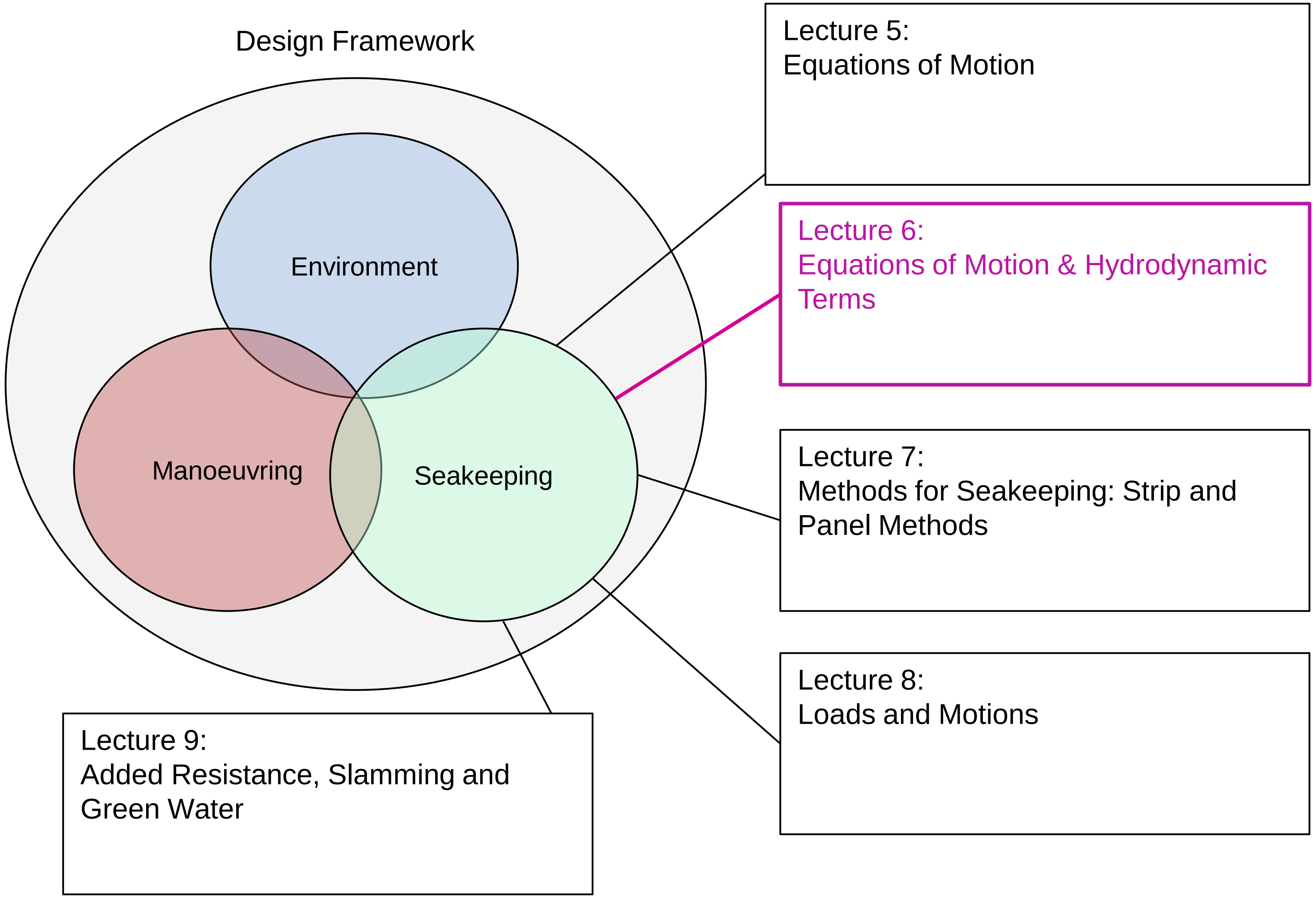
Aalto University
School of Engineering

MEC-E2004 Ship Dynamics (L)

Lecture 6 – Equations of Motion (Part II)

Aalto University
School of Engineering

Where is this lecture on the course?



Design Framework

Environment

Manoeuvring

Seakeeping

Lecture 5:
Equations of Motion

Lecture 6:
Equations of Motion & Hydrodynamic Terms

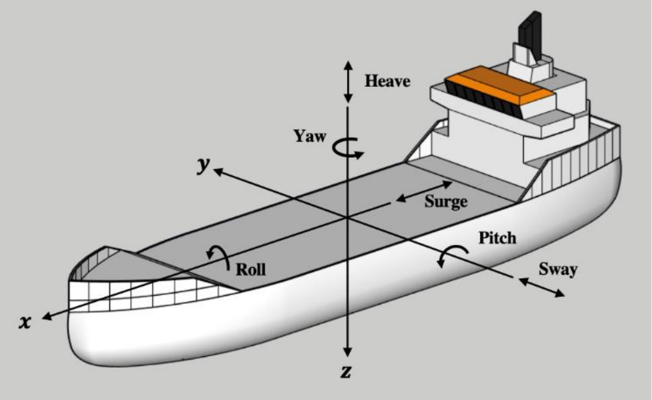
Lecture 7:
Methods for Seakeeping: Strip and Panel Methods

Lecture 8:
Loads and Motions

Lecture 9:
Added Resistance, Slamming and Green Water

Contents

- **<u>Aim:</u> To understand the physics of the components of rigid ship equations of motion in waves**
  - Basic Hydrodynamic phenomena (Radiation/Diffraction)
  - Components of Hydrodynamic EoM
  - Special cases
  - Hydrodynamic modelling
  - The experimental determination of motion components



- <u>Literature</u>
  - Journee, J.M.J., ”Introduction to Ship Hydromechanics”
  - Lloyd, A.R.J.M, ”Seakeeping – Ship Behavior in Rough Weather”, John Wiley & Sons
  - Bertram, V., ”Practical Ship Hydrodynamics”, Butterworth-Heinemann, Ch. 4.
  - Matusiak, J., ”Ship Dynamics”, Aalto University
  - Lewis, E. V. Principles of Naval Architecture. Vol. 3, ”Motions in waves and controllability”
  - Rawson, K. J., ”Basic Ship Theory. Volume 2, Ship dynamics and design - ch.12 Seakeeping & ch.13 Manoeuvrability”

Motivation

- ❑ Ship behaviour in the seaway is affected by
  - Sea state
  - Control with rudders, propulsors etc.
  - Cargo and general arrangement

- ❑ Motions and loads are evaluated by
  - Spectral methods
  - Linear dynamics
  - Potential flow methods

- ❑ Nonlinear methods are useful when motions are excessive
  - Time-domain hydrodynamics
  - Specific sea states/time-frames by using different time histories (with same spectrum, phase shift)
  - CFD methods are still under development

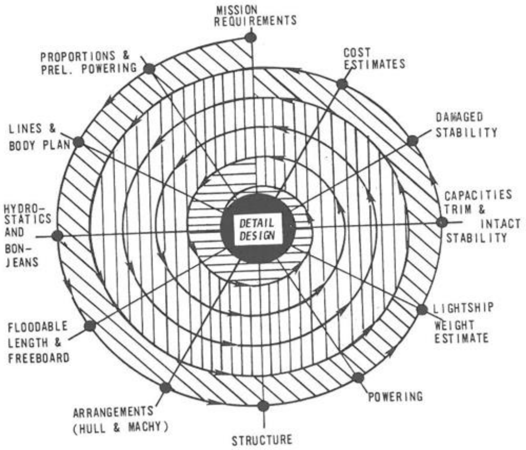
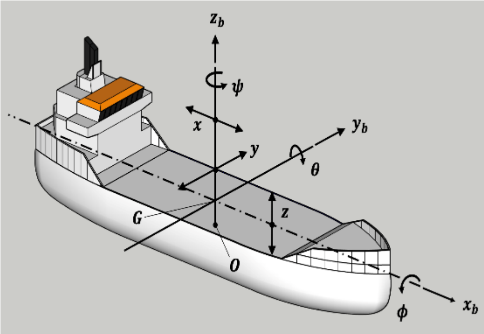


Fig. 1 Basic design spiral

Assignment 3

- **Grades 1-3:**
  - Select a book-chapter related to the ship equations of motion and read it
  - Identify the main components associated to equations of motion of your ship. How do they relate with the ship's mission *(think in operational safety terms)*
  - Discuss how the general arrangement, hull form and operational profile of your ship affect the equations of motion *(think in design for safety terms).*
  - Start using motions and loads design software (e.g. MaxSURF, Napa, etc.) and reflect how the software is related to the theory of ship motions.
- **Grades 4-5:**
  - Read 1-2 scientific journal articles related to Ship Equations of Motion
  - Reflect these in relation to knowledge from books and lecture slides
- Report and discuss the work.

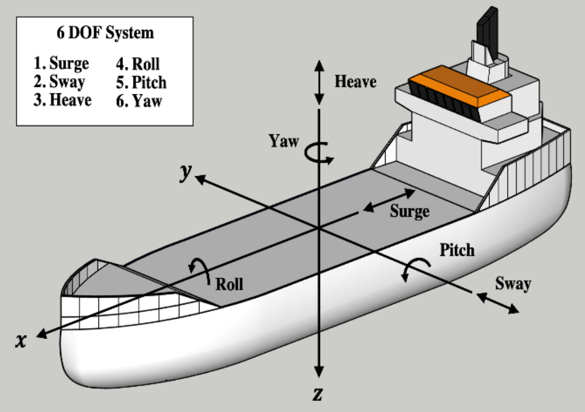


Key principles

Inputs

Fluid actions from wind and / waves

System



6 DOF ship system

Output

Ship motions and wave induced loads

- ☐ Water is denser and more viscous than air (i.e., damping is increased)
- ☐ Added mass represents the amount of total fluid accelerated by the object

Ship Equations of motion - Regular waves

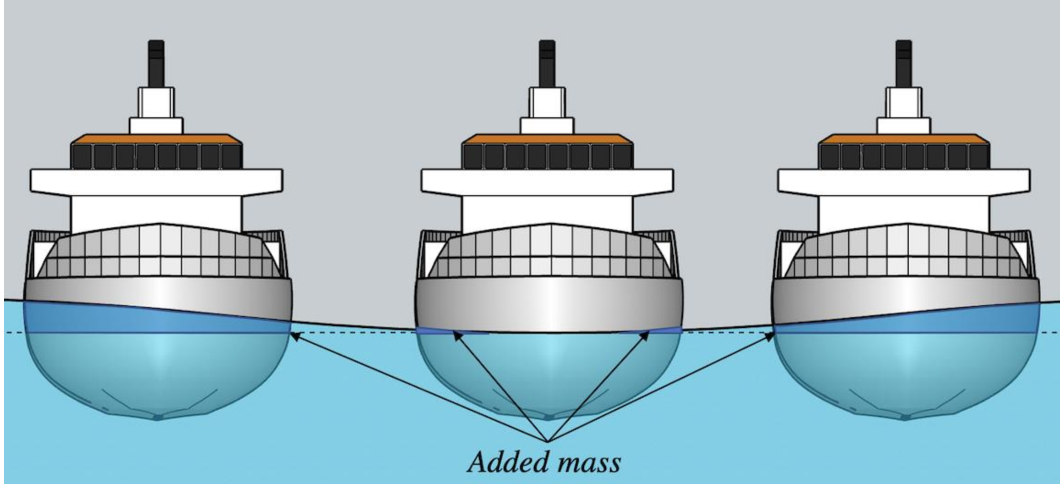
Hydro-damping Sinusoidal excitation

$$(a + m)\ddot{x} + b\dot{x} + cx = F_0 sin(\omega t)$$

added mass stiffness

- **Added mass & Hydro-damping** are functions of the frequency of oscillation.
- **Added mass** depends on the shape of the object and the type /direction of motion (linear or rotational)
- **Hydrodynamic damping** depends on fluid viscosity (frictional drag) and waves
- Each degree of freedom that has a restoring force has an associated natural frequency. These natural frequencies depend on the mass and stiffness properties of the system.

Hydrodynamic Forces



- The forces due to added mass and damping are hydrodynamic forces. They arise from pressure distribution around the oscillating hull.
- In linear hydrodynamic theory the hydrodynamic force has a component proportional to acceleration (i.e. added mass) and a component proportional to the velocity (i.e. damping coefficient).
- The following forces are used to obtain coefficients in the equations of motion :
  - **Radiation forces (or moments)**
  - **Incident wave or Froude - Krylov forces (or moments)**
  - **Diffraction forces (or moments)**

Hydrodynamic Forces

- **Radiation forces (or moments)** where the ship is assumed to oscillate in still water and the hydrodynamic added inertia and damping coefficients are determined

- **Incident wave or Froude - Krylov forces (or moments)** where the wave is considered in the absence of the ship and the corresponding wave forces (or moments) acting on the ship are determined. **NB:** *In linear hydrodynamics we assume small displacements, i.e., “true”, wetted surface is not considered.*

- **Diffraction forces (or moments)** where the effects of the presence of the ship on the waves are considered and the corresponding diffracted wave forces (or moments) are determined.

- For a ship with port-starboard symmetry the coupled motions of **heave – pitch** and **sway – roll – yaw** can be examined separately during seakeeping analysis. Of these five motions only heave pitch and roll have a restoring force or moment.

Fundamental EoM (Newton's Law)

$$\underbrace{(a+m)\ddot{x}+b\dot{x}+cx}=\underbrace{F_0 sin(\omega_e t)}$$

The LHS of EoM contains the dynamic properties of the ship and thus the forces on the ship when it is forced to oscillate in still water conditions (**hydro-mechanical**)

- Structural Mass (M) and Added Mass (a)
- Damping (b)
- Restoring forces (c)

$$[-\omega_e^2(M+A)+i\omega_e N+S]\hat{\hat{u}}=\hat{\hat{F}}_e$$

The RHS of EoM contains the forces on the ship when it is restrained from motion and subjected to regular waves (**wave exciting**)

- Incident wave Froude-Krylov
- Diffraction

$$\boxed{RAO=\frac{F_{e,0}}{N-\big(M+A(\omega_e)\big)\times\omega_e^2+iB(\omega_e)\omega_e}}$$

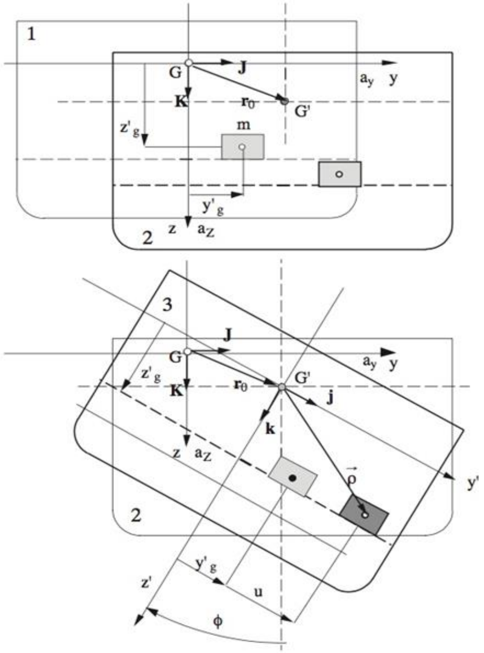
The RAO expresses the response. It is a frequency dependent function (also known as FRF). It is a complex number, i.e. it comprises of an amplitude and a phase; where *i* expresses the imaginary unit of this number. It is common to consider the absolute number of the RAO in linear seakeeping.

Structural Mass (M)

- Total mass consists of the ship structure, equipment, cargo, etc. and we assume perfectly symmetric hull along the longitudinal direction (y – axis system)
- 6 x 6 matrix corresponding to 6 dof
- Mass distributions couple with global ship motions and local actions such as sloshing liquid in tanks, openings such as swimming pools, cargo moving containers, cars.
- If we assume that the mass is perfectly attached to the CoG the cargo movement must be assessed with separate models.
- The mass moments of inertia θ relate to the origin of the ship fixed coordinate system and defined as:

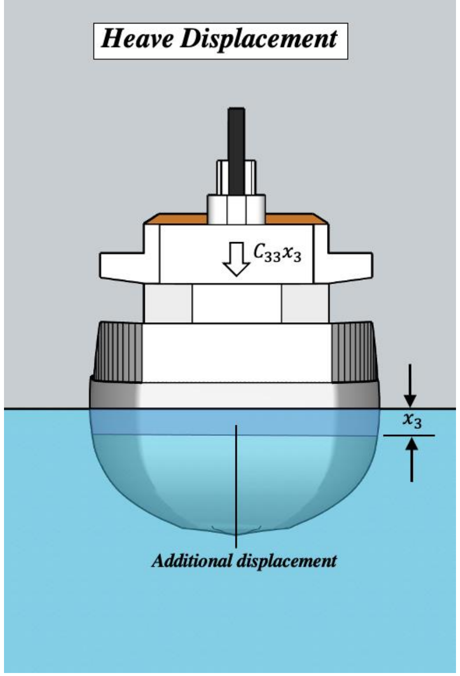
$$[-\omega_e^2(M+A)+i\omega_e N+S]\hat{\vec{u}}=\hat{\vec{F}}_e$$

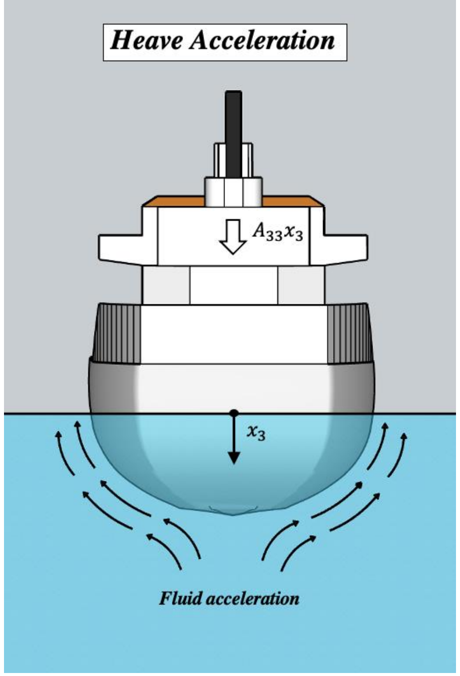
$$M=\begin{bmatrix} m & 0 & 0 & 0 & mz_g & 0 \\ 0 & m & 0 & -mz_g & 0 & mx_g \\ 0 & 0 & m & 0 & -mx_g & 0 \\ 0 & -mz_g & 0 & \theta_{xx} & 0 & -\theta_{xz} \\ mz_g & 0 & -mx_g & 0 & \theta_{yy} & 0 \\ 0 & mx_g & 0 & -\theta_{xz} & 0 & -\theta_{zz} \end{bmatrix}$$

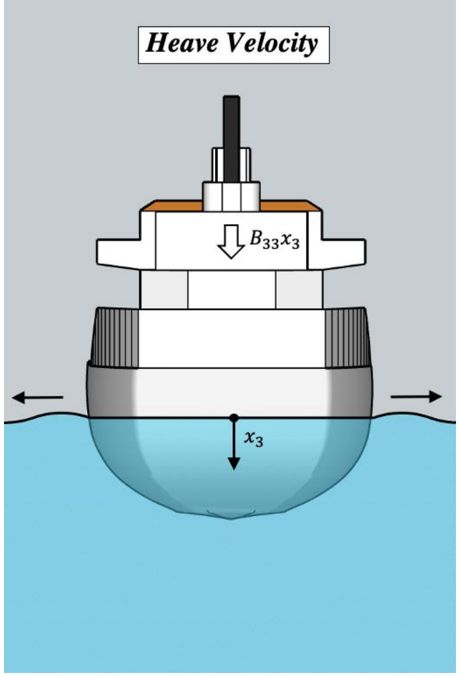


$$\theta_{xx}=\int(y^2+z^2)\,\mathrm{d}m;\quad \theta_{xz}=\int xz\,\mathrm{d}m;\quad \text{etc.}$$

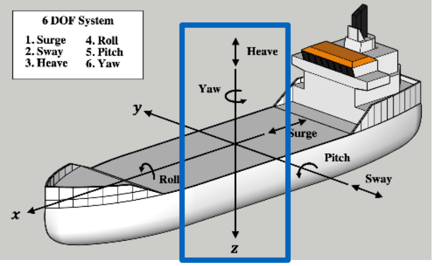
Heave Motion







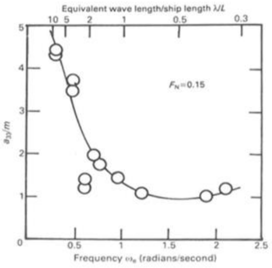
- ❑ Heave motion occurs in the z-direction and for the uncoupled case, the amount of additional displacement (water mass) during a wave-loading event causes the ship to accelerate along the z-axis due to buoyant forces
- ❑ Fluid acceleration around the hull cause waves to radiate



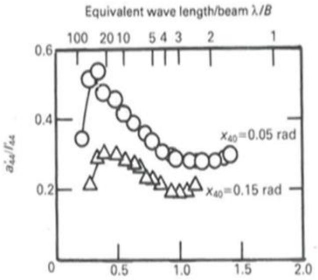
Added Mass (A)

- Added mass relates to the motion of fluids along with the ship. This affects the entire fluid domain, but the largest effect is at the neighborhood of ship.
- An easy way to define A : *the liquid that has the same acceleration and phase as the ship.*

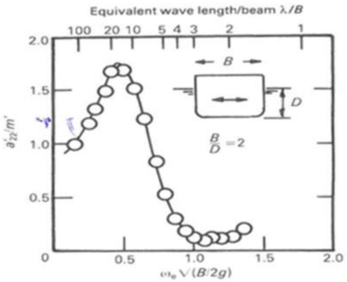
$$(\boldsymbol{A} + M)\ddot{x} + B\dot{x} + Cx = F_0 sin(\omega_e t)$$



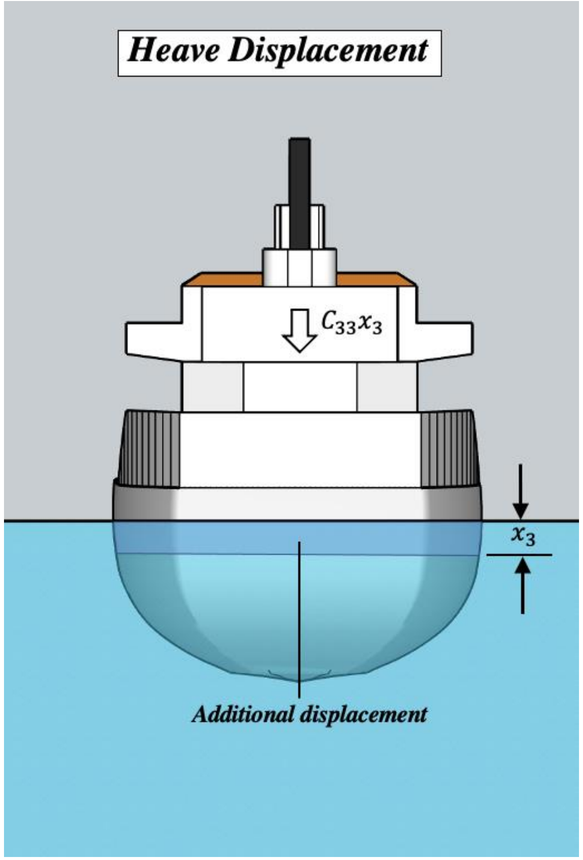
A_{heave}



A_{roll}



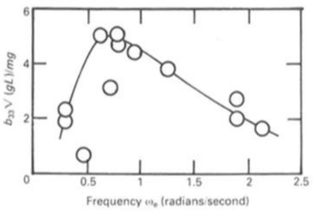
A_{sway}



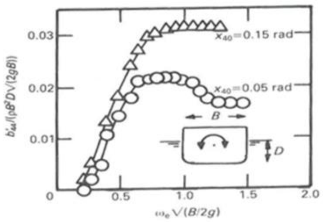
Damping (C)

- Damping (loss of energy) is caused by ship-induced waves. The waves decay at some distance from the ship
- Damping can be increased by appendices or the effects of viscosity
- The damping matrix mainly contains the effects of radiated waves.

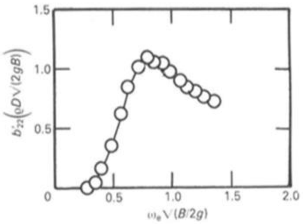
$$(A+M)\ddot{x} + B\dot{x} + \boldsymbol{C}x = F_0 sin(\omega_e t)$$



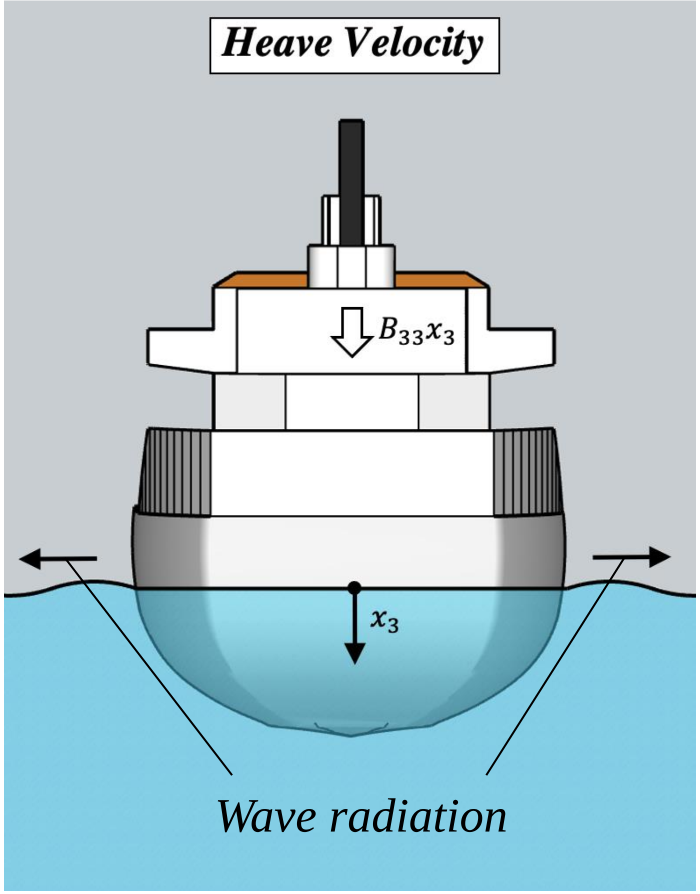
C_{heave}



C_{roll}



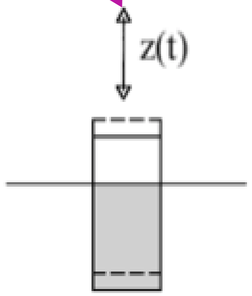
C_{sway}



Restoring matrix (C)

- The restoring forces are analogous to spring stiffness in mechanical systems
- Symmetry of the ship → main terms are zero
- When small displacements are considered, these can be derived from the hydrostatics
- The idealization presented assumes no contributions from transom stern and zero forward speed
- $\overline{GM}$: metacentric height ; $\overline{GM}_L$: longitudinal metacentric height calculated w.r.t. the origin of the coordinate system (usually amidships and not at WL), x_w the *x* coordinate of the center of the WL, A_w the WL area , ω_g the circular eigenfrequency of the control motions

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \boxed{\rho g A_w} & 0 & -\rho g A_w x_w & 0 \\ 0 & 0 & 0 & gm\overline{GM} & 0 & 0 \\ 0 & 0 & -\rho g A_w x_w & 0 & gm\overline{GM_L} & 0 \\ 0 & 0 & 0 & 0 & 0 & \theta_{zz}\omega_g^2 \end{bmatrix}$$



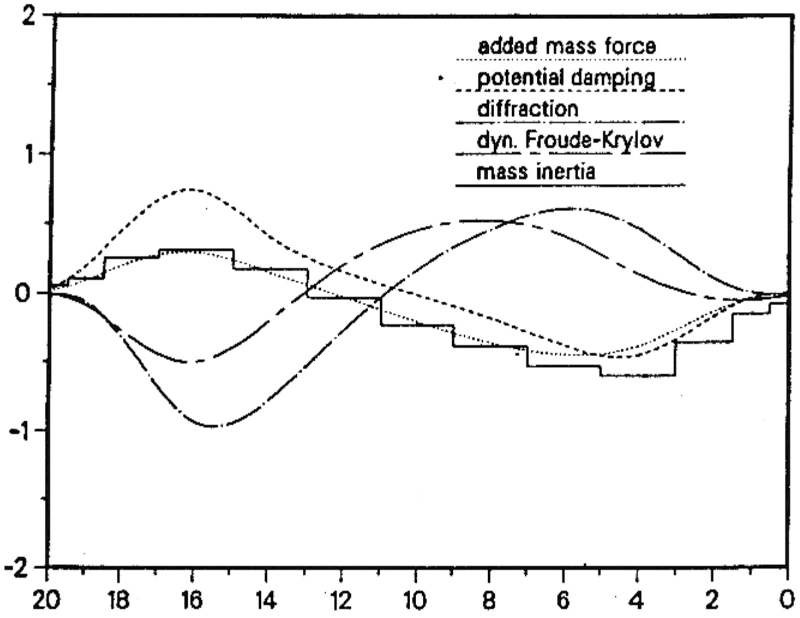
oscillation
in still water

$$\theta_{xx} = \int (y^2 + z^2)\, dm$$

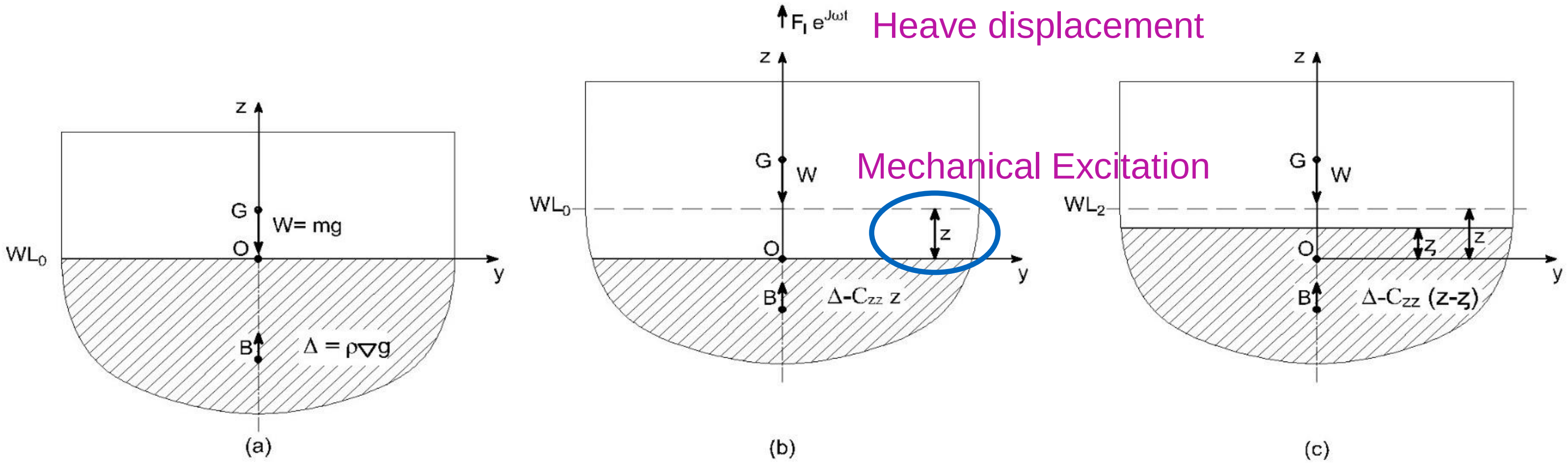
$$\theta_{xz} = \int xz\, dm$$

Froude-Krylov and Diffraction forces

- **Froude–Krylov force :** Pressure in way of the undisturbed waves integrated over the wetted surface of the ship
- **Diffraction forces :** Pressures that occur due to the disturbances in the water because of the presence of the ship
- We assume small displacements, i.e. “true”, wetted surface is not considered in linear hydrodynamics.
- J. Matusiak. (2011). **On the non-linearities of ship’s restoring and the Froude-Krylov wave load part**. International Journal on Naval Architecture and Ocean Engineering. pp: 111-115. [Journal Link]



Uncoupled Heave Motion



Heave displacement

Mechanical Excitation

(a)

Original State
Hydrostatic equilibrium
W = Δ

(b)

Mechanical excitation
Still water

(c)

Long crested regular waves

Let us assume that a ship in still water is subject to a mechanical excitation in the form of an upward force $F_z(t)$ leading to heave displacement $z(t)$ .The linear equation of motion for this 1 DOF system will be:

$$M_{zz}\ddot{z} + N_{zz}\dot{z} + C_{zz}z{=}F_z(t)$$

Uncoupled Heave Motion

For a sinusoidally varying mechanical excitation $F_z(t) = F_1 e^{j\omega t}$Assuming F_1 is a force vector of constant amplitude the response will also be sinusoidal namely $z(t) = Ze^{j(\omega t-\varepsilon)}$ where Z is the amplitude of excitation and **ε** the phase lag of the response. Accordingly:

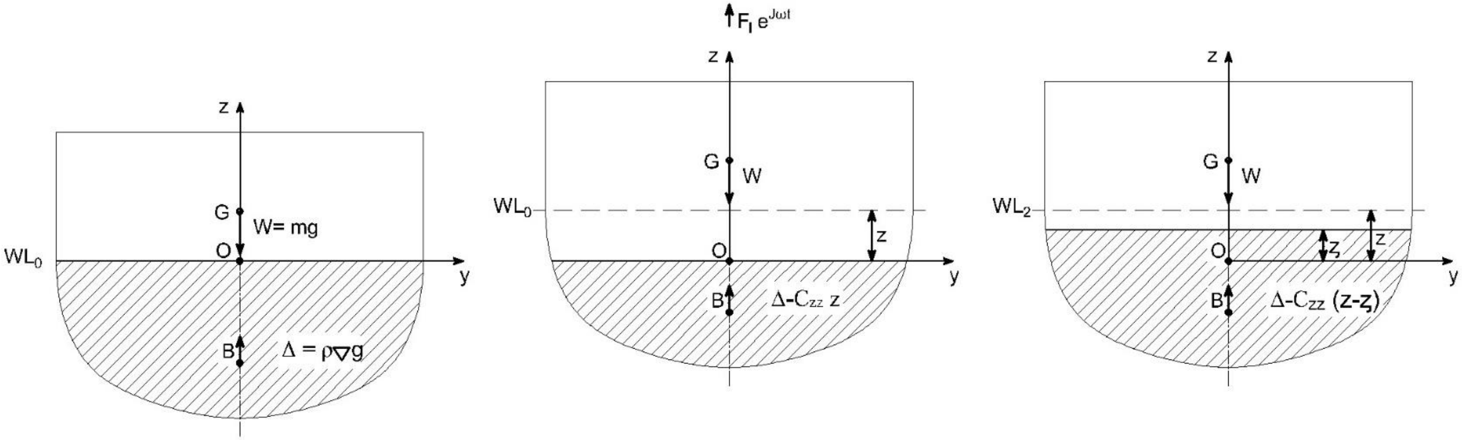
$$\mathrm{Z} = \frac{F_1}{\sqrt{(C_{zz} - \omega^2\, M_{zz}\,)^2 + (\omega N_{zz})^2}} \quad \text{and} \quad tan(\varepsilon) = \frac{\omega N_{zz}}{(C_{zz} - \omega^2 M_{zz})}$$

- ✓ $C_{zz} = \rho g A_w z$ is the hydrostatic heave restoring force with ρ representing the water density (kg/m³) ;
- ✓ g the acceleration of gravity (m/s²)
- ✓ A_w the still water lane area (m²).’
- ✓ N_{zz} is the heave damping force
- ✓ $M_{zz} = m + m_{zz}$ is the virtual mass of the ship ($m = \rho\nabla$ and m_{zz} $=$ heave added mass)

Uncoupled Heave Motion

If we assume free motions in waves it is possible to assume a constant value of added mass namely $\bar{m}_{zz} = m_{zz}(\omega_e \rightarrow \infty)$ and therefore the heave natural frequency in water is:

$$\omega_{3n} = \sqrt{\frac{c_{zz}}{m + m_{zz}}}$$

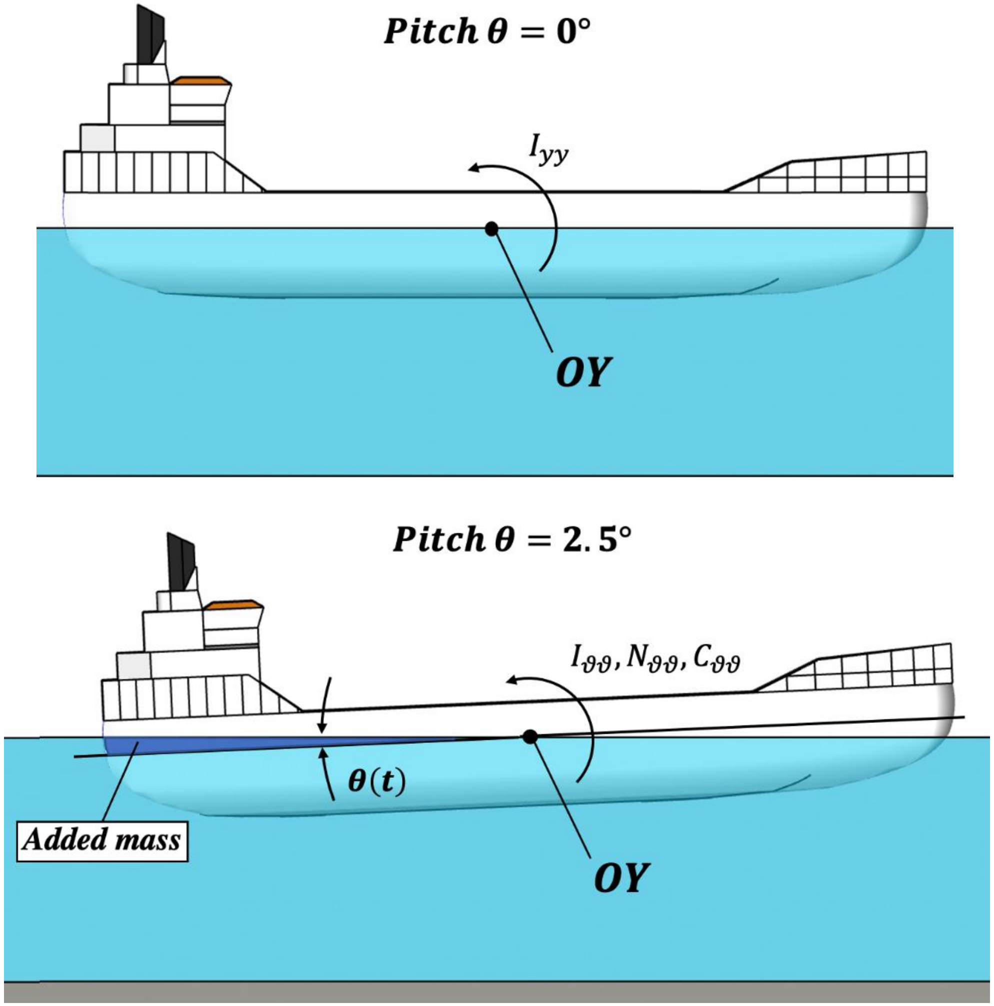


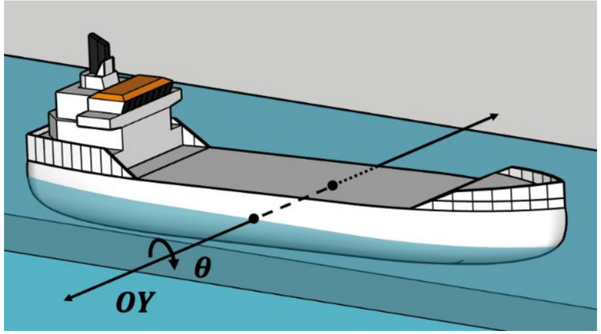
(a)

(b)

(c)

Uncoupled Pitch Motion (1/2)





Rotation about OY is only degree of freedom

When the ship rotates and introduces added mass there is a corresponding effect to the EoM coefficients

Uncoupled Pitch Motion (2/2)

- If we consider that the ship is an 1DOF system subject to pitch excitation namely $\theta(t)$

$$\left[I_{yy} + I_{\theta\theta}(\omega_e)\right]\ddot{\theta} + N_{\theta\theta}(\omega_e)\dot{\theta} + C_{\theta\theta}\theta = \bar{M}_{\theta}(\omega, \omega_e)e^{j(\omega_e t + u)}$$

  - ✓ I_{yy} is the mass moment of inertia about axis Oy
  - ✓ $I_{\vartheta\vartheta}$ is the pitch added mass moment of inertia
  - ✓ $N_{\vartheta\vartheta}$ is the pitch damping coefficient
  - ✓ $C_{\vartheta\vartheta} = \rho g I_{long}$ for $I_{long} = \text{longitudinal } 2^{\text{nd}} \text{ moment of water plane area}$
  - ✓ $\bar{M}_{\vartheta}$ is the amplitude of the wave excitation vector

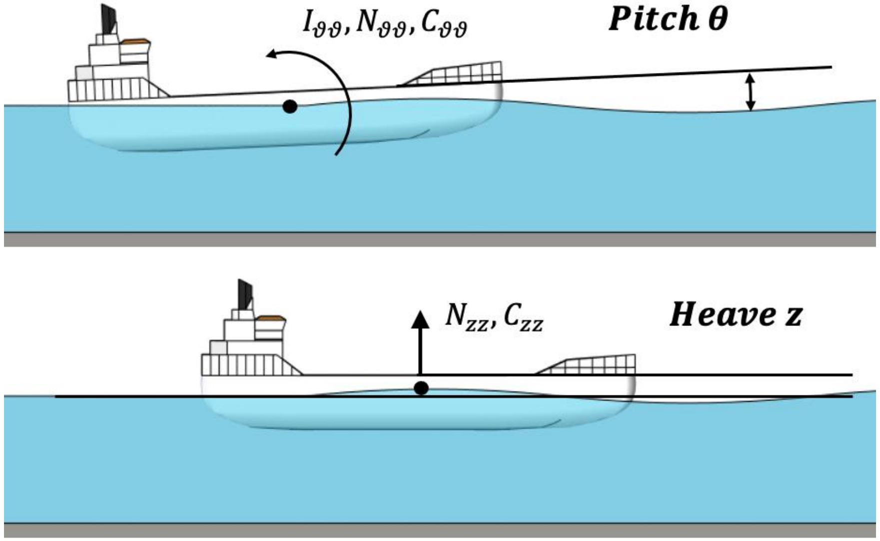
- The pitch natural frequency in water can be approximated as

$$\omega_p = \sqrt{\frac{c_{\theta\theta}}{I_{yy} + \bar{I}_{\theta\theta}}} \qquad \bar{I}_{\theta\theta} = I_{\theta\theta} \text{ as } t \rightarrow \infty$$

Coupling Heave & Pitch

$$\begin{bmatrix} m + m_{zz} & m_{z\theta} \\ m_{\theta z} & I_{yy} + I_{\theta\theta} \end{bmatrix} \times \begin{bmatrix} \ddot{z} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} N_{zz} & N_{z\theta} \\ N_{\theta z} & N_{\theta\theta} \end{bmatrix} \times \begin{bmatrix} \dot{z} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} C_{zz} & C_{z\theta} \\ C_{\theta z} & C_{\theta\theta} \end{bmatrix} \times \begin{bmatrix} z \\ \theta \end{bmatrix} = \begin{bmatrix} F_z(\omega, \omega_e) e^{j(\omega_e t + \psi)} \\ \bar{M}_\theta(\omega, \omega_e) e^{j(\omega_e t + u)} \end{bmatrix}$$

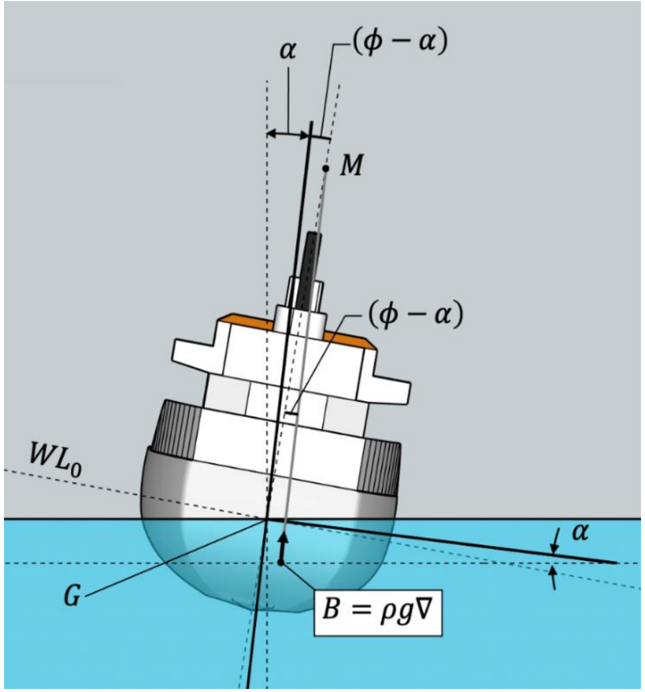
- In addition to heave added mass m_{zz} and pitch added inertia $I_{\theta\theta}$ we have **heave into pitch** (and ***pitch into heave***) added mass terms namely $m_{z\theta}$ and $m_{\theta z}$
- In addition to heave damping N_{zz} and pitch damping $N_{\theta\theta}$ coefficients we have the heave into pitch (and pitch into heave) terms defined as $N_{z\theta}$ and $N_{\theta z}$ respectively.
- The heave into pitch restoring terms are defined as $C_{z\theta} = C_{\theta z} = \rho g M_l$
- $M_l = \int_L xB(x)dx$ represents the longitudinal first moment of water plane area and $B(x)$ is the beam in way of the water line.



Roll – small amplitudes

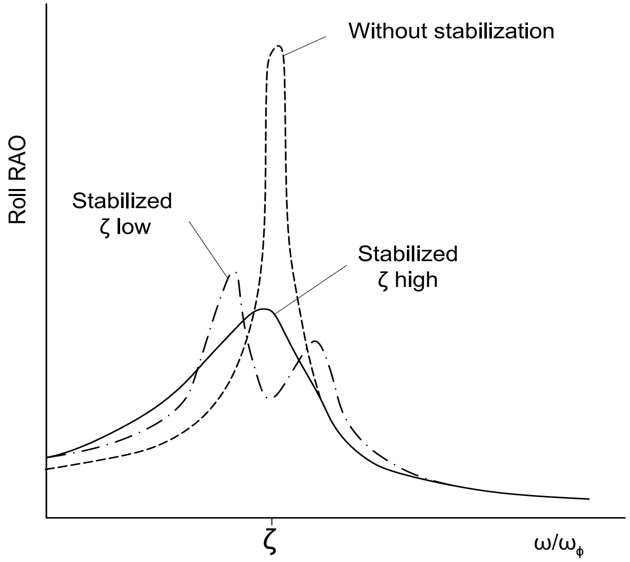
$$[J_{xx} + I_{\varphi\varphi}(\omega)]\ddot{\varphi} + N_{\varphi\varphi}(\omega)\dot{\varphi} + C_{\varphi\varphi}\varphi = K_{\varphi}(t)=K_1 e^{j\omega t}$$

- $K_{\varphi}(t)$ is a sinusoidal mechanical excitation producing a rolling moment $K_{\varphi}(t) = K_1 e^{j\omega t}$ ϕ is the angle of roll
- I_{xx} is the mass moment of inertia about the longitudinal axis through the center of mass
- $C_{\phi\phi} = \Delta GM_T = \rho g \nabla GM_T$ is the hydrostatic roll restoring coefficient
- $I_{\phi\phi}$ = roll added inertia (frequency of oscillation dependent)
- $N_{\phi\phi}$ = roll damping coefficient due to hydrodynamic effects associated with fin and tank stabilizers



Roll – large amplitudes

- Large amplitude of roll may cause discomfort compared to other motions.
- The amount of damping which is provided by the fluid is not always sufficient to reduce the roll amplitude to acceptable levels.
- Solution 1 : passive systems which make use of the roll motion and do not require any power source and control system
- Solution 2 : active systems which use power to move masses or control surfaces and a control system

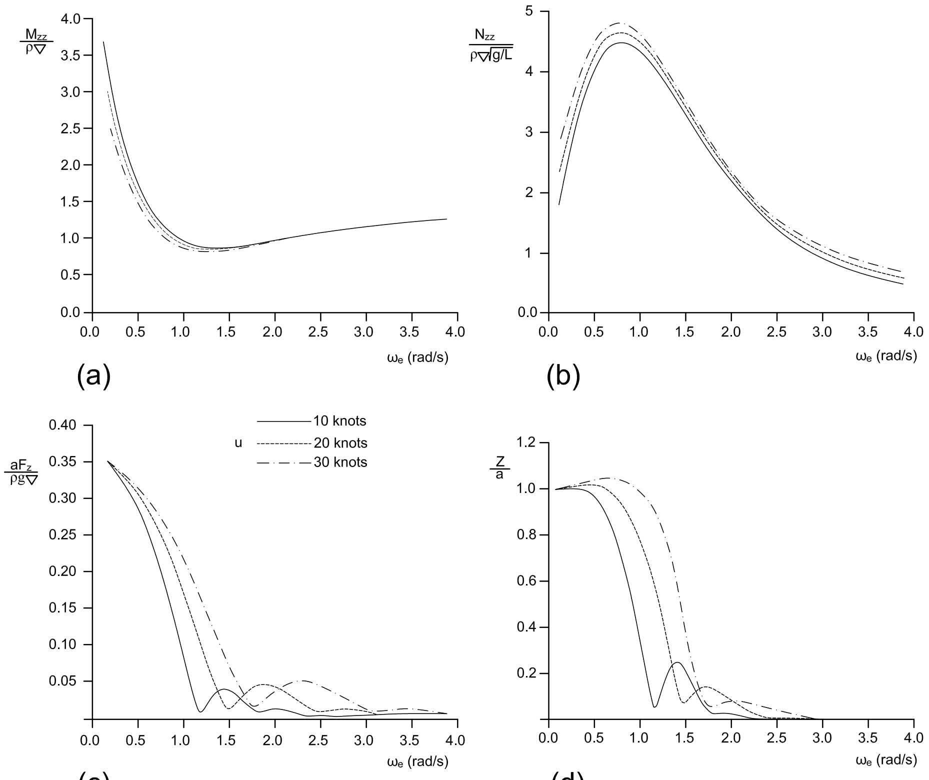


C_1 and C_3 decrease with virtual mass moment of inertia and restoring moment whilst C_2 increases the damping. C is associated with the lift generated by the fin stabilizers

$$\left[I_{xx}+I_{\varphi\varphi}(\omega)-CC_1\right]\ddot{\varphi}+\left(N_{\varphi\varphi}+CC_2\right)\dot{\varphi}+\left(C_{\varphi\varphi}-CC_3\right)\varphi=K_\varphi(t)$$

Ship Response

Variation of the nondimensionalized heave added mass, damping coefficient and excitation amplitude (for head waves of amplitude 1m)



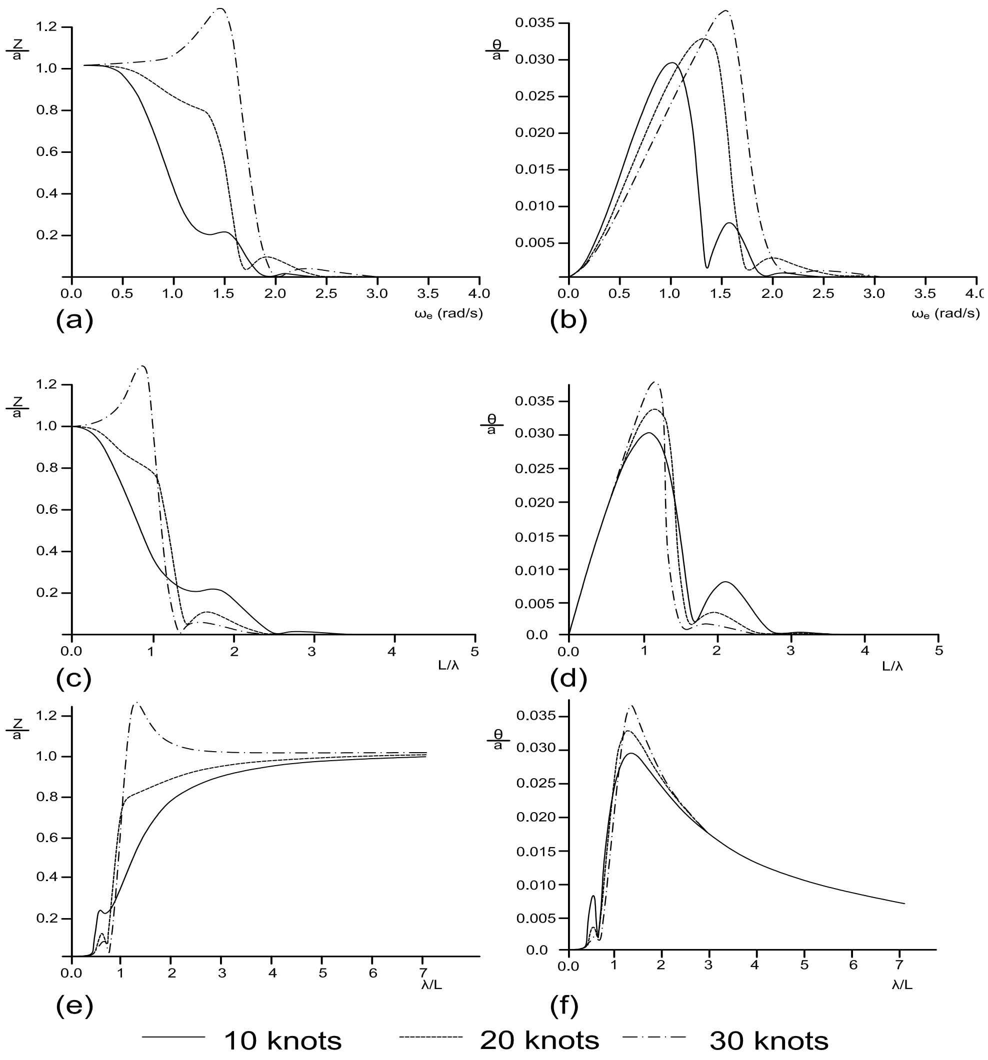
(a)

(b)

(c)

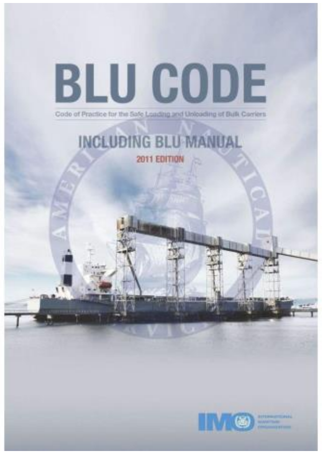
(d)

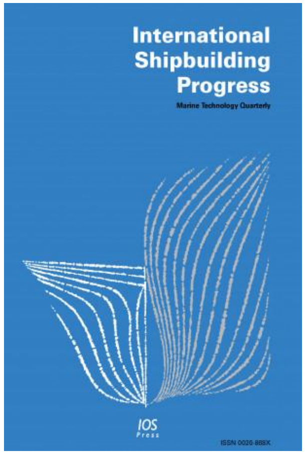
RAO demo for Heave & Pitch (Head waves)

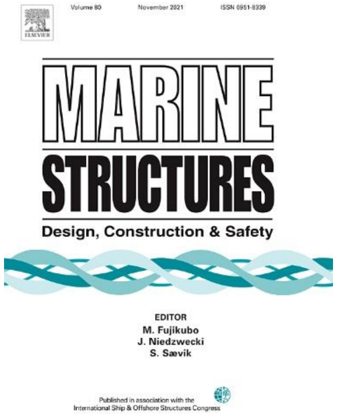


Cargo induced accelerations – Key publications

- P. Ruponen, J. Matusiak, J. Luukkonen, M. Ilus. (2009). **Experimental Study on the Behavior of a Swimming Pool Onboard a Large Passenger Ship**. Marine Technology, Vol 26, No. 1: pp. 27-33.
- J. Matusiak. (2000). **Dynamics of Cargo Shift Onboard a Ship in Irregular Beam Waves**. Int. Shipbuilding Program, Vol 47, No. 1: pp: 77-93.
- M. Acanfora, J. Montewka, T. Hinz, J. Matusiak. (2017). **On the Estimation of the Design Loads on Container Stacks Due to Excessive Acceleration in Adverse Weather Conditions**. Marine Structures Vol 53: pp105-123.
- BLU Code and BLU Manual- Code of practice for the safe loading and unloading of bulk carriers published by IMO

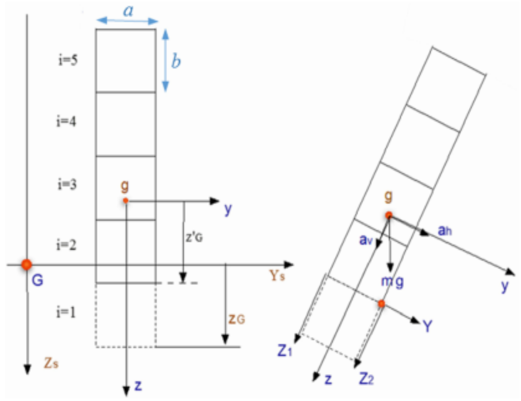




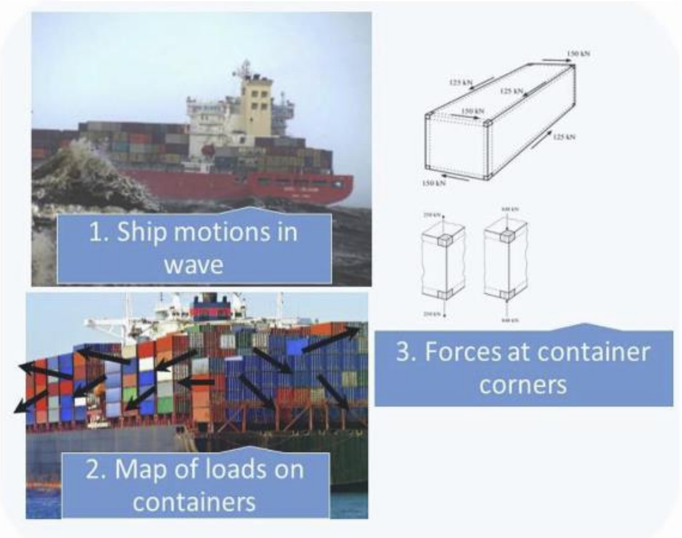


Cargo-induced accelerations - Practical example

- In recent years, many containership accidents have occurred that are coupled with violent ship motions.
- **Averse weather conditions** and **poor lashing** leading to domino effects of container stacks in waves.
- This is why the determination of accurate design accelerations and the evaluation of loads on container stacks are of primary importance for safety.
- Designers are interested to introduce safe container loadings and adequate dimensioning of the lashing equipment.
- This can be understood better by coupling seakeeping with cargo securing equations leading to avoidance of cargo shifting.

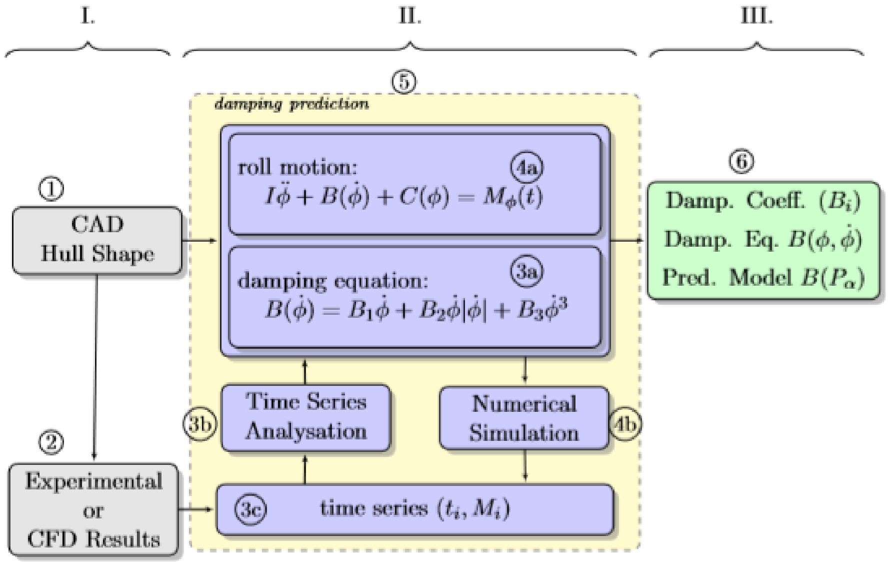


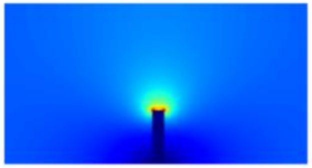
Forces at container stack – conventional arrangement.



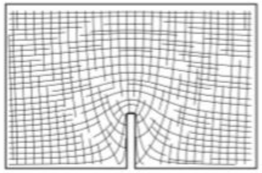
The value of experimental methods

- The coefficients for the EoM need to defined in order to obtain the solutions
- We can do this either computationally or experimentally
- Computational means are time efficient but not 100% accurate
- Model tests an efficient way to determine the coefficients. However, they are expensive to carry out and sensitive to uncertainties

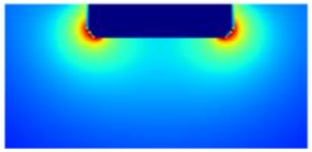




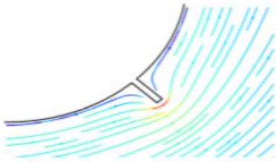
Velocity magnitude around a bilge keel



Potential- and stream-lines around a bilge keel



Velocity magnitude around a rolling hull section



Streamlines around bilge keel

Summary

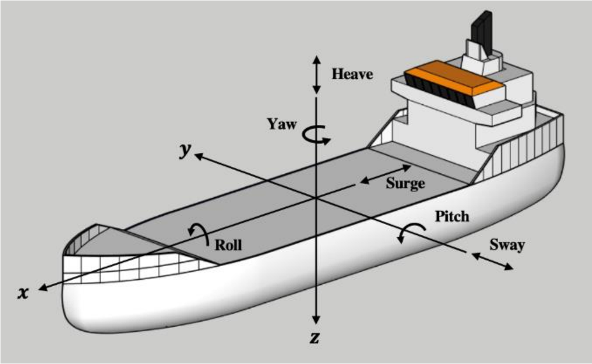
- The purpose of this lecture was to give an overview on how the linear equations of motion for rigid ships are derived.

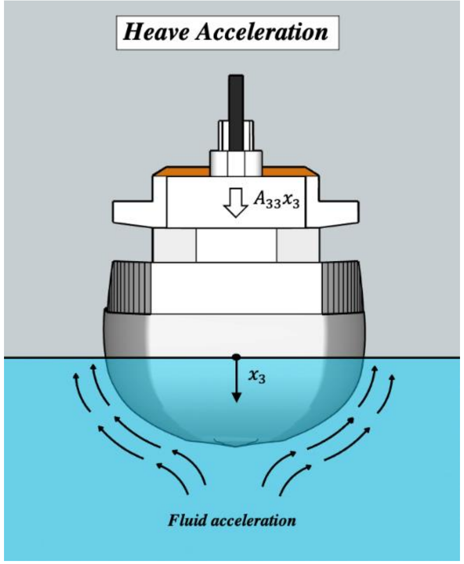
- For sinusoidal excitation we can obtain sinusoidal response with
  - ✓ *Phase differences*
  - ✓ *Amplitude of response*

- Coupling ship motions with cargo is a challenging problem that relates with ship safety

- The coefficients for equations of motion can be obtained computationally. However, using experimental techniques is important in terms of managing uncertainties

- Once we know the response for regular seas, we can move to irregular seas





Thank you !!

Next time we will talk on seakeeping methods