naval engineering (St. Denis and Pierson 1953) have seen phenomenal advances in the probabilistic analysis and prediction methodologies. As a consequence, the stochastic prediction approach has become an integrated part of modern design technology in naval and ocean engineering. However, in spite of the growing need for the application of the probabilistic approach to design, one area where there is a substantial lack of knowledge in the prediction techniques is the input information (winds, waves and currents) necessary for evaluating responses of marine systems in severe environmental conditions. For example, for the safety of marine systems operating during hurricanes, consideration of simultaneous loadings associated with winds and waves is essential. Nevertheless, little information is currently available on the concurrent severity of wind speed and sea state (significant wave height). Furthermore, information on turbulent wind spectra over a seaway and knowledge of the shape of wave spectra developed from analysis of data obtained during hurricanes are not widely known.

In order to promote more rational design of marine systems, this paper addresses recent information on marine environment (winds, waves and currents) necessary for stochastic prediction of the systems' responses in a seaway. Presented is a culmination of work carried out by the author during the past decade to clarify the stochastic properties of and to develop methods for predicting the severity of environment from stochastic analyses of available measured data. The results of

this work were presented in a variety of publications. In the present paper, that portion of the author's studies germane to design is summarized and combined with other pertinent methodologies to provide a uniform source of information on marine environment for design application.

2. WINDS

2.1 Wind Characteristics and Mean Wind Speed

It may be well to first outline the characteristics of wind speed. Figure 1 shows an example of variation of wind speed with time. As can be seen, unlike the fluctuation of wave profile in the ocean, high frequency wind speeds are superimposed on a variety of low frequency winds. In general, the wind over the sea surface may be considered as a flow having a mean speed as indicated in Figure 1 on which are superimposed turbulent fluctuations of various frequencies.

The mean wind speed is the average of the instantaneous wind speed over a certain time interval, most commonly an average over 10 minute measurements. It is a sustained constant speed, but its magnitude reduces with decrease in height above the sea level since friction associated with roughness of the sea surface retards the mean wind speed but incites the severity of turbulence. The surface-induced turbulence becomes negligible at a certain height, called the gradient height. Since the gradient

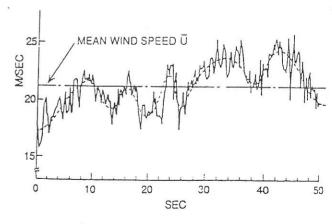


Figure 1. Example of time history of wind speed

height is on the order of 200 meters above the sea surface, the effect of turbulent winds must be considered for the design of marine systems. Thus, in general, we may write the instantaneous wind speed at height z above the sea surface as follows:

$$U_{\tau}(t) = \overline{U}_{\tau} + w_{\tau}(t) \tag{1}$$

where

 U_z = mean wind speed at height z above the sea surface

 $w_z(t)$ = turbulent wind speed.

It is common practice to consider a 10 meter height above the sea level as a standard, and the mean wind speed \bar{U}_{Z} is presented as a function of the mean wind speed at a 10 meter height, \bar{U}_{10} . There are two different conversion formulations between \bar{U}_{Z} and \bar{U}_{10} ; one called the power law velocity profile and the other, the logarithmic velocity profile. For naval and ocean engineering, however, the latter approach appears to be suitable following the results of a series of studies on wind characteristics over a sea surface carried out by Wu (1969, 1980, 1982). That is, the mean wind speed \bar{U}_{Z} can be evaluated as

$$\bar{U}_z = \bar{U}_{10} + u_* \ln(z/10)$$
 (2)

where

 u_* = shear velocity in m/sec = $\sqrt{C_{10}}$ \overline{U}_{10} C_{10} = Surface drag coefficient evaluated from wind measurements at 10 m height = $(0.8 + 0.065 \ \overline{U}_{10}) \times 10^{-3}$ (Wu 1980)

z = height above sea level in meters.

The magnitude of extreme wind speed certainly provides information very useful for the design of marine systems. Here the extreme wind speed is most commonly referred to as the largest mean wind speed estimated from statistical data accumulated over a sufficiently long period of time. In estimating the extreme wind speed from data, care has to be taken in selecting the probability distribution appropriate for analysis of the data.

If the statistical data consists of mean wind speed measured over certain time intervals, (such as every three hours for example), it is the general consensus that the data follow the Weibull probability distribution. An example of this type is shown in Figure 2 in which the cumulative probability of the mean wind speed obtained from 78,958 observations over a 26 year period in the North Sea (Bouws 1978) is plotted on Weibull probability paper. As can be seen in the figure, the data follow the Weibuil probability distribution; and hence, the probable extreme mean wind speed expected in 50 years, for example, can be estimated as 31.5 m/sec through evaluating the return period (see Ochi 1981, for example). The extreme mean wind speed for design consideration with the risk parameter $\alpha = 0.01$ becomes 38.2 m/sec for this example.

Pavia and O'Brien (1986) statistically analyzed the one-year data of mean wind speed obtained by ships over the world's oceans by applying the Weibull probability distribution, and

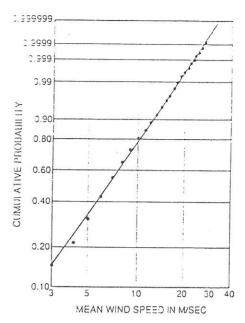


Figure 2. Comparison between cumulative distribution function of mean wind speed and Weibull distribution (Data from Bouws 1978)

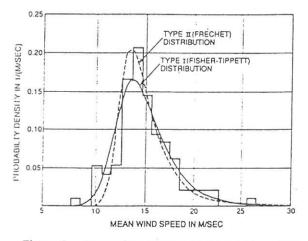


Figure 3. Comparison between histogram of monthly largest mean wind speed and asymptotic Type I, Type II extreme value distributions

presented the seasonal and latitudinal variation of the parameters of the distribution.

Another type of mean wind speed data is that consisting of monthly or yearly maximum values. In this case, theoretical probability distributions exist; hence, estimation of extreme values should be made through the asymptotic extreme value distribution; in particular, the Fisher-Tippett distribution (often called Type I asymptotic extreme value distribution). This distribution was developed for estimating extreme values of exponential-type probability distributions of which the Weibull distribution is an example. Since the sample space of the Fisher-Tippett distribution is $(-\infty, \infty)$, it is necessary to truncate the original distribution at zero and modify it so that the

sample space is limited to (0, ∞). Some use the Fréchet distribution (called Type II asymptotic extreme value distribution) for analysis of the monthly (or yearly) largest mean wind speed; Thom (1967, 1968) for example. This asymptotic distribution was developed for initial probability distributions of the Cauchy-type which have only a finite number of moments.

A comparison of probability density functions of extreme value data and Type I and Type II distributions is shown in Figure 3. The data are the monthly largest mean wind speeds obtained over a period of 97 months by the Coastal Engineering Research Center at a 19 meter height above the sea level at Duck, North Carolina. As can be seen in the figure, the Fisher-Tippett distribution reasonably represents the overall data but the tail of the distribution converges to zero much faster than the Fréchet distribution. The selection of the probability distribution appropriate for analysis of monthly (or yearly) largest mean wind speed data is yet to be determined through many more statistical analyses of data.

2.2 Turbulent Wind and Wind Spectrum

The severity of turbulent wind is often presented by the turbulent intensity defined as the ratio of the root-mean-square (rms) value of the turbulence and the mean wind speed. Another parameter representing the severity of turbulent wind is the gust factor, although its definition is not always consistent. Some define the gust factor as the ratio of the maximum wind speed (including turbulence) to the mean wind speed, while others define it as the ratio of the maximum turbulent speed to the turbulent rms-value.

Since turbulent winds consist of a wide range of frequencies, the gust factor would be extremely large if it is obtained referring to a very high frequency. Hence, it is appropriate to evaluate the gust factor after filtering out the very high frequency components. This subject was clarified by Forristall (1988); he obtained the gust factor as a function of the height and mean wind speed, z/\overline{U} , and the filtering frequency.

The turbulent intensity or the gust factor certainly provides information on the severity of turbulence. It is noted, however, that evaluation of wind-induced forces on marine systems using these parameters essentially amplifies the magnitude of mean wind speed from which the force is obtained by a static or linear dynamic method. In order to incorporate the fluctuating characteristics of turbulent wind into design, it is necessary to consider a stochastic process approach as follows:

The wind-induced drag force can be written from Eq. (1) as

$$F_D = \frac{1}{2} \rho C_D A \left\{ \overline{U} + w(t) \right\}^2 \tag{3}$$

where

 ρ = air density,

 $C_D = \text{drag coefficient},$

A = projected area of the system.

Upon expansion of the above equation, the 2nd and 3rd terms are the drag forces associated with turbulent winds and can be written as

$$F_D = \frac{2\overline{F}_D}{U}w(t) + \frac{\overline{F}_D}{\overline{U}^2}w(t)|w(t)|, \tag{4}$$

where

$$\overline{F}_D = \frac{1}{2} \rho C_D A \overline{U}^2 = \text{drag force associated with mean wind speed.}$$

Note that the squared wind velocity is expressed as w(t) |w(t)| taking into consideration the direction of wind velocity and drag force.

The first term in Eq. (4) represents an ordinary linear system which can be evaluated from a knowledge of the turbulent wind spectrum, while the second term represents the nonlinear wind force due to turbulence which can be evaluated by applying the concept of a squared random process. The energy spectral density of turbulent wind drag force may be expressed as

$$S(\omega) = \left(\frac{2\overline{F}_D}{U}\right)^2 S_{w}(\omega) + \left(\frac{\overline{F}_D}{\overline{U}^2}\right)^2 S_{w|w|}(\omega) \qquad (5)$$

where $S_{w}(\omega) = \text{turbulent wind spectrum}$

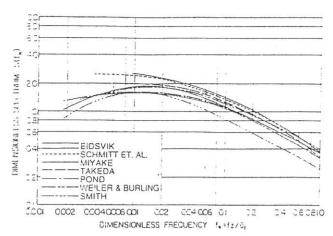
 $S_{w|w|}(\omega) = \text{spectrum for the squared wind velocity}$ w|w|.

It was found that, although the spectral density $S_{w|w|}(\omega)$ is much greater than $S_w(\omega)$, the 2nd term of Eq.(5) is negligibly small in comparison with the 1st term since the value of $(\overline{F}_D / \overline{U}^2)^2$ is extremely small (Ochi et al. 1989d). Hence, it may safely be concluded that as far as the energy spectrum of turbulent drag force is concerned, the nonlinear term can be omitted.

The turbulent wind is a random process fluctuating in three directions; the longitudinal (direction of the wind), and the lateral and vertical directions. However, the magnitude of fluctuations in the longitudinal direction is far larger than that in the other two directions; hence, it may suffice to consider the wind spectrum in the longitudinal direction only for the design of marine systems.

Several turbulent wind spectral formulations have been developed and have been applied to great extent in the design of buildings and suspension bridges, etc. It should be noted, however, that these spectral formulations were developed primarily from wind data measured on land in order to provide information on the vibration of structures with high frequency oscillation characteristics. Recently, it was found that the magnitude of turbulent wind energies at low frequencies obtained from analysis of many measurements over a seaway is much greater than that computed by any of the spectral formulations (Ochi and Shin 1988a). Knowledge of energy at the lower frequencies of the spectrum is very important for the design of some marine systems such as moored vessels, tensionleg platforms, etc., where the occurrence of resonant motions is highly probable.

Figure 4 shows turbulent wind spectra (in dimensionless form) obtained from data measured above the ocean at different geographical locations. It is noted that Eidsvik (1985) presented 17 spectra having different mean wind speeds; however, for simplicity, only the upper and lower boundary curves are shown



Comparison of turbulent wind spectra Figure 4. obtained from data measured over the ocean (From Ochi and Shin 1988a)

in the figure. For the design of marine systems, the average value of the spectral densities shown in the figure is evaluated, and the following spectral formulation (in dimensionless form) has been proposed (Ochi and Shin 1988a):

$$S(f_*) = \begin{cases} 1.75(f_*/0.003)^2 & \text{for } 0 \le f_* \le 0.003 \\ \frac{420f_*^{0.70}}{(1+f_*^{0.35})^{11.5}} & \text{for } 0.003 \le f_* \le 0.1 \\ \frac{838f_*}{(1+f_*^{0.35})^{11.5}} & \text{for } f_* \ge 0.1 \end{cases}$$
(6)

where

dimensionless frequency = fz/\bar{U}_z dimensionless spectrum = $f S(f)/u_x^2$.

Summarizing, it is highly desirable to consider wind loads induced by fluctuating turbulence along with that associated with the mean wind speed for the design of marine systems. Further, use of turbulent wind spectra measured over a seaway is essential for the design.

2.3 Hurricane Winds and Associated Sea Severity

Marine systems can be expected to experience the very critical conditions when they encounter storms, since simultaneous loadings excited by intense wind and waves may endanger their safe operation in a seaway. Examination of wind and sea severities during storms indicate that two different categories of wind and sea conditions should be considered for evaluating the responses of marine systems in storms. One is the growing stage of hurricane-generated seas in which the wind speed is increasing at an extremely high rate but the sea severity is comparatively moderate. In this situation, the wind field is moving fast and hence the wind duration is insufficient for the

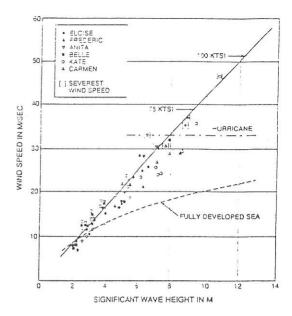


Figure 5. Mean wind speed and significant wave height during the growing stage of hurricanes (From Ochi 1993)

sea to become severe. The other is the relatively slow growing tropical storm (cyclone) in which the wind speeds are not as large as in hurricanes, but the generated sea is in an almost fully-developed condition.

Figure 5 shows the functional relationship between wind speed at a 10 meter height and significant wave height measured by NOAA buoys during the growing stage of hurricanes (Ochi 1993). Here, hurricane is defined as extra tropical cyclone with wind speed over 33.5 m/sec (75 miles per hour). Included in the figure are some hurricanes whose wind speeds did not reach the level of 33.5 m/sec when they passed over the buoys, but the wind severity reached the hurricane level later. As can be seen in the figure, the sea severity increases almost linearly with increase in wind speed during the growing stage of hurricanes. Since no appreciable scatter can be observed in the data, a probability analysis of data may not be required. By drawing the average line in Figure 5, the significant wave height during the growing stage of the hurricane can be simply obtained as a function of the mean wind speed at a 10 m level as,

$$H_{\rm s} = 0.235 \ \overline{U}_{10} \tag{7}$$

Included also in Figure 5 is the functional relationship between wind speed and significant wave height for fullydeveloped seas obtained from Pierson-Moskowitz spectrum. Although the wind speed in the Pierson-Moskowitz spectrum is referred to a 19.6 m height above the sea level, the wind speeds are converted to those at a 10 m height. The relationship is given as

$$H_s = 0.237 \left(U_{10}^2 / g \right) \tag{8}$$

It can be seen in Figure 5 that the sea severity during the growing stage of a hurricane for a given wind speed is much less than that for fully developed seas. This is because the time duration of a given wind speed is extremely short during hurricanes in comparison with that required for fully developed seas.

Another environmental condition which should be considered for marine system design is the sea condition resulting from continuous winds of mild severity blowing for one week or longer then followed by a storm, usually a tropical cyclone. In this case, the sea becomes severe. For example, before the tropical cyclone GLORIA passed near by the NOAA buoy in 1985, the sea condition (significant wave height) in that area was 2.5 - 4.0 meters for 10 days with consistently blowing winds of 7-11 m/sec. That is, the sea had the potential of being easily augmented its severity when GLORIA came to the area.

Figure 6 shows the relationship between the mean wind speed and significant wave height of GLORIA measured by the NOAA buoy. Included also in the figure is the functional relationship applicable for fully-developed seas given in Eq. (8). As can be seen, the wind velocity-sea severity relationship is very close to that applicable for a fully-developed situation in this example. This does not imply, however, that the shape of the wave spectrum is very close to that of a fully-developed sea.

It is known, in general, that a sufficiently long time duration is prerequisite for generating the fully-developed wave spectrum given by Pierson-Moskowitz. For example, at least 42 hours is required at a mean wind speed, \bar{U}_{10} , of 20 m/sec for a sea to become fully-developed and to have the spectral shape given by Pierson et al. (1955). It is not realistic to expect such a long sustained wind speed during a tropical cyclone. Therefore, the shape of the spectrum is different from that for a fully-developed sea as will be discussed in Section 3.2.

In summary, for the design of marine systems in hurricanes and tropical cyclones, we may consider two different situations: (i) the severest wind speed in a hurricane and the associated sea severity for which Eq. (7) may be applied, and (ii) the severest sea expected in tropical cyclones and the associated wind speed for which Eq. (8) may be considered. In the latter case, the

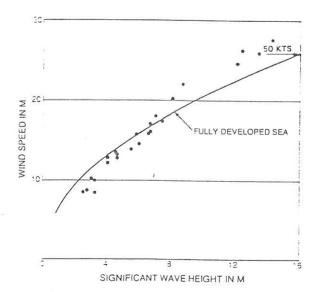


Figure 6. Mean wind speed and significant wave height during the growing stage of tropical cyclone GLORIA (From Ochi 1993)

possibility of pre-existing sea condition, though not severe but lasting for a week or longer prior to the storm, is included in Eq. (8).

As a practical application, suppose a hurricane wind speed of 100 knots (51.4 m/sec) is chosen as the design wind speed for a marine structure in a seaway, then it is necessary to consider (i) a sea of significant wave height of 12.1 meters along with the wind speed of 100 knots (see Figure 5), and (ii) a sea of significant wave height of 16.0 meters along with the wind speed of 50 knots (see Figure 6). In both cases, evaluation of simultaneous loadings excited by wind and waves is highly desirable. For estimating the wind loads, significance of the turbulent wind force was emphasized in the previous section. The shape of wave spectra specifically applicable for sea states in hurricanes will be presented in Section 3.2.

3. WAVES

3.1 Estimation of Lifetime Sea Severity

Estimation of the most extreme sea state which a marine system will encounter in her lifetime provides information vital for the design of the system. The severity of sea state depends to a great extent on the geographical location; hence, it is necessary to statistically analyze long-term significant wave data accumulated in the area(s) where the marine system will be operated.

Since there is no scientific basis for selecting a specific probability law to represent the statistical distribution of sea state, various probability distributions have been proposed which appeared to best fit particular sets of observed data. These include (a) log-normal distribution (Ochi 1978a), (b) modified log-normal distribution (Fang and Hogben 1982), (c) three-parameter Weibull distribution (Burrows and Salih 1986, Mathisen and Bitner-Gregersen 1990) (d) combined exponential and power distribution (Ochi and Whalen 1980) and (e) modified exponential distribution (Thompson and Harris 1972), etc.

Because of the different probability distributions used in the analyses, it is extremely difficult to compare results of analyses of sea state data obtained at various geographical locations, including extreme values. It is highly desirable, therefore, to develop a more rational probability distribution which will well represent most of the data obtained anywhere in the world.

In order to develop a probability distribution appropriate for analysis of significant wave height data, various probability distributions as well as long-term significant wave height data were standardized so that a comparison of the distributions and data could be made under the uniform condition of zero mean and unit variance. From results of the analyses it was found that the following generalized gamma distribution best represents long-term significant wave height data obtained at various geographical locations including those obtained in areas where the water depth is finite (Ochi 1992):

Probability density function

$$f(x) = \frac{c}{\Gamma(m)} \lambda^{cm} x^{cm-1} \exp\{-(\lambda x)^c\}, \quad 0 \le x < \infty$$
 (9)

Cumulative distribution function

$$F(x) = \Gamma\left\{m, (\lambda x)^{C}\right\} / \Gamma(m) \tag{10}$$

As an example, comparison between the cumulative distribution of data obtained off Norwegian coast and that of the generalized gamma distribution is shown in Figure 7. Included also in the figure are the values of the three parameters of the distribution evaluated from the data.

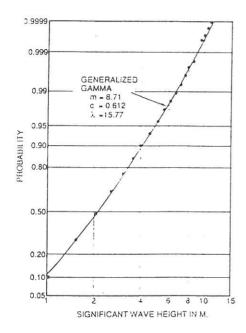


Figure 7. Comparison between cumulative distribution functions of data and generalized gamma distribution (Data from Mathisen and Bitner-Gregersen 1992)

A method to estimate the parameters involved in the generalized gamma distribution is discussed by Stacy and Mihram (1965). Although their method is mathematically correct, some difficulty is often encountered in determining the parametric values in practice. It is noted that since the sample size of significant wave height data is usually quite large (on the order of several thousand), the parameters may be estimated simply by equating the sample moments to the theoretical moments. Furthermore, it was found through statistical analysis of many samples of significant wave height data that the higher order moments (2nd, 3rd and 4th) yield a cumulative distribution which well represents the observed data. Thus, it is suggested that the parameters m, c and λ to be determined from the following equation:

$$E[x_j] = \frac{1}{\sqrt{j}} \frac{\Gamma(m + \frac{j}{c})}{\Gamma(m)}, \quad j = 2, 3 \text{ and } 4$$
 (11)

where $E[x_j]$ = the j-th sample moment obtained from data.

Upon evaluating the parameters involved in the generalized gamma distribution, the extreme sea state expected to occur in a specified time period, 50 years for example, can be estimated by applying order statistics. Here, we may consider (a) the probable extreme sea state most likely to occur, \bar{y}_n , and (b) the extreme sea state for design consideration with the risk parameter α , $\hat{y}_n(\alpha)$. There are two different approaches for estimating the extreme values of the generalized gamma distribution; the one, from the concept of return period; the other, by application of Cramér's asymptotic extreme value statistics (Ochi 1978b). Formulae for estimating these extreme values are summarized below. The extreme values \bar{y}_n and $\hat{y}_n(\alpha)$ are the values of y_n which satisfy each equation.

- (a) Probable extreme sea state (significant wave height)
 - (i) From the concept of return period

$$\frac{1}{1 - \frac{\Gamma\left\{m, (\lambda y_n)^c\right\}}{\Gamma(m)}} = N \tag{12}$$

where N = number of significant wave heights expected in a specified time period.

(ii) From application of Cramér's asymptotic theorem

$$\frac{1}{\Gamma(m)} u_n^{m-1} e^{-u_n} = \frac{1}{N} \left\{ 1 - \frac{1}{u_n} \left(m - \frac{1}{c} \right) \right\}$$
 (13)

where $u_n = (\lambda y_n)^c$. Note that the left side of the above equation is the gamma probability distribution, and hence solution of the equation with respect to u_n can easily be obtained for a given m, c, and λ .

- (b) Extreme sea state for design consideration with risk parameter α
 - (i) From the concept of return period

$$\frac{1}{1 - \frac{\Gamma\left\{m, (\lambda y_n)^C\right\}}{\Gamma(m)}} = \frac{N}{\alpha}$$
(14)

(ii) From applications of Cramér's asymptotic theorem

$$\frac{1}{\Gamma(m)} u_n^{m-1} e^{-u_n} = \frac{\alpha}{N} \left\{ 1 - \frac{1}{u_n} \left(m - \frac{1}{c} \right) \right\}$$
 (15)

As an example, Figure 8 shows the probable extreme and design extreme sea states as a function of time using the Norwegian data shown in Figure 7. The estimation was made using Equations (13) and (15), respectively. As can be seen, the magnitude of extreme values does not increase significantly with increase in time. However, the extreme sea state for design consideration with risk parameter $\alpha=0.01$ is substantially larger (approximately 40 percent) than the probable extreme sea state in this example.

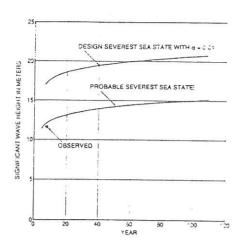


Figure 8. Probable extreme significant wave height and design extreme significant wave height with risk parameter $\alpha = 0.01$ (Data from Mathisen and Bitner-Gregersen 1992)

3.2 Wave Spectra for Design

Spectral Formulations

For predicting responses of marine systems in a seaway, spectral analysis in the frequency domain is most commonly carried out. This approach has a significant advantage over the deterministic approach in that system response can be evaluated for all frequencies including those at which possible resonant conditions with waves will take place. Specifically for design of a marine system, it is ideal to evaluate its responses by employing wave spectra representing various sea conditions in the area(s) where the system will be operated. However, usually this is not done; instead, the analyses are performed by applying conveniently available spectral formulations.

Although quite a few spectral formulations are currently available, it is highly desirable for design consideration to employ formulations developed from analysis of measured data and which yield spectra with minimal specifications (inputs) such as significant wave height or significant wave height and average wave period, etc. Under these restrictions, the number of available spectral formulations is somewhat limited as shown below.

(1) Pierson-Moskowitz Spectrum

The formulation was developed based on approximately 70 spectra associated with fully-developed seas under sustained wind speeds in the North Atlantic (Pierson-Moskowitz 1964). The original formulation is given as a function of the mean wind speed at 19.5 meter height above sea level.

$$S(\omega) = \frac{8.10}{10^3} \frac{g^2}{\omega^5} \exp\left\{-0.74 \left[\frac{g/\overline{U}_{19.5}}{\omega}\right]^4\right\}$$
 (16)

Under the assumption of a narrow-band spectrum, the formula can be presented as a function of significant wave

height, H. That is,

$$S(\omega) = \frac{8.10}{10^3} \frac{g^2}{\omega^5} \exp\left\{-0.032 \left[\frac{g/H_s}{\omega^2}\right]^2\right\}$$
 (17)

Since the spectrum represents a fully developed sea, the following relationships hold between the mean wind speed, significant wave height and wave modal frequency, ω_m , defined as the frequency where the spectrum peaks:

$$H_S = 0.21 \left(\overline{U}_{19.5}^2 / g \right)$$
, and $\omega_m = 0.4 \sqrt{g/H_S}$ (18)

(2) Two-Parameter Spectrum

In order to represent fully as well as partially developed wind-generated seas, Bretschnider (1959) developed a spectral formulation as a function of mean wind speed, mean wave height and average wave period. Although his original formulation has not been used in practice, the concept of a wave spectrum defined by two parameters has significantly contributed in the development of several formulations. Among others, the formulation presented by a function of significant wave height and modal frequency is given by

$$S(\omega) = \frac{1.25}{4} \frac{\omega_m^4}{\omega^5} H_s^2 \exp\left\{-1.25 \left[\frac{\omega_m}{\omega}\right]^4\right\}$$
 (19)

It is noted that the above spectral formulation yields the Pierson-Moskowitz spectrum if ω_m is replaced by the functional relationship given in Eq. (18). In applying the two-parameter spectral formulation for design, it may be well to use available statistical data (contingency tables) which provide information on the frequency of occurrence of the average zero-crossing period T_O for a specified significant wave height. Here, the average period T_O can be converted to the modal frequency by $\omega_m = 4.46/T_O$ assuming the spectrum is narrow-banded.

The following two spectral formulations (ISSC and ITTC spectra) are not based on measured data; instead, they are empirical formulations. However, it may be well to summarize them here, since they have been proposed by the international committees for use in evaluating responses of marine systems.

The International Ships & Offshore Structures Congress (ISSC) proposed the following spectral formulation whose inputs are visually observed wave height and period (ISSC 1964):

$$S(f) = 0.11 H_{\rm v}^2 T_{\rm v} (T_{\rm v} f)^{-5} \exp\left\{-0.44 (T_{\rm v} f)^{-4}\right\}_{(20)}$$

where $H_{\rm v}$, $T_{\rm v}=$ visually observed wave height and period, respectively. Later, in 1979, $H_{\rm v}$ and $T_{\rm v}$ were replaced by significant wave height and significant period, respectively. It is noted, however, that if $T_{\rm v}$ is replaced by $T_m/1.30$, and the formulation is presented as a function of circular frequency, ω , then we can derive

$$S(\omega) = 0.314 \frac{\omega_m^4}{\omega^5} H_s \exp\left\{-1.257 \left[\frac{\omega_m}{\omega}\right]^4\right\} (20')$$

which is identical with the spectral formulation given in Eq.(19).

On the other hand, the International Towing Tank
Conference (ITTC) proposed the following spectral formulation
(ITTC 1978):

$$S(\omega) = 173 \frac{H_s^2}{\bar{T}^4 \omega^5} \exp\left\{-\frac{691}{(\bar{T}\omega)^4}\right\}$$
 (21)

where T = mean wave period. If a narrow-band spectrum is assumed, we can write T = 0.77 $T_m = (0.77)(2\pi)/\omega_m$ By using this relationship, Eq. (21) becomes

$$S(\omega) = 0.316 \frac{\omega_m^4}{\omega^5} \exp\left\{-1.261 \left(\frac{\omega_m}{\omega}\right)^4\right\}$$
 (21')

which is also almost identical with that given in Eq. (19).

(3) Six-Parameter Spectral Family

This formulation carries six parameters, but in reality significant wave height is the only input to the formulation; the other five parameters are given as functions of significant wave height. The formulation yields 11 different spectra (including spectra with double-peaks) for a specified sea severity. The source of the data is the same as that for the Pierson-Moskowitz spectrum, but analysis was carried out on 800 spectra including partially-developed sea states (Ochi and Hubble 1976). The

formulation is given by,

$$S(\omega) = \frac{1}{4} \sum_{j} \frac{\left\{ (4\lambda_{j} + 1) \omega_{mj}^{4} / 4 \right\}^{\lambda_{j}}}{\Gamma(\lambda_{j})} \frac{H_{sj}^{2}}{\omega^{4\lambda_{j}} + 1}$$

$$\times \exp \left\{ -\left[\frac{4\lambda_{j} + 1}{4} \right] \left[\frac{\omega_{mj}}{\omega} \right]^{4} \right\} \qquad (22)$$

where j = 1 and 2 represent the lower and higher frequency components, respectively. Values of the six parameters are given in Table 1 as a function of significant wave height.

The advantage of using a family of spectra for design is that one of the family members yields the largest response for a specified sea severity, while another yields the smallest response with confidence coefficient of 0.95. Hence, by connecting the largest and smallest values, respectively, obtained in each sea severity, the upper and lower-bounded response can be established. It was found that the bounds cover the variation of responses computed using the measured spectra in various locations in the world, although the formulation is based on data measured in the North Atlantic (Ochi 1978c). Thus, it may safely be concluded that the upper-bound of the response evaluated by applying Eq. (22) can be used for design consideration of marine systems.

(4) JONSWAP Spectrum

This formulation is based on an extensive wave measurement program known as the Joint North Sea Wave Project (Hasselmann et al. 1973). It represents wind-generated seas with a fetch limitation, and thereby wind speed and fetch length are inputs to this formulation. The original formulation is given as a function of frequency f in H_{\perp} as follows:

Table 1. Parameters of six-parameter family spectra (From Ochi and Hubble 1976)

	H H e 2		ω _{m1}	ω _{m2}	λ ₁	λ,	
Most probable spectrum	0.84 Н	0.54 H	0.70 e ^{-0.046} H _a	1.15 e ^{-0.039} H _s	3.00	1.54 e ^{-0.062} H _s	
	0.95 H	0.31 Н	0.70 e ^{-0.046 H}	1.50 e ^{-0.046} H _a	1.35	2.48 e ^{-0,102} H _s	
	0.65 Н	0.76 н	0.61 e ^{-0.039} H _s	0.94 e ^{-0.036} H _s	4.95	2.48 e ^{-0.102 H} s	
	0.84 H	0.54 H	0.93 e ^{-0.056} H _s	1.50 e ^{-0.046} H _s	3.00	2.77 e ^{-0.112 H} s	
	0.84 H	0.54 H	0.41 e ^{-0.016} H _s	0.88 e ^{-0.026 H} s	2.55	1.82 е-0.089 н	
U.95 confidence spectra	0.90 н	0.44 H	0.81 e ^{-0.052 H} s	1.60 e ^{-0.033 H} s	1,80	2.95 e ^{-0.105} H _s	
	0.77 н	0.64 Н	0.54 e ^{-0.039 H} s	0.61	4.50	1.95 e ^{-0.082} H _a	
	0.73 H	0.68 н	0.70 e ^{-0.046} H _g	0.99 e ^{-0.039 H} s	6.40	1.78 e ^{-0.069} H _s	
	0.92 H	0.39 н	0.70 e ^{-0.046} H	1.37 e ^{-0.039 H} _s	0.70	1.78 e ^{-0.069 H} s	
	0.84 н	0.54 Н	0.74 e ^{-0.052 H} s	1.30 e ^{-0.039 H} s	2.65	3.90 e ^{-0.085} H _s	
	0.84 H	0.54 H	0.62 e ^{-0.039 H} s	1.03 e ^{-0.030 H} s	2.60	0.53 e ^{-0.069 H} s	

$$S(f) = \alpha \frac{g^2}{(2\pi)^4} \frac{1}{f^5} \exp\left\{-1.25(f_m / f)^4\right\}$$

$$\times \gamma^{\exp\left\{-(f - f_m)^2 / 2(\sigma f_m)^2\right\}}$$
(23)

where

 γ = peak-shape parameter, 3.30 as an average α = 0.076 $(\bar{x})^{-0.22}$

 $\sigma = 0.07 \text{ for } f \le f_m, \text{ and } 0.09 \text{ for } f > f_m$ $f_m = 3.5(g/\bar{U})(\bar{x})^{-0.33}$ $\bar{x} = \text{ dimensionless fetch } = gx/\bar{U}^2, x = \text{ fetch length,}$ and $\bar{U} = \text{mean wind speed.}$

It is noted that the peak-shape parameter y obtained from analysis of the original data varies approximately from 1 to 6 even for a constant wind speed. It is actually a random variable which is normally distributed with mean 3.30 and variance 0.62. Hence, it is possible to generate a family of spectra for various \gamma-values with their weighting factors based on the probability distribution of γ (Ochi 1979a).

It is also noted that Eq. (23) is given as a function of wind speed. It is very convenient in practice if the spectrum is presented in terms of significant wave height. For this, a series of computations was carried out on Eq. (23) for various combinations of fetch length and wind speed, and the following relationship was derived (Ochi 1979a):

$$\bar{U} = k x^{-0.615} H_s^{1.08}$$
 (24)

where k = 83.7 for $\gamma = 3.30$, \overline{U} in m/sec, x in Km, H_s in meters. From Eqs. (23) and (24), the JONSWAP spectrum can be obtained for a specified sea severity and fetch length.

Wave Spectra during Hurricanes

It is of considerable interest to examine the shapes of wave spectra during hurricanes, since the rate of change of wind speed is much faster during hurricanes than with that observed during ordinary storms. Many studies on wave spectra in hurricane seas have been made primarily through hindcasting and forecasting techniques. These include Cardone, Pierson and Ward (1976), Bretschneider and Tamaye (1976), Young and Sobey (1981), Ross and Cardone (1978) and Young (1988), among others. Although these studies provide valuable information for design consideration, it is not possible to draw general conclusions regarding the shape of wave spectra during hurricanes.

One may expect that the shape of wave spectra during hurricanes is extremely random; however, the results of analysis on approximately 400 wave spectra obtained from measured data in the open ocean have shown that there is a consistent trend in their shape, particularly in severe sea conditions. That is, the spectra can be well represented by the JONSWAP spectral formulation, but the values of parameters involved in the formulation are quite different from those given in Eq. (23)

(Foster 1982, Ochi 1993). For instance, from the results of analysis, the scale parameter, α , can be well presented as a function of significant wave height and modal frequency. That

$$\alpha = 4.5 H_s^2 f_m^4$$
 (25)

where the constant carries the units of (sec²/meter)².

Furthermore, the peak-shape parameter, v, substantially deviates from the mean value 3.3 as given in Eq. (23). Figure 9 shows the histogram of the peak-shape parameter which is evaluated by fitting the measured spectra by the JONSWAP formulation. As can be seen in the figure, the values of γ for hurricane-generated seas range from 0.60 to 4.0 which is significantly smaller than that obtained in the JONSWAP Project. It is noted that values of γ greater than 3.0 shown in the figure are primarily from the decay stage of a specific hurricane ---- Hurricane Belle.

Figure 10 shows the peak energy density, $S(f_m)$, obtained from measured data in hurricanes as a function of significant wave height. As can be seen, there is a strong correlation between the peak energy and significant wave height, and it can be presented by

$$S(f_m) = 0.75 H_s^{2.34} (26)$$

On the other hand, the peak energy density of the JONSWAP spectrum can be obtained from Eq. (23) as

$$S(f_m) = \alpha \frac{g^2}{(2\pi)^4 \int_{-\infty}^5 e^{-1.25} \cdot \gamma$$
 (27)

Thus, from Eqs. (25) through (27), the peak-shape parameter can be expressed as a function of significant wave height and modal frequency as follows:

$$\gamma = 9.5 \ H_s^{0.34} f_m \tag{28}$$

By using these relationships, we may derive a wave spectral formulation specifically applicable for hurricane-generated seas

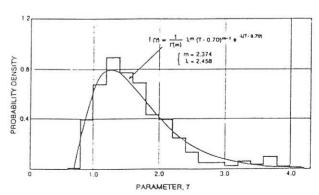


Figure 9. Histogram of peak-shape parameter γ obtained from analysis of hurricane data (From Ochi 1993)

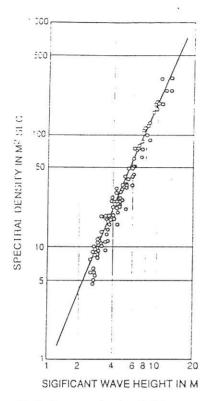


Figure 10. Peak energy density $S(f_m)$ versus significant wave height (From Foster 1982)

in the form of the JONSWAP formulation and as a function of significant wave height and modal frequency as follows:

$$S(f) = \frac{4.5}{(2\pi)^4} (H_5 g)^2 (f_m^4 / f^5) \exp\{-1.25 (f_m / f)^4\}$$

$$\times (9.5 H_5^{0.34} f_m)^{\exp} \{-(f - f_m)^2 / 2(\sigma f_m)^2\}$$
(29)

where, the units are in meters and second. The above formula can also be written as a function of frequency ω as follows:

$$S(\omega) = \frac{4.5}{(2\pi)^4} (H_s g)^2 \left(\omega_m^4 / \omega^5\right) \exp\left\{-1.25(\omega_m / \omega)^4\right\}$$

$$\times \left[\frac{9.5}{2\pi} H_s^{0.34} \omega_m\right] \exp\left\{-\left(\omega - \omega_m\right)^2 / 2(\sigma \omega_m)^2\right\}$$
(29')

Figure 11 shows an example of comparison between Hurricane Eloise and the spectral formulation given in Eq. (29').

Directional Spectrum

The wave spectra discussed in the previous two sections represent the energy of all waves coming from various

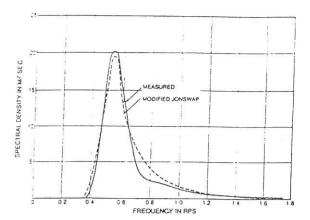


Figure 11. Comparison between wave spectrum obtained during hurricane ELOISE and spectrum formulation given in Eq.(29') (From Ochi 1993)

directions at a given location in the sea. They are called point spectra. If statistical prediction of wave characteristics is carried out using the point spectrum, the assumption is made that all wave energy is propagating in one direction. This is certainly not true in reality; but instead, waves are propagating in different directions although the predominant wave energy is in line with the wind direction.

Ideally, random seas should be presented by a twodimensional spectrum. However, for convenience, the directional wave spectrum, $S(\omega, \theta)$, is commonly presented by the product of the point spectrum, $S(\omega)$, and a directional spreading function, $D(\omega, \theta)$. That is,

$$S(\omega, \Theta) = S(\omega) D(\omega, \Theta)$$
 (30)

where

$$\int_{-T}^{T} D(\omega, \Theta) \ d\Theta = 1$$

Several directional spreading functions have been proposed from results of analyses of measured data. The following formulation developed by Longuet-Higgins et al. (1961) with the parameters determined by Mitsuyasu et al. (1975) appears to be the best formulation currently available to the profession. By using the values of parameters obtained by Mitsuyasu, the spreading function is given by

$$D(\omega, \Theta) = \frac{1}{\pi} 2^{2s-1} \frac{\{\Gamma(s+1)\}^2}{\Gamma(2s+1)} \left| \cos \frac{\Theta}{2} \right|^{2s}$$
$$-\pi < \Theta < \pi$$
(31)

where

$$s = \begin{cases} 11.5 \ \overline{\omega}^{-2.5} & \text{for } \overline{\omega} \ge \overline{\omega}_m \\ \\ 11.5 \ \overline{\omega}^5 \ \overline{\omega}_m^{-7.5} & \text{for } \overline{\omega} \le \overline{\omega}_m \end{cases}$$

$$\bar{\omega} = \text{dimensionless frequency} = \omega \, \bar{U}_{10} / g$$

$$\overline{\omega}_m = {
m dimensionless\ modal\ frequency} = \omega_m\,\overline{U}_{10}$$
 / g

For a quick evaluation of the spreading function for design consideration, it may suffice to use the following two spreading functions recommended by the International Ship and Offshore Structures Congress:

$$D(\omega, \Theta) = \frac{2}{\tau} \cos^2 \Theta \quad |\Theta| \le \tau / 2 \tag{32}$$

or

$$D(\omega, \Theta) = \frac{8}{3\pi} \cos^4 \Theta \quad |\Theta| \le \pi / 2$$
 (33)

3.3 Stochastic Properties of Random Waves

Wave Height

For design of marine systems, it is common practice to consider the following Rayleigh probability distribution to represent the statistical properties of ocean waves:

$$f(x) = \frac{2x}{R} e^{-\frac{x^2}{R}}, \quad 0 \le x < \infty$$
 (34)

where

$$R = \begin{cases} 2m_0 & \text{for } x \text{ is amplitude} \\ 8m_0 & \text{for } x \text{ is height} \end{cases}$$

 m_0 = area under the wave spectrum.

Strictly speaking, however, the probability density function given in Eq. (34) is valid for waves in deep water with a narrow-band spectrum which implies that the wave profile has a single peak or trough during a one-half cycle and waves are considered to be a Gaussian random process. The latter assumption is admissible even for very severe wave conditions if the water is sufficiently deep.

On the other hand, waves in a sea of finite water depth may be considered as a Gaussian random process only when the sea is mild. The wave profile generally transforms to that of a non-Gaussian random process with increase in sea severity due to limited water depth. The results of analysis of wave data obtained during storms (including hurricanes) in nearshore areas in the Gulf of Mexico, where the water depth is approximately 100 meters, show non-Gaussian properties and the Rayleigh distribution substantially overpredicts the magnitude of the highest recorded waves (Haring et al. 1976, Foristall 1978).

For predicting the magnitude of wave peaks for a non-narrow-band spectrum, a probability density function was developed by Cartwright and Longuet-Higgins (1956). Although the probability density functions applicable to wave amplitude with narrow-band and non-narrow-band spectra are different, it can be proved that the magnitude of extreme wave amplitudes

expected in a specified time are the same (Ochi 1973). That is, the probable extreme wave amplitude which is most likely to occur in T-hours is given by,

$$\begin{pmatrix}
\text{Probable extreme} \\
\text{amplitude}
\end{pmatrix} =
\sqrt{2 \ln \left\{ \frac{(60)^2}{2\pi} \frac{T}{m_0} \right\} \sqrt{m_0}}$$

(35)

where T = time in hours

 m_0 = area under the wave spectrum

 $m_2 = 2$ nd moment of wave spectrum.

On the other hand, the design extreme wave amplitude with risk parameter α is given by

$$\left(\begin{array}{c} \text{Design extreme} \\ \text{amplitude} \end{array}\right) = \sqrt{2 \ln \left\{\frac{(60)^2 T}{2\pi\alpha} \sqrt{\frac{m_2}{m_0}}\right\}} \sqrt{m_0}$$
(36)

An example of extreme wave heights computed by using Equations (35) and (36) for wave spectrum obtained in the North Atlantic is shown in Figure 12 as a function of T.

It is generally assumed that wave height in random seas is equal to twice the wave amplitude. This assumption is not correct in a strict sense; however, it appears that the prediction of the magnitude of wave height will not be significantly affected by removal of this assumption. A detailed study on this subject was made by Tayfun (1981) in which the probability distribution applicable to the sum of successive peaks and troughs was derived.

It is also generally assumed that all individual wave amplitudes are statistically independent. In reality, however, the magnitude of each wave amplitude is by and large influenced by the severity of the previous wave amplitude. A theoretical analysis on the effect of statistical dependence of wave peaks on the magnitude of extreme wave height was carried out under the condition that the peaks are subject to the Markov chain condition (Ochi 1979b). The results of this study show that the

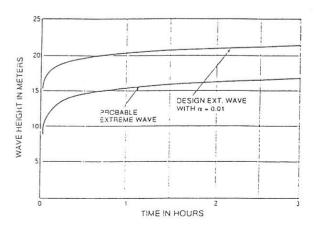


Figure 12. Extreme wave heights as a function of time

effect of statistical dependence is prominent in predicting extreme wave height for waves whose spectral band width parameter is less than 0.5 and that extreme wave heights with the Markov chain concept are approximately 10 percent greater than those predicted without the Markov chain concept.

Although the prime interest in statistical wave characteristics lies in the magnitude of the peaks and troughs or in peak to trough excursions crossing the zero-line (wave height), it is of interest to evaluate the statistical properties of every excursion, x₁, x₂, — illustrated in Figure 13. Statistical analysis of the vertical distance (displacement) between successive peak to trough excursions or vice versa is called half-cycle excursion ansiysis. As can be seen in the figure, the half-cycle excursions can be categorized as those crossing the zero-line and those located above or below the zero-line. Furthermore, there are ascending and descending excursions for each case. Buckley (1984, 1991) analyzed wave records and constructed diagrams called half-cycle matricies (HACYM) indicating the frequency of magnitude associated with ascending and descending halfcycle excursions. His approach provides significant insight in clarifying the statistical characteristics of random waves.

In order to evaluate the statistical properties (including extreme values) of half-cycle excursions, the probability distribution for an arbitrarily given wave spectrum was derived by Ochi and Eckhoff (1984a) based on the Gaussian wave assumption. In their approach, the probability density function applicable to excursions crossing the zero-line (Type I excursion) and that without crossing the zero-line (Type II excursion) are multiplied by theoretically derived weighting factors. Formulae necessary for computing the probability density function are summarized as follows:

The probability density function of excursions crossing the zero-line, defined as Type I excursion is the same as that given in Eq. (34). For convenience, the probability density function is denoted here as $h_1(\zeta)$ and is expressed in dimensionless form by letting $x = \sqrt{R/8} \zeta$ in Eq. (34). The probability density function of Type II excursions can be evaluated by

$$h_{2}(\zeta) = \int_{0}^{\infty} \frac{\left[f_{1}(\mu)\right]_{\mu = \zeta + \nu}}{1 - \left[F_{1}(\mu)\right]_{\mu = \nu}} \left[f_{1}(\mu)\right]_{\mu = \nu} d\nu \quad (37)$$

where $f_1(\mu)$ and $f_2(\mu)$ are density functions of positive maxima (Point A in Figure 13) and positive minima (Point B in the figure), respectively, given by

$$\begin{cases}
f_1(\mu) \\
f_2(\mu)
\end{cases} = \frac{2}{1 \pm \sqrt{1 - \varepsilon^2}} \left[\frac{\varepsilon}{\sqrt{2\pi}} e^{-\frac{\mu^2}{2\varepsilon^2}} \right] \\
\pm \sqrt{1 - \varepsilon^2} \mu e^{-\frac{\mu^2}{2}} \times \left\{ 1 - \phi \left[\mp \frac{\sqrt{1 - \varepsilon^2}}{\varepsilon} \mu \right] \right\} \right]$$
(38)

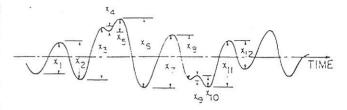


Figure 13. Definition of half-cycle excursion

where
$$F_{1}(\mu) = \int_{0}^{\mu} f_{1}(\mu) d\mu$$

$$\epsilon = \left(1 - m_{2}^{2} / m_{0} m_{4}\right)^{1/2}$$

$$\Phi(\mu) = \int_{-\infty}^{\mu} \frac{1}{\sqrt{2\pi}} \exp\left\{-\left(z^{2} / 2\right)\right\} dz$$

 $m_{j} = j$ -th moment of wave spectrum.

By taking the frequency of occurrence of Type I and Type II excursions into consideration, the probability density function of all excursions for the short-term waves can be written as.

$$h(\zeta) = \frac{p_1^2}{p_1^2 + p_2} h_1(\zeta) + \frac{2p_2}{p_1^2 + p_2} h_2(\zeta)$$
 (39)

where

$$\begin{split} p_1 &= \sqrt{1 - \epsilon^2} \ / \left(1 + \sqrt{1 - \epsilon^2} \right) \\ p_2 &= \left(1 - \sqrt{1 - \epsilon^2} \right) / \left(1 + \sqrt{1 - \epsilon^2} \right) \end{split}$$

Figure 14 shows a comparison between the theoretical probability density function and the histogram constructed from a wave record of significant wave height 12.3 meter during

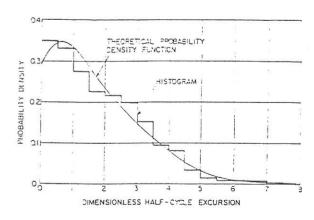


Figure 14. Comparison between observed histogram of half-cycle excursions (dimensionless) and theoretical probability density function (From Ochi and Eckhoff 1984a)

Hurricane Camille. The data show weakly non-Gaussian properties in this sea severity. The excursions are presented in dimensionless form by dividing by $\sqrt{m_O}$, where m_o is the area under the spectrum. It is noted that half-cycle analysis yields somewhat greater extreme values (approximately 5-6 percent) than those computed by applying Eqs. (35) and (36). It is also noted that the probability density function given in the above equation can also be used for evaluating lifetime responses of a marine structure such as fatigue loads (see Section 3.5).

Wave Height and Associated Period

The statistical properties of wave height discussed in the previous section certainly provides valuable information for the design of marine systems. However, more important information is the joint statistical distribution of wave height and period. This is of particular significance for the design of floating structures, because the magnitude of response of a system may become critical if the wave period is close to the system's natural period; namely, under the resonant condition. In designing a floating system, therefore, it is very important to consider extreme wave height with periods that are critical for the system. This requires the joint statistical distribution of individual wave height and associated period. It is also noted that the joint statistical distribution provides information necessary for the time domain prediction of a marine system's response in a seaway.

The joint probability distribution of wave height and period has been developed by several researchers. Longuet-Higgins developed a joint distribution in 1975 but his distribution function was superseded by his second paper (1983) in which the asymmetry in the probability distribution of wave period is incorporated, resulting in good agreement with observed data. Cavanie et al. (1976) developed a joint distribution function of wave peaks and associated time interval between peaks. Although the shape of this probability density function resembles that for wave amplitude and period, there is a substantial difference between them. The Cavanie's joint density function has contributed significantly in derivation of further advanced statistical distribution functions such as probability functions applicable for evaluating frequency of occurrence of two different types of breaking waves in random seas. Lindgren (1972), Lindgren and Rychlik (1982) also developed distribution functions which are mathematically elaborate and very complex, requiring a great deal of computation in practice. Here, for design consideration of marine systems, the Longuet-Higgins joint distribution is presented since it is relatively simple and demonstrates the joint statistical properties of wave height and period with sufficient accuracy.

Longuet-Higgins' joint probability density function of wave amplitude and period is given in dimensionless form by

$$f(\xi,\eta) = L(\nu) \frac{2}{\sqrt{\pi} \nu} \left(\frac{\xi}{\eta}\right)^2 e^{-\xi^2 \left\{1 + \left(1 - \frac{1}{\eta}\right)^2 / \nu^2\right\}}$$

$$0 \le \xi < \infty, \quad 0 \le \eta < \infty$$
(40)

where $L(\nu) = \text{normalization factor} = 1/2 \left\{1 + (1 + \nu^2)^{-1/2}\right\}$

 $\frac{1}{T} = \text{mean period} = 2\pi m_0 / m_1$ $\frac{1}{T} = \text{spectral width parameter} = \sqrt{\frac{(m_2 m_0 / m_1^2) - 1}{(m_2 m_0 / m_1^2) - 1}}$

It is convenient in practice to convert the above joint probability distribution to that of wave height and period. For this, let us define the dimensionless wave height $\zeta = H/\sqrt{m_o}$. The density function becomes,

j-th moment of the spectrum.

$$f(\zeta,\eta) = \frac{L(\nu)}{4\sqrt{\pi}\nu} \left(\frac{\zeta}{\eta}\right)^2 e^{-\frac{\zeta^2}{8} \left\{ 1 + \left(1 - \frac{1}{\eta}\right)^2 / \nu^2 \right\}}$$

$$0 \le \zeta < \infty, \quad 0 \le \eta < \infty$$
(41)

This joint density function will be applied for evaluating the frequency of occurrence of breaking waves in Section 3.4. Figure 15 shows an example of the joint probability density function given Eq. (41) for the spectrum width parameter $\nu = 0.4$.

Longuet-Higgins suggests that Eq. (41) is applicable for data with spectral width parameter $\nu \leq 0.6$; however, it is reported that good agreement with measured data was obtained up to $\nu = 0.7$ in mild seas (Srokosz and Challenor 1987) and up to $\nu = 0.4$ in severe seas (Shum and Melville 1984).

The probability density function of wave period can be derived as the marginal probability density function of Eq. (40)

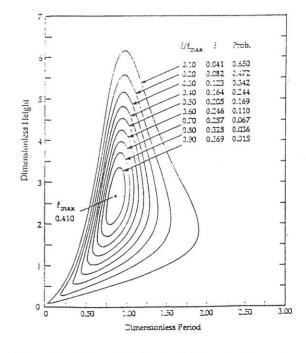


Figure 15. Joint probability density function (Longuet-Higgins 1983) of wave height and period (dimensionless) for $\nu = 0.4$

or Eq. (41). Longuet-Higgins shows the following formula for dimensionless period:

$$f(\eta) = \frac{L(\nu)}{2 \nu \eta^2} \left\{ 1 + \left(1 - \frac{1}{\eta}\right)^2 / \nu^2 \right\}^{-\frac{3}{2}}, \quad (42)$$

$$0 \le \eta < \infty$$

It is noted that, unlike the statistical prediction of wave height, wave period needed for design of a marine system is neither the extreme wave period (defined as the longest wave period) nor the probable wave period most likely to occur in a given sea; instead, it is the most probable wave period associated with the extreme wave height. Therefore, for evaluating design wave period, the probability density function of wave period alone is not sufficient; instead, the conditional probability density function of wave period for a given height is required. It is given by

$$f(\eta \mid \zeta) = \frac{1}{2\sqrt{2\pi}\nu} \frac{\zeta}{\eta^2} \frac{1}{\Phi(\zeta/2\nu)}$$

$$\times \exp\left\{-\frac{\zeta^2}{8} \left[1 - \frac{1}{\eta}\right]^2/\nu^2\right\} \tag{43}$$

From the above density function the most probable wave period associated with extreme wave height can be evaluated.

It is also noted that the conditional distribution of wave height for a specified wave period can be extremely useful for the design of marine systems. This is because the extreme wave with period critical for possible resonant motion of the system is significant, and it can be estimated from the conditional distribution. The conditional distribution of wave height for a specified period can be derived from Eqs. (41) and (42) as follows:

$$f(\zeta | \eta) = \frac{\zeta^2}{4\sqrt{2\pi}} \left\{ 1 + \left(1 - \frac{1}{\eta} \right)^2 / \nu^2 \right\}^{3/2}$$

$$\times \exp \left\{ -\frac{\zeta^2}{8} \left[1 + \left(1 - \frac{1}{\eta} \right)^2 / \nu^2 \right] \right\}$$
 (44)

3.4 Special Wave Events

Marine systems operating in random seas often encounter unusual wave conditions which may easily endanger their safety or cause structural damage. These unusual wave conditions, called special wave events, include breaking waves, group waves and freak waves. Although the existence of freak waves has been reported (Sand, et al. 1989, for example) the details of freak waves are still not clearly known; hence, the following discussion is limited to breaking waves and group waves.

Breaking Waves

The breaking wave phenomenon observed in the open ocean

occurs whenever a momentarily high wave crest reaches an unstable condition. It occurs intermittently, and the frequency of occurrence depends on the sea severity and the shape of the wave spectrum. The significance of information on breaking waves cannot be overemphasized in the design of marine systems, since breaking waves are always associated with steep waves and they exert the largest wave-induced forces in the form of impacts on marine structures. Hence, it is highly desirable to evaluate the frequency of occurrence and severity of breaking wave impact force for design.

It is known from Mitchell's theory that wave breaking occurs when wave height exceeds 14.2 percent of the Stokes limiting wave length which is 20 percent greater than that of ordinary sinusoidal waves of the same frequency. This criterion is correct in so far as breaking of regular waves is concerned. It was found, however, that the breaking criterion for irregular waves is substantially different from that for regular waves in that irregular waves are more susceptible to breaking due to instability characteristics. From the results of experiments on breaking of irregular waves generated in the tank, the following relationship between wave height H and period T was derived as the criterion for breaking of irregular waves (Ochi and Tsai 1983):

$$H \ge 0.020 \ g T^2$$
 (45)

The validity of this criterion has been further confirmed by experimental studies carried out by Xu et al. (1986) and Ramberg and Griffin (1987).

Wave breaking in random seas occurs under two different situations; one that takes place along an excursion crossing the zero-line, the other that occurs along an excursion above the zero-line. The excursions may be called Type I and Type II, respectively, as illustrated in Figure 16. An analytical method to evaluate the frequency of occurrence of breaking waves for these two types of excursions for a given wave spectrum was developed by applying the joint probability distribution of excursions and time intervals (Ochi and Tsai 1983). Results of computations have shown that the frequency of occurrence of breaking waves associated with Type I excursion is much greater than that associated with Type II excursions. Furthermore, Type I breaking induces a severe impact force when it strikes a marine structure. Therefore, it may be sufficient to consider the breaking associated with Type I excursion only for the design of marine systems. A method to

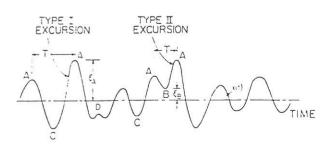


Figure 16. Explanatory sketch of Type I and Type II excursions

evaluate the frequency of occurrence of breaking waves for a given spectrum by applying the joint probability distribution of wave height and period given in Eq. (41) is as follows:

Let us first write the breaking criterion given in Eq. (45) in dimensionless form following the definition given in Eqs. (40) and (41). That is,

$$\zeta > (0.02 g) \left(\frac{\overline{T^2}}{\sqrt{m_O}}\right) \eta^2 \tag{46}$$

In order to evaluate the frequency of occurrence of breaking waves, the criterion given in Eq. (46) is incorporated in the joint probability density function, $f(\zeta, \eta)$, as shown in Figure 17. The location of the line representing Eq. (46) depends on the shape of the spectrum. In the figure, breaking takes place for wave heights exceeding the criterion line for a given period (this region is denoted by the breaking zone in the figure). Therefore, the volume of the joint probability density function in the breaking zone represents the probability of occurrence of wave breaking in a given sea. It can be evaluated by

$$\left(\begin{array}{c}
\text{Probability of} \\
\text{wave breaking}
\right) = \int_{0}^{\infty} \int_{0}^{\infty} f(\zeta, \eta) \, d\zeta \, d\eta \qquad (47)$$

where
$$a = 0.02g \frac{\overline{T}^2}{\sqrt{m_o}} \eta^2$$

It may be of interest to present the effect of spectral shape on breaking waves. It can be proved theoretically that the frequency of occurrence depends on the magnitude of the 4th moment of the wave spectrum. Figure 18 shows the probability

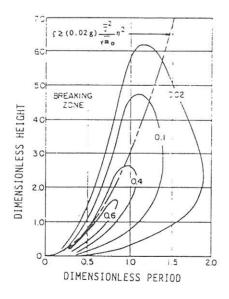


Figure 17. Joint probability density functions of wave height and period (dimensionless) and wave breaking line (From Ochi and Tsai 1983)

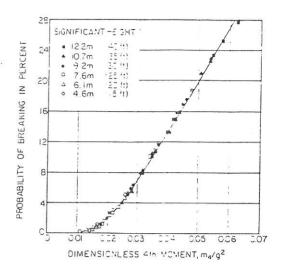


Figure 18. Probabilities of occurrence of breaking waves plotted against dimensionless 4th moments of wave spectra (From Ochi and Tsai 1983)

of occurrence of breaking waves of the six-parameter wave spectral family in various sea states plotted against the dimensionless 4th moment of each spectrum. As can be seen in the figure, the frequency of breaking increases significantly with increase in the 4th moment irrespective of sea severity. This implies that more breaking can be expected in a sea with substantial energy in shorter length waves.

As stated earlier, breaking waves exert large forces on marine structures in the form of impacts. A method for evaluating the magnitude of impact pressure (including extreme values) caused by wave breaking on the front face of a circular cylinder in a random sea was developed by Ochi and Tsai (1984b) for the two different types of breakings shown in Figure 16. The method applicable to Type I breaking will be outlined below for application to marine system design.

In evaluating the magnitude of impact pressure on a marine structure caused by breaking waves, impacts caused by two different conditions are considered. One is the impact associated with waves breaking in close proximity to the structure, the other is an impact caused by waves approaching the structure after they have broken. These impacts are called breaking wave impact and broken wave impact, respectively.

The functional relationship between impact pressure and wave period was found from an analysis of experimental data to

$$p = \rho C \left(1.092 \frac{gT}{2\pi} \right)^2 \tag{48}$$

where
$$C = constant =$$

$$\begin{cases}
1.38 \text{ for breaking wave impact} \\
1.34 \text{ for broken wave impact.}
\end{cases}$$

As can be seen in the above equation, the impact pressure is almost the same for breaking wave impact and for broken wave impact. We may write the above equation in terms of the dimensionless wave period $\eta = T/\overline{T}$, where \overline{T} is the mean period defined in Eq. (40). That is,

$$p = \phi \eta^2$$
, where $\phi = \rho C \left(1.092 \frac{gT}{2\pi}\right)^2$. (48')

The probability distribution of impact pressure can be derived by using the relationship given in Eq. (48') in conjunction with the probability distribution of wave period. Care has to be taken, however, that the probability distribution of wave period is not that for periods of all waves in a given sea but for periods associated with breaking wave only. Therefore, it is necessary to derive the probability density function of breaking wave period from the joint probability density function of wave height and period taking the breaking criterion into consideration. For this, we first evaluate the joint probability density function in the breaking zone, denoted by $f_*(\zeta, \eta)$, from Eqs. (41) and (47) as follows:

$$f_*(\zeta, \eta) = \frac{f(\zeta, \eta)}{\left(\begin{array}{c} \text{Probability of breaking} \\ \text{waves given in Eq.(47)} \end{array}\right)}$$
 (49)

where
$$(0.02g) \frac{\overline{T}^2}{\sqrt{m_o}} \eta^2 < \zeta < \infty, \quad 0 < \eta \le \infty$$

Then, the probability density function of breaking wave period (in dimensionless form) denoted by $f_*(\eta)$, can be obtained as the marginal density function of Eq. (49). That is,

$$f_*(\eta) = \int_{a}^{\infty} f_*(\zeta, \eta) d\zeta \tag{50}$$

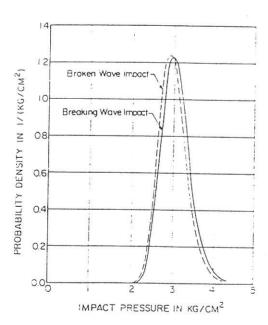


Figure 19. Probability density functions of extreme impact pressure in 50 breaking waves in a sea of significant wave height 10.8 m (From Ochi and Tsai 1984b)

where
$$a = 0.02 g \frac{\overline{T}^2}{\sqrt{m_O}} \eta^2$$

From Eqs. (48') and (50), the probability density function of impact pressure can be derived by applying the technique of transformation of random variables. That is,

$$f(p) = \left[f_*(\eta) \right] \times \left(1 / 2 \sqrt{p \phi} \right) \tag{51}$$

where $\eta = \sqrt{p/\phi}$

Various statistical information (including extreme values) on impact pressure associated with breaking waves can be obtained from the probability density function given in Eq. (51). An example of the probability density function of impact pressure associated with wave breaking on the front face of a circular cylinder in a sea of significant wave height 10.8 meter is shown in Figure 19. In this example, the probability density functions for breaking wave impact and broken wave impact are computed including both Type I and Type II breakings. As can be seen in this example, the probability density functions for breaking and broken wave impacts are almost identical.

Group Waves

Another special wave event observed in random seas is a sequence of high waves having nearly equal periods commonly known as group waves. Group waves can cause serious problems for the safety of marine systems when the periods of individual waves in the group are close to the marine system's natural motion period. This is not because wave height is exceptionally large but occurs primarily because of motion augmentation due to resonance with the waves which can result in capsizing. As another example of potential problem imposed by group waves is that of a moored marine system which tends to respond to successive high waves with low frequencies which induce a slow drift oscillation of the system resulting in large forces on the mooring lines.

One way to evaluate the various statistical properties of group waves is to consider the phenomenon as a level-crossing problem associated with the envelop of a random process as shown in Figure 20. However, the mathematical derivation of the probability density function applicable to the time duration ($\tau_{\alpha+}$ in Figure 20) and the time interval between successive groups (τ_{α} in Figure 20) is extremely difficult. Hence, only the

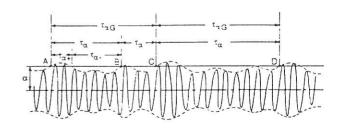


Figure 20. Level crossing of envelope of a random process

mean (average) value of the number of waves in a group and mean value of the length of time the wave group persists, etc. are evaluated; Nolte and Hsu (1972), Ewing (1973) and Goda (1976), among others.

It should be noted that the basic concept of identifying a wave group by the exceedance of the envelope crossing is not a sufficient condition. If the time duration, $\tau_{\alpha+}$, shown in the figure is relatively short, there may be only one wave crest or none in time $\tau_{\alpha+}$ which obviously does not constitute a wave group though the wave envelope exceeds the specified level. Numerical computations on group waves indicate that only 10-20 percent of the crossing time $\tau_{\alpha+}$ contains at least two wave peaks. Therefore, consideration of (i) exceedance and (ii) at least two wave peaks in time $\tau_{\alpha+}$ must be considered in evaluating the statistical properties of group waves.

The probability density function of the time duration $\tau_{\alpha+}$, taking into consideration the above two conditions, was analytically developed by Ochi and Sahinoglou (1989a). Since derivation of the probability density function for a given wave spectrum is rather complicated, only the procedure for evaluating the stochastic properties of group waves is outlined below.

First, the probability density function applicable to the time duration $\tau_{\alpha+}$ associated with the envelope exceeding a specified level α for a given spectrum is derived. Then, the density function is modified so that it addresses only with the time interval during which two or more wave peaks are present. This leads to the desired probability function of the time interval associated with a group, and therefrom the probability of occurrence of a specified number of peaks in a group can be evaluated. Figure 21 shows an example of the probability density function associated with wave groups exceeding specified levels for a two-parameter wave spectrum having a significant wave height of 10 meters and a modal frequency of 0.42 rps.

As an example of the application of the formulations on group waves, computations were carried out using wave data measured at sea. The data were taken in the North Sea off Norway by the Norwegian Marine Technology Research Institute. Table 2 shows a comparison between observed and

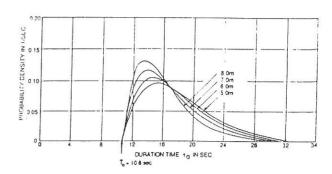


Figure 21. Probability density function of time duration associated with wave group at various levels (From Ochi and Sahinoglou 1989a)

computed average time durations associated with the wave envelop exceeding a specified level. Comparisons are made for two categories --- one for time duration of level crossing in which there may be none or one or more peaks present, the other for time duration of level crossing in which at least two or more peaks are present (wave group). Included also in the table are the results computed by a formula commonly used for evaluating the average time duration derived from the joint probability distribution of envelop and its velocity. As can be seen in the table, the computed average time duration associated with a wave group reasonably agrees with that obtained from the records. On the other hand, the average time duration computed based on the concept of the joint probability of envelope and its velocity are shorter than the values obtained from the records by a considerable amount.

Another area to address regarding group waves is the evaluation of how many times a group will occur in a certain time period, say 30 minutes or one hour, in a specified sea state. For this, the probability distribution of the time interval between successive wave groups is necessary. It is noted again that the time interval between every envelop up-crossing, τ_{α} , shown in Figure 20 does not necessarily represent the time interval of a wave group. Suppose the up-crossing of the

Table 2. Comparison between observed and computed average time durations associated with envelope exceeding a specified level. Data measured in North Sea off Norway for 17 minutes, $H_g = 8.13$ m, level $\alpha = 4.0$ m

		COMPUTED					
LEVEL	OBSERVED	Based on probability function of Ta+	Based on modified probability function of Ta+ applicable to wave group				
	Including one	wave peak or none	during level crossi	ng			
4.0 m	15.9	9.1	 	3.9			
5.0 m	10.8	7.9		3.1			
	At least two	peaks during level	crossing (Wave grou	ıp)			
4.0 m	19.0		17.9				
5.0 m	13,7		17.0				

envelope at Point B in the figure is not that associated with at least two peaks during the crossing time, then the time interval AC (instead of AB or BC) should be considered as the time interval between wave groups. Similarly, if the up-crossing of the envelope at Point C constitutes the same situation as at Point B, then we have to consider the interval AD for the probability of occurrence of group waves. Several studies on the frequency of occurrence of group waves have dealt with every up-crossing of the envelope resulting in an extremely large probability of occurrence of group waves.

A method for evaluating the probability of occurrence of group waves in a specified sea state, taking into consideration the time interval in which only two or more peaks are present, was developed by applying the renewal theory (Ochi and Sahinoglou 1989b). The procedure for evaluating the frequency of occurrence of wave groups in random seas is as follows:

First, the probability density function of the time interval T of successive envelope up-crossings for a given level α is derived. Then, the sum of the probability density function of two time intervals of τ_{α} , three time intervals of τ_{α} , etc., are obtained by applying the convolution integral, and these functions are accumulated by multiplying by a weighting factor. This is the probability density function of the time interval between successive wave groups, $\tau_{\alpha G}$, in Figure 20. The probability density function was further approximated by the gamma probability distribution function and therefrom the probability of n-occurrences of a wave group in a certain time period was evaluated. As an example of the application of the prediction method, the probabilities of occurrence of group waves were computed by using wave data measured in the North Sea off Norway. The recording time was 17 minutes in a sea of significant wave height of 8.13 meters. Table 3 shows the probability of group waves at a level of 4 meters. As can be seen in the table, the probability of the occurrence of 4, 5 and 6 wave groups is high; specifically, the highest probability is 0.204 for 5 occurrences in 17 minutes, and the predicted result

Table 3. Predicted probability of occurrence of wave groups and comparison with measured number of wave groups. Data measured in North Sea off Norway for 17 minutes, $H_3 = 8.13$ m, level $\alpha = 4.0$ m (From Ochi and Sahinoglou 1989b)

Predicted prob	Measured number of wave groups		
Fr { No group }	=	0.001	
Pr { 1 group }	=	0.012	
Pr { 2 groups }	=	0.050	
Pr { 3 groups }	=	0.116	
Pr { 4 groups }	=	0.180	
Pr (5 groups)			5
Pr { 6 groups }	=	0.178	
Pr { 7 groups }	=	0.126	
Pr { 8 groups }	=	0.073	
Pr { 9 groups }	=	0.036	
Pr { More than }	=	0.024	

agrees well with the measurements. Previously existing formulae for evaluating the average time interval between successive envelope crossings at a specific level yield a very large number of wave groups, 37.7 occurrences in 17 minutes.

3.5 Estimation of Wave-Induced Design Loads

In order to evaluate the wave-induced loads for design of marine systems, one major area of concern is the selection of sea states to be considered for evaluation. Considerable controversy has existed regarding the evaluation of lifetime extreme values as to whether the estimation should be made through a long-term or a short-term prediction approach. The long-term prediction approach deals with an accumulation of all lifetime responses and hence it may appear that estimation by this method is superior to that of the short-term prediction approach. However, long-term estimation includes a considerable percentage of small responses in relatively mild seas which do not contribute to the design extreme value.

Computations were carried out on the transverse force acting on the cross-structure of a semi-submersible offshore platform through long-term, intermediate-term, and short-term prediction approaches (Ochi 1987). The platform had two large submerged horizontal hulls upon which a total of six vertical cylinders were distributed to support the bridging structure. In evaluating the response through the long-term approach, various sea states (significant wave heights) at 1-meter intervals up to 13 meters were considered, and each sea condition was weighted in accordance with data obtained in the North Sea. The sixparameter wave spectral family was used for each sea state, and five different wave directions, taken at 22 1/2 degree intervals from head to beam seas, were considered assuming that all directions have an equal probability of occurrence. Thus, a total of 572 short-term computations were accumulated for constructing the probability density function of the lifetime

In the intermediate-term approach, computations were carried out only in the severest sea, but for five wave directions again using the six-parameter wave spectral family. In this case total of 44 short-term responses were accumulated. In the short-term approach, the responses were evaluated only in the severest sea, for the severest wave direction (beam sea in this example), and for the wave spectrum of the spectral family which yielded the largest response. Evaluation of extreme values was made based on the narrow-band assumption in all of the computations.

Table 4. Comparison of probable extreme forces expected in 50 years predicted through short, intermediate and long term approaches (From Ochi 1987)

	LONG-TERM		INTERMEDIATE- TERM	SHORT-TERM
Number of stort-terms :			44	1
Number of responses	3.06 x 10 ³	:	2.J9 x 10 ⁵	4.08 x 10 ³
Probable extreme force; expected in 53 years ; in tons	7,750	1	7,400	7,120

Table 4 shows a comparison of probable extreme values of wave-induced loads expected in 50 years obtained through the three different approaches. As can be seen in the table, the extreme value estimated through the long-term approach is approximately 8-9 percent greater than that estimated through the short-term approach. However, it should be noted that there are reservations regarding the accuracy of the estimated extreme values expected in 50 years through the long-term approach, in general. For this example, the number of lifetime responses for the long-term approach is on the order of 3 x 108. This results in the extreme value being determined for a probability of exceedance of 1 - F(x) = 3.4×10^{-9} , which is so small as to render the results doubtful. On the other hand, the extreme values for the short-term is determined for a probability of exceedance of 2 x 10-4 which appears to yield a far more reasonable result than the long-term approach. Furthermore, the short-term approach has a distinct advantage due to its simplicity. Thus, as far as estimation of the lifetime extreme value of response is concerned, the short-term approach appears to be appropriate.

In selecting the sea condition for evaluating the extreme responses such as motions, wave-induced bending moments, etc., through the short-term approach, we may choose the severest sea state expected in the operational area of the marine system, since the magnitude of these responses increases with increase in sea severity. However, this is not the case for estimating the extreme value of accelerations. Large accelerations of a system do not occur in the severe sea state, in general; instead, it occurs in a moderate sea state. This is because the spectral density of the acceleration is given by ω^4 S(ω), where S(ω) is the spectral density of the system's motion. Therefore, although the spectral densities of the motion are relatively large for low frequencies in severe seas, acceleration spectral densities become small due to ω^4 .

In estimating possible fatigue failure which might occur in the lifetime of a marine system, the long-term approach is indispensable. In particular, it is highly desirable to evaluate every excursion of the response by applying the method presented in connection with half-cycle excursion analysis, since the number of small excursions expected in a lifetime is extremely large. As an example, Figure 22 shows the results of

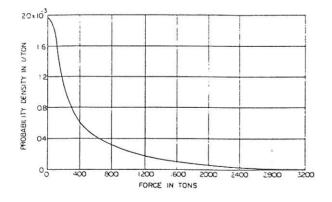


Figure 22. Lifetime probability density function of transverse force acting on the cross-structure of a semi-submersible platform obtained through half-cycle excursion analysis (From Ochi 1985)

computations on transverse force acting on the cross-structure of a semi-submersible offshore platform by applying half-cycle analysis (Ochi 1985). Computations were carried out for various sea severities, headings to waves and wave spectral shapes. As can be seen in the figure, the number of small responses is extremely large which might reach the critical level established in the fatigue criterion such as Palmgren-Miner criterion. Note that many cycles of small responses can be attributed to the responses belonging to the Type II excursion considered in the half-cycle excursion analysis.

Evaluation of nonlinear responses of marine systems such as motions of tension-leg platform and moored ships in random seas is an extremely important subject to be considered for design. These nonlinear responses are low frequency, non-Gaussian random processes. However, a particular input (wave condition) is not required for evaluating the responses. The most appropriate currently available approach for evaluating the statistical properties of nonlinear responses is based on Volterra's 2nd-order stochastic series expansion with application of Kac-Siegert's method (see Naess 1985, 1986, Kato et al. 1990, among others).

4. CURRENTS

4.1 Stochastic Properties of Offshore Currents

Because information on offshore current velocity often plays a significant role in design of marine systems, extensive current measurements have been carried out during the past decade. The magnitude of current velocity, its frequency and direction are all changing continuously; hence, statistical prediction of its extreme value is not as simple as it may first appear.

Current velocity measured in offshore areas, in general, results from currents primarily related with astronomical tide and those induced by local wind. Current velocity associated with ocean circulation and storm surge, etc., may also be significant depending on the geographical location. All of these components can exist simultaneously, and hence the measured data demonstrate that the magnitude of the current velocity fluctuates in a random fashion with frequencies covering a wide range from 0.0008 to 0.08 cycles per hour (period range from 1200 to 12 hours). Furthermore, the direction of the current velocity changes with time; it rotates clockwise in the northern hemisphere due to the rotation of the earth and the coriolis force. The angular frequency of rotation, however, is random. It varies from 0.31 to 0.70 radians per hour. Thus, the current velocity is considered to be a random process with varying frequency and direction.

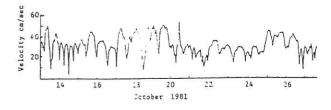


Figure 23. Example of time history of current velocity

Figure 23 shows a portion of current velocities recorded at a 26 meter depth off Newfoundland, Canada. The data represent the hourly mean current velocities measured at 10 minute intervals in a dominant hourly current direction. It is somewhat difficult, in general, to separate the measured current velocities into various components such as the wind-generated, tidal, ocean current components, etc. However, one way to analyze the velocities is from a consideration of frequency. That is, the high-frequency components are considered to be currents associated with tides, while the low-frequency components are those attributed to all other environmental conditions which may be called the residuals.

It is not possible to obtain statistical information on high frequency (tidal) and low frequency (residual) current velocities simply by separating the time history shown in Figure 23 into high and low frequency components, since the components thusly obtained merely represent the high and low frequency current velocities in the dominant hourly current direction. Note that the tidal and residual currents flow independently in constantly changing direction.

Recently, a method was developed to evaluate the statistical properties of tidal and residual components for design

consideration from measured current velocities (Ochi and McMillen 1988b, Ochi 1990). In this method, the measured current velocity was first decomposed into two rectangular components as shown in Figure 24; for convenience, the East-West and North-South directions were considered. Each of these two velocity components, denoted by U and V, respectively, is then further decomposed into high frequency (tidal) and low frequency (residual) velocity components by choosing an appropriate cutoff frequency. In the example shown in Figure 23 this was set at 0.035 cycles/hour since the results of spectral analysis of the records showed a small amount of energy existed beyond the 24 hour period (0.042 cycles/hour). Thus, the time history of current velocity obtained from measurements at one hour intervals was decomposed into the following four components:

In the East-West direction
High frequency (tidal) component, H_u Low frequency (residual) component, L_u In the North-South direction
High frequency (tidal) component, H_v Low frequency (residual) component, L_v .

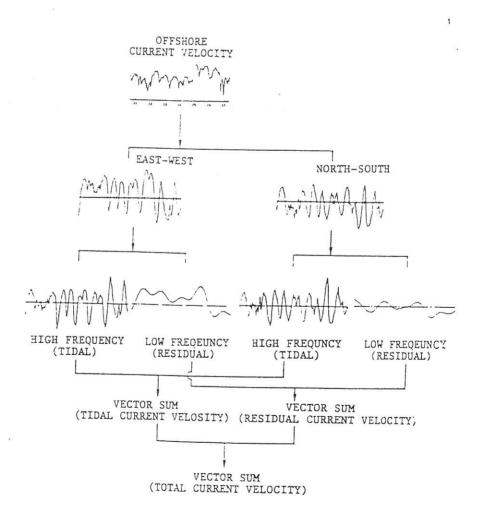


Figure 24. Diagram illustrating analysis of offshore current velocity data (From Ochi 1990)

$$F(C) = \alpha \left[\frac{\sqrt{R_H R_L}}{2 \left(\sqrt{2R_H} - \sqrt{R_L} \right) \left(\sqrt{2R_L} - \sqrt{R_H} \right)} \right]$$

$$\times \left(1 - \exp\left\{ - \left[\frac{2}{R_H R_L} C^2 \right] \right] \right]$$

$$+ \frac{2R_H^2}{\left(R_H - R_L \right) \left(2R_H - R_L \right)} \left[1 - \exp\left\{ - \frac{C^2}{R_H} \right\} \right]$$

$$- \frac{2R_L^2}{\left(R_H - R_L \right) \left(2R_L - R_H \right)} \left[1 - \exp\left\{ - \frac{C^2}{R_L} \right\} \right] \right].$$
(54)

Second, the probable extreme current velocity most likely to occur in a specified time period is evaluated as a solution of the following equation:

$$\frac{1}{1 - F(C)} = n \tag{55}$$

where n = number of samples in a specified time period.The solution is most commonly obtained graphically.

As stated in regard to Eqs. (14) and (15), the extreme current velocity for design consideration with risk parameter α can be evaluated by letting the right side of Eq. (55) be n/α . By using the number of samples n=1,911 in 4 months for the example considered, the extreme values in 4 months, 20 years and 50 years are estimated as shown in Figure 26. For convenience, the logarithm of Eq. (55) is shown in the figure. The estimated extreme current velocity in 4 months is 76 cm/sec as compared with 74 cm/sec observed in the data. The extreme values expected in 20 and 50 years are 92.0 cm/sec and 94.5 cm/sec, respectively, as indicated in the figure.

4.3 Currents and Sea Severity

It is of considerable interest to examine whether or not a simultaneous occurrence of extreme current velocity and extreme sea state can be expected during a storm. Heideman et al. (1989) presented the results of an analysis of wave and current measurements carried out over 7 years off the Norwegian shelf. They claim that maximum waves maximum current rarely occurs at the same time; only one occurrence was observed in 38 storms, each having their largest significant wave height exceeding 7 meters. Although the frequency of occurrence is very low, the simultaneous occurrence of extreme sea and extreme current observed by Heideman et al. can not be disregarded.

It is noted that the tidal (high frequency) current velocity becomes large almost regularly twice (or once) everyday; hence, there is a possibility that the extreme sea state during hurricane may occur at the same time when the high frequency current velocity takes place. Therefore, for the design of offshore structure, it is appropriate to consider the simultaneous occurrence of extreme sea state and extreme current velocity associated with the tide. The latter information may be

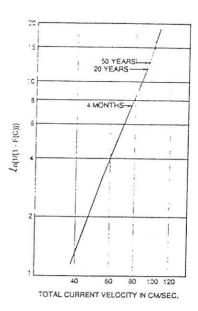


Figure 26. Estimation of extreme current velocity (From Ochi 1990)

evaluated from Equations (54) and (55) by letting the low frequency component R_{L} be zero.

Another area of interest is the interaction between current and waves; in particular the effect of currents on the wave spectrum. In general, if the current direction is in line with the wave direction, the wave amplitude decreases and its period becomes longer, but if the current direction is opposite to that of the waves, the situation is reversed (Longuet-Higgins and Stewart 1961). Many studies on the interaction between current and waves have been carried out. These studies, for the most part, deal primarily with the interaction of monochromatic waves with currents, leaving information on practical design application extremely limited. However, studies on the effect of steady current on wave spectra were carried out by Huang et al. (1972) and by Tung and Shyu (1992) particularly for opposing directions between waves and current.

CONCLUSIONS

The stochastic prediction approach has become an integral part of modern design technology in naval and ocean engineering. In order to promote a more rational design of marine systems, this paper presents a state-of-the-art summary of information on environmental conditions (winds, waves and currents) relevant to the design of marine systems. From the results of the review the following conclusions are drawn:

• In evaluating wind-induced force on marine systems, drag force associated with fluctuating turbulent winds should be considered along with that associated with the mean wind speed. The spectral shape of turbulent winds over a seaway is substantially different from that over land at low frequencies. The spectral formulation given in Equation 6 is derived from analysis of many spectra obtained from data measured over an open ocean.

- For design of marine systems operating in hurricanes and tropical cyclones, two different situations may be considered: (i) the severest wind speed and associated sea severity given in Equation 7, and (ii) the severest sea expected in tropical cyclones and associated wind speed given in Equation 8. In the latter case, the possibility of pre-existing sea condition, though not severe but lasting for a week or longer prior to the storm, is included in Equation 8. In both cases evaluation of simultaneous loadings excited by wind and waves is highly desirable.
- The most severe sea state which a marine system will encounter in her lifetime can be estimated by applying the generalized gamma probability distribution to long-term significant wave height data.
- The magnitude of extreme wave height expected to occur in a specified time period based on narrow-band and non-narrowband spectra are the same. The design wave height should be estimated for a specified risk parameter, since the probability that the extreme wave height exceeds the estimated probable extreme height is theoretically very large.
- The probability density function applicable for all wave excursions (half-cycle excursion analysis) is given in Equation 39. Extreme values based on half-cycle excursion analysis are only 5-6 percent greater than those estimated without the half-cycle analysis concept. However, the half-cycle excursion analysis method should be considered for estimating the long-term response of marine systems; in particular, for estimating possible fatigue failure which may occur in the lifetime of the system.
- It is highly desirable to employ wave spectral formulations developed from analysis of measured data and which yield spectra with minimum specifications (inputs) such as significant wave height and modal frequency, etc. Under these restrictions, four spectral formulations given in Section 3.2 are appropriate.
- Wave spectra during hurricanes show a consistent trend in shape, particularly in severe sea conditions. The spectra can be well represented by the JONSWAP formulation, but the values of parameters involved in the formulation are quite different from those given in the original JONSWAP formulation (see Equation 29).
- As to the wave energy spreading function associated with directional spectra, the formulation developed by Longuet-Higgins et al. with parametric values determined by Mitsuyasu (Equation 31) appears to be best. However, for a quick evaluation of the spreading function it may suffice to consider either the cosine-square or cosine-4th-power formulations.
- Application of the joint probability density function of wave height and period (Equation 41) to design is highly recommended irrespective of whether design is undertaken in the time domain or in the frequency domain. Knowledge of (a) wave period associated with the extreme wave height, and (b) the extreme wave height with period critical for possible resonant motion of the marine systems, for example, provides information vital for design, and these can be evaluated form the

- conditional probability density function given in Equations 43 and 44, respectively.
- The probability of occurrence of wave breaking in a given sea state can be evaluated by Equation 47 with the aid of the joint probability density function given in Equation 41. The magnitude of impact (including extreme values) associated with wave breaking on the front face of a circular cylinder can be evaluated by Equation 51.
- Methods to statistically evaluate (a) time duration of a wave group in which two or more wave peaks are present, and (b) number of occurrences of wave groups in a specified sea state have been recently developed. The computed results reasonably agree with those obtained from measured data.
- The results of computations of extreme value of response of an offshore structure through the long, intermediate, and shortterm approaches demonstrate that the short-term approach is adequate.
- The magnitude of responses of marine systems in a seaway increases with increase in sea severity in general. Hence, the severest sea state may be considered for estimating the extreme responses. However, this is not the case for estimating the extreme value of accelerations. Large accelerations of a system do not occur in a severe sea state; instead, they occur in a moderate sea state.
- A method to estimate the extreme magnitude of current velocity has been developed from analysis of measured data. That is, the energy spectrum of current velocity in each of two rectangular directions is separated into high and low frequency components and then variances of each component are obtained from the area under each spectrum. The magnitude of extreme current velocity for design consideration can then be estimated from knowledge of these variances (Equations 54 and 55).
- For the design of offshore structure, it is appropriate to consider the simultaneous occurrence of extreme sea state and extreme current velocity associated with the tide. The latter information can be evaluated from Equations 54 and 55 by letting the low frequency component $R_{\rm L}$ be zero.

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