That stuff about principal values?

Say your function $f(\omega)$ has some nastiness at $\omega = c$. 

$$\overset{a}{|}\!\!-\!\!-\!\!-\!\!\overset{c}{|}\!\!-\!\!-\!\!-\!\!\overset{b}{|}$$

Cauchy principal value, defined by

$$\lim_{\varepsilon \to 0^+} \int_a^{c-\varepsilon} f(\omega)\,d\omega + \int_{c+\varepsilon}^{b} F(\omega)\,d\omega$$

How is this related to lecture 7?

Well, there we have integrals of type

$$I = \int_{-\infty}^{\infty} d\omega \int_0^{\infty} dt\, f(\omega) e^{i\omega t}$$

how do we deal with this so that integral doesn't go crazy?

Trick: $\omega \to \omega + i\omega$ (so we get $e^{-\varepsilon t}$ term)

$$\Rightarrow \lim_{\varepsilon \to 0^+} \int_0^{\infty} \left( \int_{-\infty}^{\infty} e^{i\omega t - \varepsilon t} f(\omega)\,d\omega \right) dt \quad \left( \begin{matrix} f(\omega) \text{ continuous} \\ \text{on the real line} \end{matrix} \right)$$

$$= \lim_{\varepsilon \to 0^+} \int_{-\infty}^{\infty} d\omega \left( \int_0^{\infty} e^{i\omega t - \varepsilon t}\,dt \right) f(\omega)$$

$$= \lim_{\varepsilon \to 0^+} \int_{-\infty}^{\infty} \frac{i f(\omega)}{\omega + i\varepsilon}\,d\omega$$

Then this becomes: $\pi \lim_{\varepsilon \to 0^+} \int_{-\infty}^{\infty} \frac{\varepsilon}{\pi(\omega^2 + \varepsilon^2)} f(\omega)\,d\omega$ ← approaches $\delta(\omega)$

$$+ i \lim_{\varepsilon \to 0^+} \int_{-\infty}^{\infty} \frac{\omega^2}{\omega^2 + \varepsilon^2} \frac{f(\omega)}{\omega}\,d\omega$$

$$\frac{\omega^2}{\omega^2 + \varepsilon^2} = \begin{cases} 1, & |\omega| \gg \varepsilon \\ 0 & |\omega| \ll \omega \end{cases} + \text{symmetric} \Rightarrow \text{Cauchy principal value.}$$