

ECON-C4100 - Econometrics I

Lectures 10&11: Causal parameters part II - Instrumental variables regression

Otto Toivanen

Learning outcomes

- At the end of these lectures, you understand
 - 1 what simultaneous causality means
 - 2 what is meant by an **endogeneity** problem
 - 3 why it causes bias in the parameters
 - 4 what an instrumental variable is and why it solves the endogeneity problem
 - 5 what characteristics are required of an instrumental variable
 - 6 what one should pay attention to when using an instrumental variable

Learning outcomes

- At the end of these lectures, you understand
- 7 what a **reduced form** equation/parameter is
 - 8 what a **structural** equation/parameter is
 - 9 how to "manually" estimate a model with simultaneous causality
 - 10 what **2SLS** estimation means, how you do it and why it is used

Overview

- Demand experiment, market data analysis.
- Simultaneous causality.
- Instrumental variable (= IV) regression and 2SLS.
- NOTE: Instrumental variables are used in a large variety of contexts.
- We are exploring it in a particular but historically and practically very important setting.
- In *Applied Microeconometrics I* and *II* you will learn more about IV, its use and the interpretation of results.

Simultaneous causality

- Your task is to estimate the demand function for a homogeneous good sold at unit price.
- How would you do this in an experiment?

Simultaneous causality

- Your task is to estimate the demand function for a homogeneous good sold at unit price.
- How would you do this in an experiment?
- By changing the price yourself ("at random") and observing how many units are sold at each price.

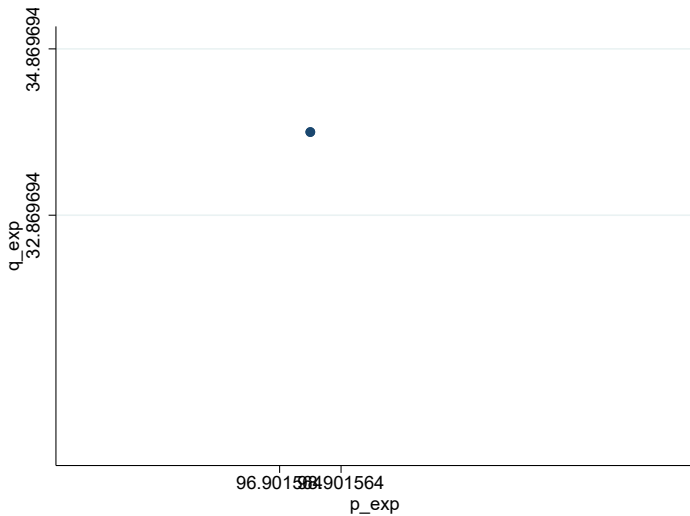
Experiment

- What does “choosing prices” at random mean?
 - ① We offer different randomized prices to individual consumers.
 - ② We offer different randomized prices each to a group of consumers.
 - ③ Think either of geographically separate markets, or a given market over time.

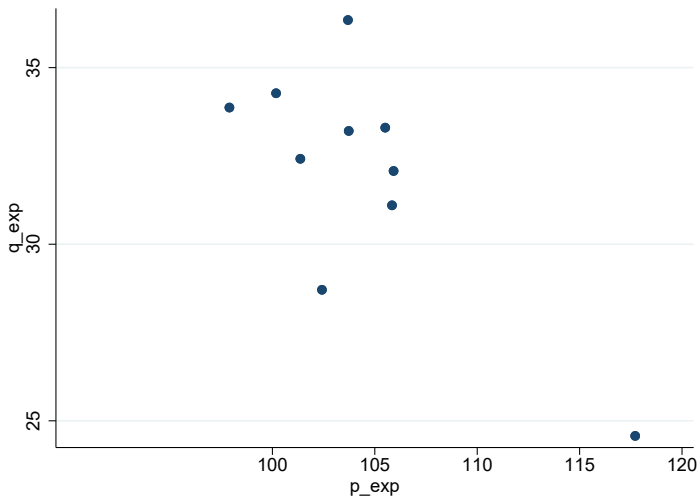
Experiment

- We record quantity sold at different prices.
- We study the outcomes.
- For illustration, I have conducted such an experiment in my computer.
- We will get to the details of how I do it later, but now just imagine I have conducted the experiment in a real market.

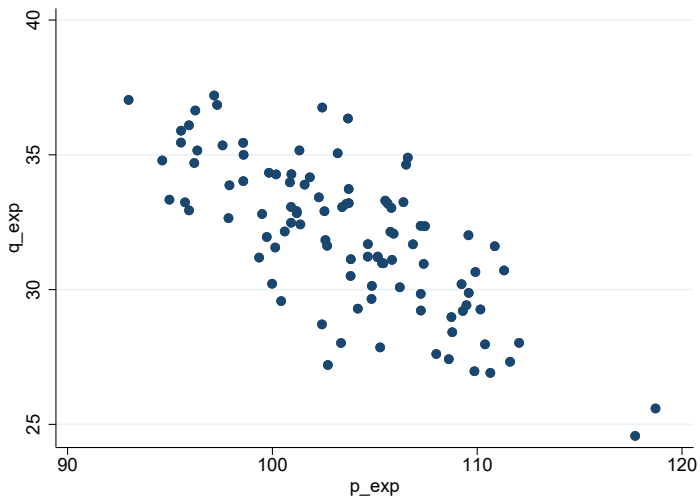
Experiment



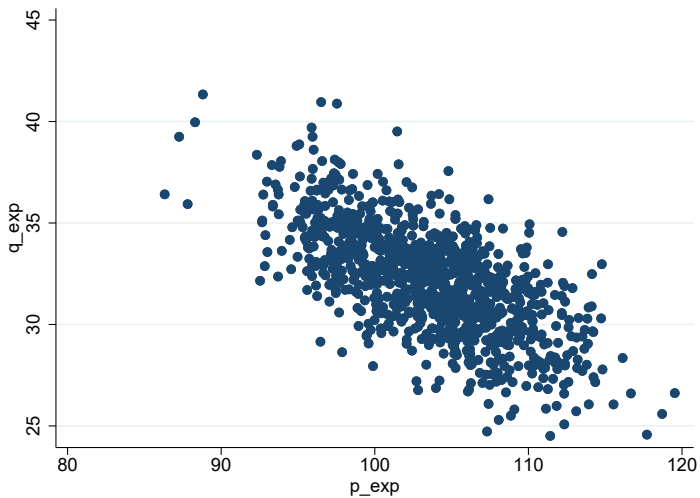
Experiment



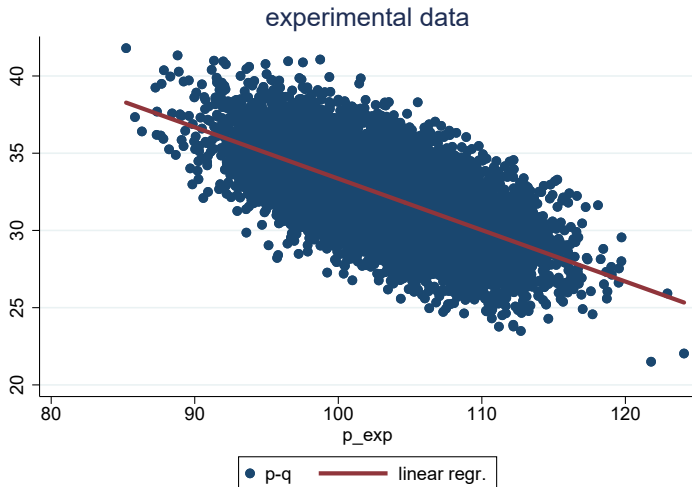
Experiment



Experiment



Experiment



Experiment

- Question: why does sold quantity vary between two experiments where the prices are identical?

Experiment

- Question: why does sold quantity vary between two experiments where the prices are identical?
- Answer: Demand is stochastic from the viewpoint of the econometrician.
- Let's study a simple set-up (the one I used in the experiment) in more detail.

Linear demand

- Demand function

$$Q_i = a - bP_i + \epsilon_i$$

- a = average intercept.
- b = slope.
- ϵ_i = market specific deviation from the average intercept.
- i = a particular market realisation.
- Question: Where does this demand function come from?
- Answer: From consumers making utility-maximizing choices.
- Exercise: What does the utility function look like that produces a linear demand function?

Linear demand

- Inverse demand function

$$P_i = \frac{a}{b} - \frac{1}{b}Q_i + \frac{1}{b}\epsilon_i = \alpha + \beta Q_i + \tilde{\epsilon}_i$$

Regression analysis

```
. regr q_exp p_exp
```

Source	SS	df	MS	Number of obs	=	10,000
Model	27500.3163	1	27500.3163	F(1, 9998)	=	6935.10
Residual	39645.8875	9,998	3.96538183	Prob > F	=	0.0000
Total	67146.2038	9,999	6.71529191	R-squared	=	0.4096
				Adj R-squared	=	0.4095
				Root MSE	=	1.9913

q_exp	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
p_exp	-.3329958	.0039986	-83.28	0.000	-.3408339	-.3251577
_cons	66.65341	.4145481	160.79	0.000	65.84081	67.466

Robustness analysis

```
. gen p_exp2 = p_exp^2
```

```
. regress q_exp p_exp*
```

Source	SS	df	MS	Number of obs	=	10,000
Model	27506.3195	2	13753.1598	F(2, 9997)	=	3468.48
Residual	39639.8843	9,997	3.96517798	Prob > F	=	0.0000
Total	67146.2038	9,999	6.71529191	R-squared	=	0.4096
				Adj R-squared	=	0.4095
				Root MSE	=	1.9913

q_exp	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
p_exp	-.4806486	.1200668	-4.00	0.000	-.7160038	-.2452935
p_exp2	.0007134	.0005798	1.23	0.219	-.0004231	.0018498
_cons	74.27605	6.208917	11.96	0.000	62.10532	86.44677

Parameters for inverse demand function

```
. scalar alpha_exp      = - _b[_cons] / _b[p_exp]
. scalar beta_exp       = -1 / _b[p_exp]

. scalar list alpha_exp beta_exp
alpha_exp = 200.16291
beta_exp  = 3.003041
```

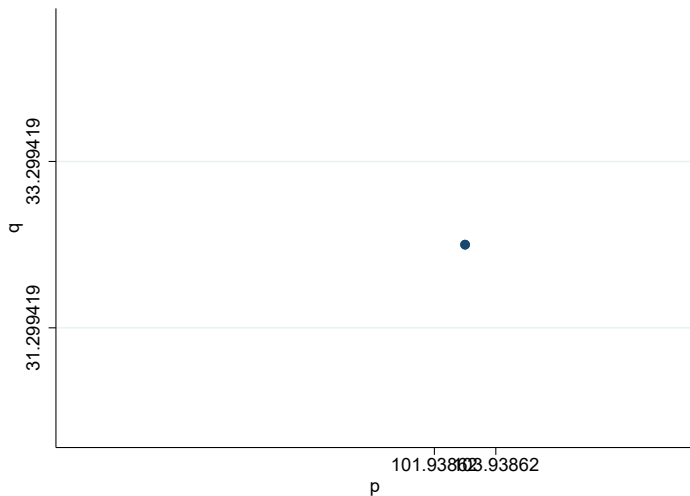
Market outcomes

- Assume you are an outside observer of a market, say, a prospective buyer of a firm or the competition authority.
→ you cannot run experiments.
- You would still want to know demand (to calculate e.g. price cost margins, consumer surplus).

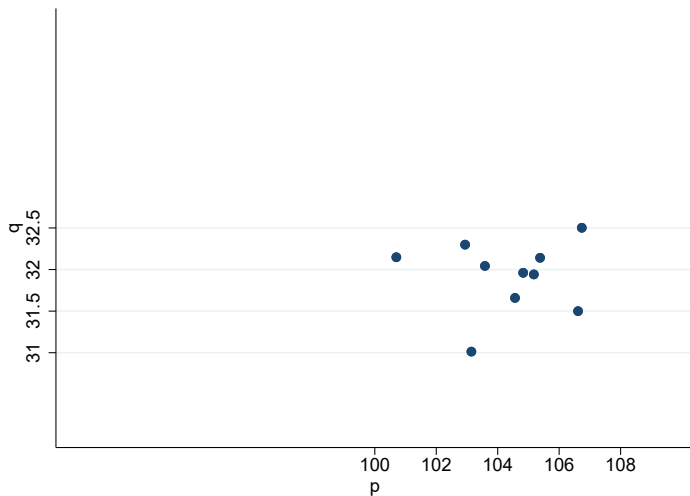
Market outcomes

- We collect data from the market.
- We observe pairs (P_i, Q_i) , $i = \text{market}$.
- Let's think how such pairs are determined, using a simple monopoly model.
- Let's allow a monopolist to choose prices instead in the same market.

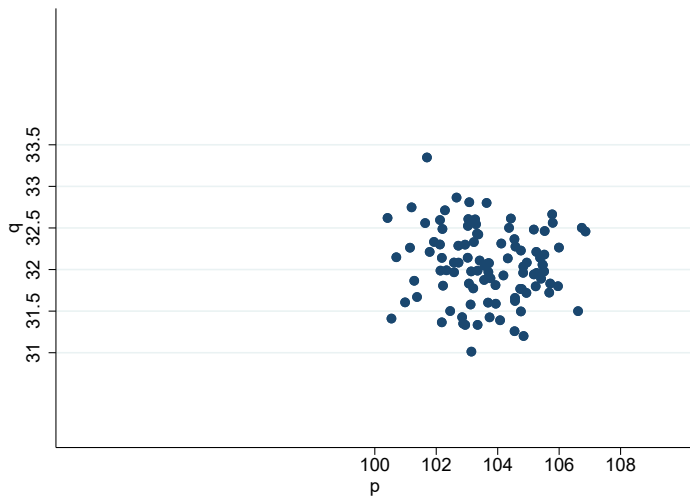
Market outcome data



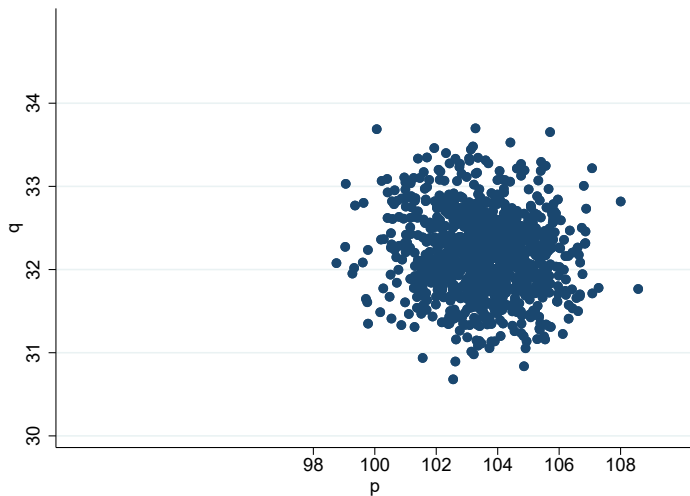
Market outcome data



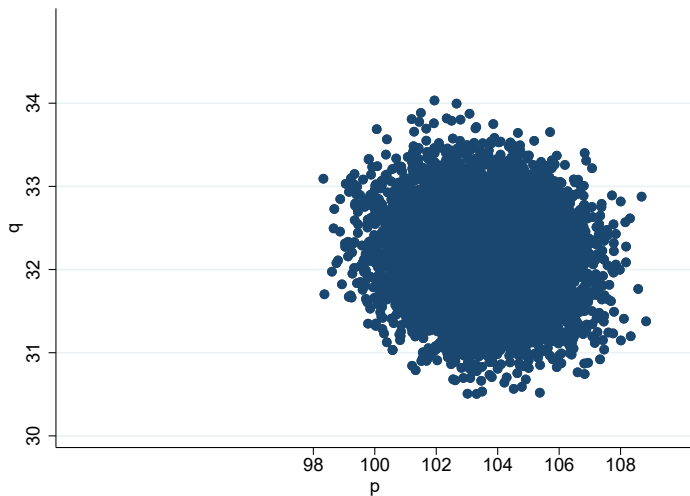
Market outcome data



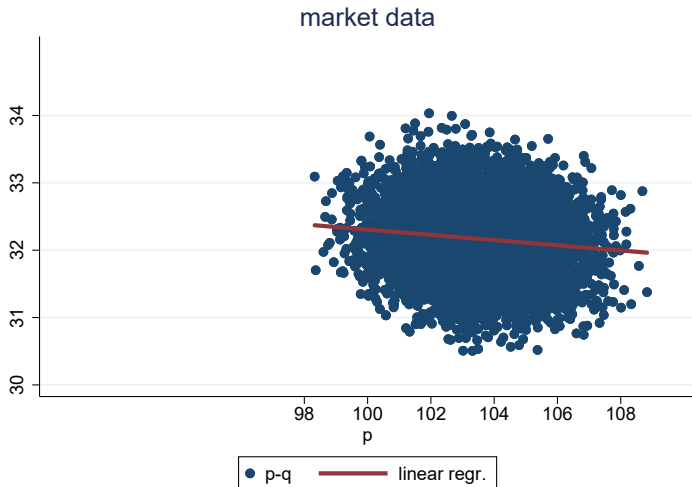
Market outcome data



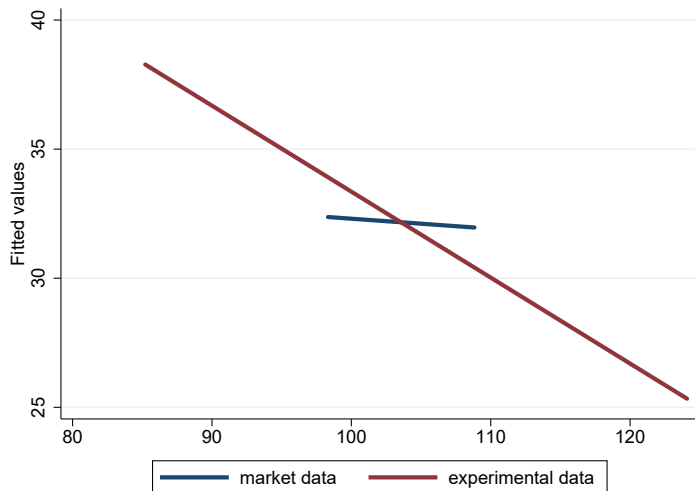
Market outcome data



Market outcome data



Market outcome data



Regression using market data

```
. regr q p
```

Source	SS	df	MS	Number of obs	=	10,000
Model	33.3509286	1	33.3509286	F(1, 9998)	=	135.06
Residual	2468.76463	9,998	.246925848	Prob > F	=	0.0000
				R-squared	=	0.0133
				Adj R-squared	=	0.0132
Total	2502.11556	9,999	.250236579	Root MSE	=	.49692

q	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
p	-.0386712	.0033275	-11.62	0.000	-.0451938	-.0321487
_cons	36.17252	.3444748	105.01	0.000	35.49728	36.84776

Parameter comparison for demand and inverse demand functions

```
. scalar alpha_ols          = -_b[_cons] / _b[p]
. scalar beta_ols           = -1 / _b[p]
. scalar a_ols              = _b[_cons]
. scalar b_ols              = -_b[p]
. scalar list a_exp b_exp a_ols b_ols
  a_exp = 66.653405
  b_exp = .33299579
  a_ols = 36.172515
  b_ols = .03867121
. scalar list alpha_exp beta_exp alpha_ols beta_ols
alpha_exp = 200.16291
beta_exp = 3.003041
alpha_ols = 935.38611
beta_ols = 25.859029
```

Challenge with market data

- Price quantity pairs are a leading example of simultaneous causality.
- This generalizes to more complicated markets with:
 - ① differentiated goods
 - ② multiproduct firms
 - ③ endogenous entry and exit
 - ④ dynamic considerations (e.g. collusion, durable goods, ...)
 - ⑤ advertising
 - ⑥ ...

Challenge with market data

- Need to address simultaneous causality.
- → need to understand and exploit determinants of price and quantity.
- How did the experiment solve the problem?
- By having the researcher shift (=change) prices instead of the firm.

Linear monopoly

- Demand function

$$Q_i = a - bP_i + \epsilon_i$$

- a = average intercept.
- b = slope.
- ϵ_i = market specific deviation from the average intercept.
- NOTE: We assume the firm observes **all** these parameters.
- Question: What if the firm did not observe our "unobservable", i.e., ϵ_i ?

Linear monopoly

- Inverse demand function

$$P_i = \frac{a}{b} - \frac{1}{b}Q_i + \frac{1}{b}\epsilon_i = \alpha + \beta Q_i + \tilde{\epsilon}_i$$

How to get the supply function?

- Inverse demand function.
- Need to specify costs of production: constant marginal cost

$$c_i = c_0 + c_1 z_i + \eta_i$$

- Note: Here one should have an understanding of the production technology.

How to get the supply function?

$$c_i = c_0 + c_1 z_i + \eta_i$$

- c_0 = average of marginal cost when $z_i = 0$.
- z_i = a component of marginal cost that varies across markets (cost of raw materials / unit of output, cost of labor / unit of output, .., with c_1 being the number of units of the input needed to produce one unit of output).
- η_i = "shock" to average marginal cost / deviation from the avg. This is observed by the firm but not by the econometrician.
- If the firm did not observe η_i , how could it take it into account in its decision?

How to get the supply function?

The monopolist's problem:

$$\max_{P_i} \pi_i = (P_i - c_i) \times Q_i$$

Equilibrium price:

$$\begin{aligned} P_i &= \frac{a}{2b} + \frac{1}{2}c_i + \frac{1}{2b}\epsilon_i \\ &= \frac{a}{2b} + \frac{1}{2}(c_0 + c_1z_i + \eta_i) + \frac{1}{2b}\epsilon_i \end{aligned}$$

How to get the supply function?

Equilibrium quantity

$$Q_i = \frac{a}{2} - \frac{b}{2}(c_0 + c_1 z_i + \eta_i) + \frac{1}{2} \epsilon_i$$

How to get the supply function?

Equilibrium price and equilibrium quantity are functions of:

How to get the supply function?

Equilibrium price and equilibrium quantity are functions of:

- 1 (fixed) demand parameters a and b

How to get the supply function?

Equilibrium price and equilibrium quantity are functions of:

- ① (fixed) demand parameters a and b
- ② (fixed) supply side parameters c_0 and c_1

How to get the supply function?

Equilibrium price and equilibrium quantity are functions of:

- 1 (fixed) demand parameters a and b
- 2 (fixed) supply side parameters c_0 and c_1
- 3 variable cost determinant z_i

How to get the supply function?

Equilibrium price and equilibrium quantity are functions of:

- 1 (fixed) demand parameters a and b
- 2 (fixed) supply side parameters c_0 and c_1
- 3 variable cost determinant z_i
- 4 cost shock η_i

How to get the supply function?

Equilibrium price and equilibrium quantity are functions of:

- 1 (fixed) demand parameters a and b
- 2 (fixed) supply side parameters c_0 and c_1
- 3 variable cost determinant z_i
- 4 cost shock η_i
- 5 demand shock ϵ_i

Market data challenge

- Both eq. price and eq. quantity are functions of:

① demand shock ϵ_i

② supply (cost) shock η_i

→ simultaneous causality.

→ price is an endogenous variable (ϵ_i and η_i are the "omitted" variables that affect both price and quantity).

Market data solution

- We want to learn the demand curve.
- Could we mimic the experimental approach with market data?
- Needed: Something that shifts firm's (supply) decision at "random".
- Random = without being affected by demand shock ϵ_i .

Experiment #2

- Imagine the firm still sets the price,
- but we choose (=randomize) $z_i = z_i^{exp}$.
- Recall that $c_i = c_0 + c_1 z_i^{exp} + \eta_i$.
→ we "shift" firm's marginal cost.

Experiment #2

- Now the firm sets each time the price

$$P_i = \frac{a}{2b} + \frac{1}{2}(c_0 + c_1 z_i^{\text{exp}} + \eta_i) + \frac{1}{2b} \epsilon_i$$

→ equivalent to running an experiment.

- Change in price due to (known) change in z_i^{exp} .

Experiment #2

- Imagine we raise z_i^{exp} by 1 unit.
- By how much does
 - ① marginal cost $c_i = c_0 + c_1 z_i + \eta_i$ change? Answer: c_1 .
 - ② price change? Answer: $\frac{1}{2}c_1$ (by the equilibrium price equation).
 - ③ demand change? Answer: $-b\frac{1}{2}c_1$ (by the equilibrium quantity equation).

Experiment #2

- How could we get the slope of the demand function from these changes?
- Yes, by dividing the change in quantity by the change in demand:

$$-\frac{b\frac{1}{2}c_1}{\frac{1}{2}c_1} = -b$$

Experiment #2

- How could we get those numbers?

1. Regress P_i on z_i^{exp} to get $\frac{1}{2}c_1$.

$$\begin{aligned}P_i &= \frac{a}{2b} + \frac{1}{2}(c_0 + c_1 z_i^{exp} + \eta_i) + \frac{1}{2b}\epsilon_i \\&= \left(\frac{a}{2b} + \frac{1}{2}c_0\right) + \frac{1}{2}c_1 z_i^{exp} + \left(\frac{1}{2}\eta_i + \frac{1}{2b}\epsilon_i\right) \\&= \gamma_0 + \gamma_1 z_i + e_i \\ \gamma_1 &= \frac{1}{2}c_1, \gamma_0 = \left(\frac{a}{2b} + \frac{1}{2}c_0\right)\end{aligned}$$

Experiment #2

2. Regress Q_i on z_i^{exp} to get $-b\frac{1}{2}c_1$.

$$Q_i = \mu_0 + \mu_1 z_i + w_i$$

$$\mu_1 = -b\frac{1}{2}c_1$$

- One can link the Q_i equation to the equilibrium Q_i -expression just like was done for P_i on the previous slide.
- The P_i and Q_i regression equations on this and previous slide are called **reduced form** equations.

Experiment #2

- The regression equations we estimated, i.e.,

$$P_i = \gamma_0 + \gamma_1 z_i + e_i$$

$$Q_i = \mu_0 + \mu_1 z_i + w_i$$

- are called **reduced form** equations.

What are reduced form equations?

- Proper definition: A **reduced form equation** is an equation whose parameters are functions of the **structural** parameters.
- In our model, structural parameters are a , b , c_0 and c_1 .
 - ① They are building blocks of the theory model
 - ② They are determined outside our model
 - ③ They are not functions of any other parameters (or variables) of the model
- The parameters (γ_0 , γ_1 , μ_0 , μ_1) of the two regressions (P_i on z_i and Q_i on z_i) we ran are functions of the structural parameters (= those in the theoretical model, i.e., a , b , c_0 , c_1).

What are reduced form equations?

- Commonly used meaning: A **reduced form equation** is an equation that is not derived from a theoretical model.
- Examples: The regressions in the papers we have studied, i.e.,
 - Bronnenberg et al., 2015.
 - Kleven et al., 2011.

Experiment #2

- When would this work?
 - ① z_i^{exp} **has to have** an impact on the decision of the firm, i.e., have an effect on c_i .
→ c_1 cannot be (insignificantly different from) zero.
 - ② z_i^{exp} **may not have** an effect on Q_i directly, but only via c_i .

Let's regress P on z .

```
. regr p z
```

Source	SS	df	MS	Number of obs	=	10,000
Model	9783.4964	1	9783.4964	F(1, 9998)	=	7814.03
Residual	12517.9151	9,998	1.25204192	Prob > F	=	0.0000
				R-squared	=	0.4387
				Adj R-squared	=	0.4386
Total	22301.4115	9,999	2.23036418	Root MSE	=	1.1189

p	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
z	.4986242	.0056407	88.40	0.000	.4875673	.5096812
_cons	101.0138	.0304066	3322.11	0.000	100.9542	101.0734

```
. scalar red_1 = _b[z]
```

Let's regress Q on z .

```
. regr q z
```

Source	SS	df	MS	Number of obs	=	10,000
Model	1121.21997	1	1121.21997	F(1, 9998)	=	8117.89
Residual	1380.89558	9,998	.138117182	Prob > F	=	0.0000
				R-squared	=	0.4481
				Adj R-squared	=	0.4481
Total	2502.11556	9,999	.250236579	Root MSE	=	.37164

q	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
z	-.1687997	.0018735	-90.10	0.000	-.1724721	-.1651273
_cons	33.01561	.0100991	3269.17	0.000	32.99581	33.0354

```
. scalar red_2 = _b[z]
```

Let's calculate b .

```
. scalar b_red    = red_2 / red_1  
. scalar list b_red  
   b_red = -.33853094
```

Instrumental variable

- **Instrumental variable** = a variable that **causes variation** in explanatory variable X (price) but **does not affect** dependent variable Y (demand) directly.
- If the variable cost component z_i varies "at random", i.e., without affecting demand directly,

→ market data allows us to use the "experimental approach" indirectly.

Approach #2

- Could we proceed differently?
 - ① Regress P_i on z_i . Calculate predicted price $\hat{P}_i = \hat{\gamma}_0 + \hat{\gamma}_1 z_i$.
 - ② Regress Q_i on \hat{P}_i to get b (and a).
- Equation $Q_i = a - bP_i + \epsilon_i$ is a **structural** equation. Why?

Approach #2

- Could we proceed differently?
 - ① Regress P_i on z_i . Calculate predicted price $\hat{P}_i = \hat{\gamma}_0 + \hat{\gamma}_1 z_i$.
 - ② Regress Q_i on \hat{P}_i to get b (and a).
- Equation $Q_i = a - bP_i + \epsilon_i$ is a **structural** equation. Why?
- Because it is a function of structural parameters only (+ P_i which is determined within the model).

Approach #2

- The parameters of a structural equation are part of the **model primitives**, i.e.,
 - they are not determined within the model
 - they are not functions of other parameters of the model
- **Reduced form parameters** are functions of structural parameters.

Regress P on z , create predicted values

```
. regr p z
```

Source	SS	df	MS	Number of obs	=	10,000
Model	9783.4964	1	9783.4964	F(1, 9998)	=	7814.03
Residual	12517.9151	9,998	1.25204192	Prob > F	=	0.0000
				R-squared	=	0.4387
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Total	22301.4115	9,999	2.23036418	Root MSE	=	1.1189

p	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
z	.4986242	.0056407	88.40	0.000	.4875673	.5096812
_cons	101.0138	.0304066	3322.11	0.000	100.9542	101.0734

```
. predict p_hat
```

```
(option xb assumed; fitted values)
```

Regress Q on \hat{P}

```
. regress q p_hat
```

Source	SS	df	MS	Number of obs	=	10,000
Model	1121.21997	1	1121.21997	F(1, 9998)	=	8117.89
Residual	1380.89558	9,998	.138117182	Prob > F	=	0.0000
				R-squared	=	0.4481
				Adj R-squared	=	0.4481
Total	2502.11556	9,999	.250236579	Root MSE	=	.37164

q	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
p_hat	-.3385309	.0037573	-90.10	0.000	-.345896 - .3311659
_cons	67.21192	.3889483	172.80	0.000	66.4495 67.97433

Why / how do these approaches work?

- 1 Regress P_i on z_i to get $\frac{1}{2}c_1 = \frac{\text{cov}(P_i, z_i)}{\text{var}(z_i)}$.
 - 2 Regress Q_i on z_i to get $-b\frac{1}{2}c_1 = \frac{\text{cov}(Q_i, z_i)}{\text{var}(z_i)}$.
- $$\rightarrow -b = \frac{\text{cov}(Q_i, z_i)}{\text{cov}(P_i, z_i)}.$$

Regress Q on \hat{P}

$$b = \frac{\text{cov}(Q_i, \hat{P}_i)}{\text{var}(\hat{P}_i)} = \frac{\text{cov}(Q_i, \hat{\gamma}_0 + \hat{\gamma}_1 z_i)}{\text{var}(\hat{\gamma}_0 + \hat{\gamma}_1 z_i)}$$

$$\rightarrow b = \frac{\hat{\gamma}_1 \text{cov}(Q_i, z_i)}{\hat{\gamma}_1^2 \text{var}(z_i)} = \frac{1}{\hat{\gamma}_1} \frac{\text{cov}(Q_i, z_i)}{\text{var}(z_i)}$$

$$\text{because } \hat{\gamma}_1 = \frac{\text{cov}(P_i, z_i)}{\text{var}(z_i)} \rightarrow$$

$$b = \frac{\text{var}(z_i)}{\text{cov}(P_i, z_i)} \frac{\text{cov}(Q_i, z_i)}{\text{var}(z_i)} = \frac{\text{cov}(Q_i, z_i)}{\text{cov}(P_i, z_i)}$$

2SLS / instrumental variables regression

- In practice, want to use the so called Two Stage Least Squares (2SLS) or instrumental variables regression command. In Stata, `ivregress` or from SSC `ivreg2`.
- Manual and `ivregress` command(s) produce same point estimates, but the latter corrects the standard errors.
- This is important, as the manual approach yields too small standard errors: It ignores the uncertainty in the parameters $\hat{\gamma}_0$ and $\hat{\gamma}_1$ used to calculate \hat{P}_i .

2SLS estimation of demand

```
. ivregress 2sls q (p = z)
```

```
Instrumental variables (2SLS) regression      Number of obs   =    10,000
                                             Wald chi2(1)    =    2506.07
                                             Prob > chi2     =    0.0000
                                             R-squared       =    .
                                             Root MSE       =    .66888
```

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
q						
p	-.3385309	.0067624	-50.06	0.000	-.351785	-.3252769
_cons	67.21192	.7000301	96.01	0.000	65.83988	68.58395

```
Instrumented:  p
Instruments:   z
```

Requirements for an instrument

- Think of our normal regression $Y = \beta_0 + \beta_1 X + u$.
 - ① **Instrument relevance:** The instrument Z has to affect the (endogenous) explanatory variable X of the equation of interest ("2nd stage equation") in the equation

$$X = \alpha_0 + \alpha_1 Z + v.$$

- ② **Instrument exogeneity:** The instrument Z may not be correlated with the error term of the equation of interest, i.e.,

$$\text{cov}(Z, u) = 0.$$

Instrument relevance / Weak instrument

- Relevance = instrument Z needs to be "correlated enough" with the endogenous explanatory variable X .
- What happens when $cov(Z, X) \rightarrow 0$?
- β_1 becomes undefined!

Instrument relevance / Weak instrument

→ you want to check that your instrument is relevant.

= you don't have a **weak instrument**.

- Rule of thumb: F-statistic of Z when you regress X on Z (and possible further controls) > 10 .
- Note: With 1 instrument, F-test is the square of the t-test.
- Notice that the test for weak instruments is stricter than our usual 5% confidence level (t-stat 2).

2SLS estimation

```
. estat firststage
```

First-stage regression summary statistics

Variable	R-sq.	Adjusted R-sq.	Partial R-sq.	F(1,9998)	Prob > F
p	0.4387	0.4386	0.4387	7814.03	0.0000

Minimum eigenvalue statistic = 7814.03

Critical Values # of endogenous regressors: 1
Ho: Instruments are weak # of excluded instruments: 1

	5%	10%	20%	30%
2SLS relative bias	(not available)			
2SLS Size of nominal 5% Wald test	16.38	8.96	6.66	5.53
LIML Size of nominal 5% Wald test	16.38	8.96	6.66	5.53

Instrument relevance / Weak instrument

- There are more sophisticated tests.
- There are ways of allowing for weak instruments.
- We leave all that for later courses.
- Good instruments are hard to find...

Instrument correlated with error

= "**exogeneity**" assumption or **exclusion restriction**:

$$\mathbb{E}[u|\mathbf{X}] = 0$$

If this condition does not hold \rightarrow biased estimate of β_1 .

- Similar to omitted variable bias.

Instrument correlated with error

- What can be done?
 - ① Strong story for why no correlation between instrument and error.
 - ② With multiple instruments, may do tests.
 - ③ There are ways of allowing for (some) correlation to check robustness of your results (for later).