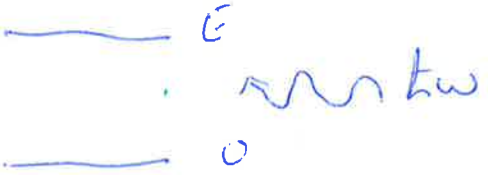
Light shift to energy levels:



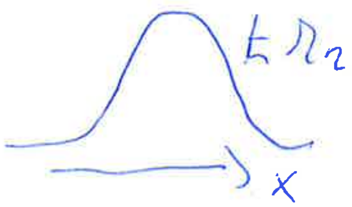

detuning $\Delta = \hbar\omega - E$

$$\Rightarrow \quad H = \begin{pmatrix} -\Delta/2 & \hbar\Omega_2 \\ \hbar\Omega_2 & \Delta/2 \end{pmatrix} \qquad |\psi\rangle = \begin{pmatrix} |e\rangle \\ |g\rangle \end{pmatrix}$$

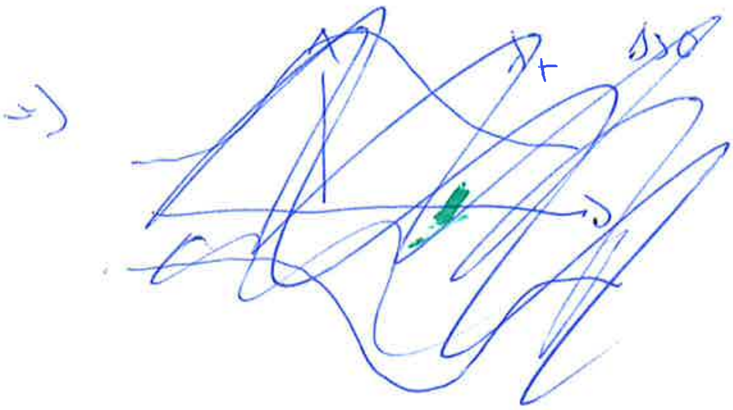
eigenstates: $-\left(\frac{\Delta}{2}+\lambda\right)\left(\frac{\Delta}{2}-\lambda\right) - (\hbar\Omega_2)^2 = 0$

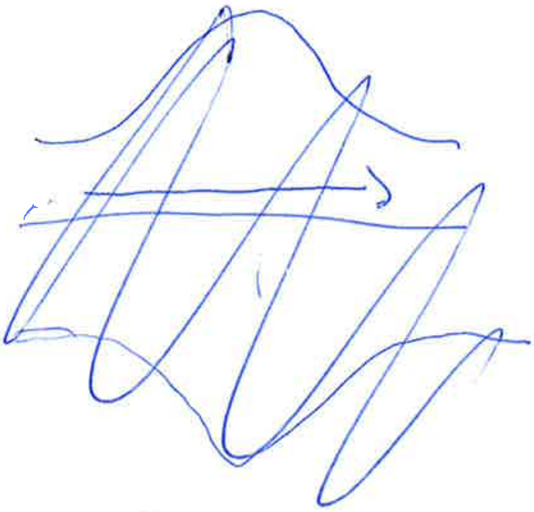
$$\lambda^2 - \left(\frac{\Delta}{2}\right)^2 - (\hbar\Omega_2)^2 = 0$$

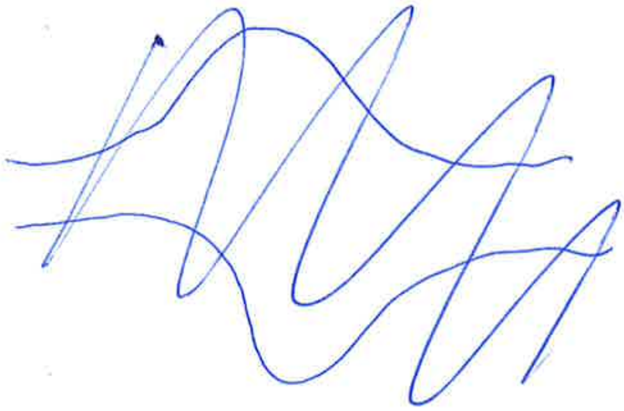
$$\Rightarrow \lambda = \pm\sqrt{\left(\frac{\Delta}{2}\right)^2 + (\hbar\Omega_2)^2}$$

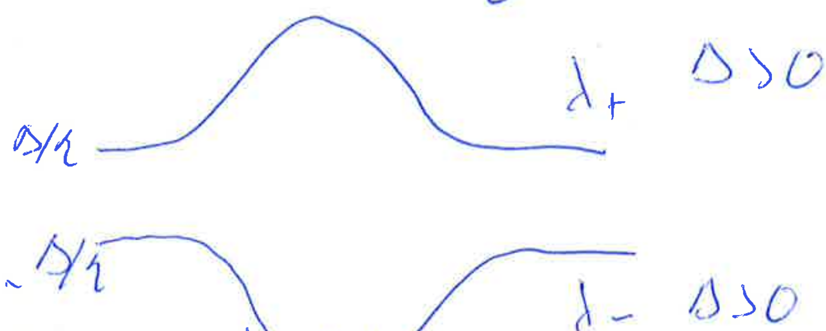



$\Rightarrow$








If $\Delta < 0$ the $|g\rangle$ is the ground state
sees the laser intensity $\propto (\Omega_2)^2$ as a potential dip.

$$\lambda = \pm\frac{\Delta}{2}\sqrt{1+\left(\frac{2\hbar\Omega_2}{\Delta}\right)^2} \approx \pm\frac{\Delta}{2}\left[1 + \frac{2\hbar^2\Omega_2^2}{\Delta^2}\right]$$

For large detunings depth goes like $\Omega_2^2/\Delta$

However, if atoms end up in the excited state new complications such as spontaneous decay might appear. How much population we have in the excited state when we are in the dressed eigenstate?

Sol

$$H\begin{pmatrix} a \\ b \end{pmatrix} = \lambda_\pm \begin{pmatrix} a \\ b \end{pmatrix} \quad \text{for eigenstates}$$

$$\Rightarrow \begin{pmatrix} -\frac{\Delta}{2} a + \hbar\Omega_2 b \\ \hbar\Omega_2 a + \frac{\Delta}{2} b \end{pmatrix} = \pm\sqrt{\left(\frac{\Delta}{2}\right)^2 + (\hbar\Omega_2)^2} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\left(-\frac{\Delta}{2} \mp \frac{\Delta}{2}\sqrt{1+\left(\frac{2\hbar\Omega_2}{\Delta}\right)^2}\right) a + \hbar\Omega_2 b = 0$$

$$\hbar\Omega_2 a + \left(\frac{\Delta}{2} \mp \frac{\Delta}{2}\sqrt{1+\left(\frac{2\hbar\Omega_2}{\Delta}\right)^2}\right) b = 0$$

for large $\Delta$

$$\Rightarrow \left[-\frac{\Delta}{2} \mp \frac{\Delta}{2}\left(1 + \frac{2(\hbar\Omega_2)^2}{\Delta^2}\right)\right] a + \hbar\Omega_2 b = 0$$

$$\hbar\Omega_2 a + \left(\frac{\Delta}{2} \mp \frac{\Delta}{2}\left(1+\frac{2(\hbar\Omega_2)^2}{\Delta^2}\right)\right) b = 0 \qquad b = \sqrt{1-a^2}$$

Lower sign: $+\dfrac{(\hbar\Omega_2)^2}{\Delta} a + \hbar\Omega_2 b = 0 \quad (*)$

$$\hbar\Omega_2 a + \left(\Delta + \frac{(\hbar\Omega_2)^2}{\Delta}\right) b = 0 \quad (**)$$

$$\frac{\hbar\Omega_2}{\Delta} a + \sqrt{1-a^2} = 0 \Rightarrow \left(\frac{\hbar\Omega_2}{\Delta}\right)^2 a^2 = 1 - a^2 \Rightarrow a^2 = \frac{1}{1+\left(\frac{\hbar\Omega_2}{\Delta}\right)^2}$$

$\Delta$ large

$$\Rightarrow a^2 \approx 1 - \left(\frac{\hbar \Omega_R}{\Delta}\right)^2$$

upper sign $a^2 \approx \left(\frac{\hbar \Omega_R}{\Delta}\right)^2$

Key point:

population of the other level goes like $\propto 1/\Delta^2$

Energy shift was $\propto 1/\Delta$

So we can have a combination of small population on the other level but relevant energy shift on ~~the~~ lower level!