ECON-L1350 - Empirical Industrial Organization PhD II: Topics

Static Entry Models: Lecture 2

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Estimating a model with predetermined order of entry (Toivanen and Waterson, 2005)

Ordered entry

- Used in several papers, notably Berry (1992).
- How does this solve the problem in a 2-firm game?
- By imposing a entry order, the set of (u_1, u_2) yields a unique equilibrium because the first mover enters.
- Problem with BR approach: limited in what can be identified, only allows firm heterogeneity in a limited way (Schaumans and Verboven, 2015).

The Toivanen and Waterson (2005) set-up

- UK fast food 1991 - 1995, some 450 markets.
  1. Market well defined in terms of goods.
  2. Entry centrally decided in both firms.
  3. Firms (McD, BK) are expanding (and no exit). $\rightarrow$ can assume existing outlets predetermined.
  4. Good proxies for local markets + data on them.
  5. Market can reasonably be though of as a duopoly.
- Multi"plant" duopoly.
- Seems reasonable to assume McD moves first.

The data

TABLE 2 Statistical Information on Fast Food Outlets

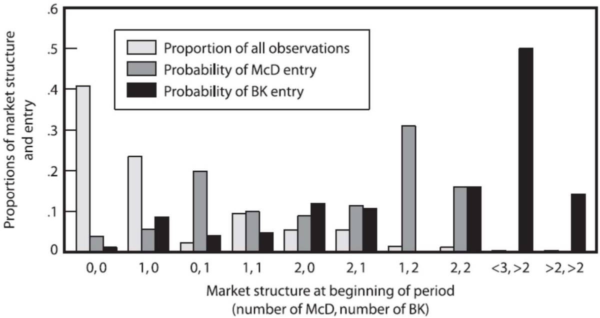
All Districts	BK	McD
Total number of outlets at end of 1995	392	637
Transit outlets	98	45
Three London boroughs	21	27
Total number of exits since chain started	N.K.	4
Estimation sample		
(452 districts, nontransit outlets)		
Stock at end of 1995	273	561
Number of new outlets 1991–1995	175	196
Number of districts entered in 1991–1995	126	148
Proportion of outlets franchised	.73	.2
Mean number of outlets/district by		
beginning of year		
1991	.217	.809
	(.477)	(1.085)
1992	.316	.905
	(.595)	(1.229)
1993	.354	.976
	(.655)	(1.309)
1994	.416	1.073
	(.747)	(1.396)
1995	.487	1.175
	(.854)	(1.473)
End of 1995	.603	1.239
	(.975)	(1.522)

Standard deviations in parentheses.

The raw data

FIGURE 1

MARKET STRUCTURE AND ENTRY



Challenges to identification

- Some markets are better in terms of (time-invariant) unobservables.
- Solutions:
  1. Random effects probit. Assumes RE uncorrelated with X.
  2. LPM with market FE.
  3. (Chamberlinian) Logit with market FE.
- Results robust to estimator and a number of other things.

Reduced form Probit results

TABLE 4 **Marginal Effects of Market Structure Indicators**

Market Structure Dummies	BK	McD
$M1, B0$	.1089***	.0413***
	(.0315)	(.0150)
$M1, B1$	.0185	.0589**
	(.218)	(.0247)
$M0, B1$	.0528	.0345
	(.0404)	(.0294)
$M2, B0$	.1834***	.1448***
	(.0553)	(.0428)
$M2, B1$	.0442	.1759***
	(.0313)	(.0524)
$M0, B2$	—	.1585**
		(.0847)
$M1, B2$	—	.1233
		(.0798)
$M2, B2$	−.0074	.2341**
	(.0299)	(.1120)
$M3, B0$	.2011***	−.0037
	(.0935)	(.0066)
$M3, B <$	.0291	.2584***
	(.0281)	(.0693)
$M <, B3$	−.0102	
	(.0316)	—
ρ^{a}		
Number of observations	2,260	2,260

Structural model

- TW follow Bresnahan (1992) in terms of specifying the profit function.
- They allow both (expected) market size $S(.)$ and variable profits per customer $V(.)$ to be functions of the # of own and rival outlets.
- They assume McD is the leader and BK the follower $\rightarrow$ McD takes into account what effect its own entry has on BK entry decision.

Structural model

- Estimation algorithm:
  1. Estimate BK entry decision taking existing outlets, including McD entry in period t, as given.
  2. Calculate $\hat{BK}(|McD)$ based on the estimates.
  3. Estimate McD entry decision taking expected BK entry decision (conditional on McD entry/no entry) into account. This necessitates simulation.
- TW include a RE into $S(.)$ + project it onto # own and rival existing outlets (to capture potential correlations).
- Notice they only report standard probit results (as including RE made no difference).
- Model estimated using simulated maximum likelihood.

Structural model results

TABLE 5 **Structural Estimations**

Function/Variable	(1) BK Standard Probit	(4) McD Standard Probit
Market size		
β_{i_1}/Youth	$-.2848^{***}$	$-.2282^{**}$
	(.0884)	(.1052)
β_{i_2}/Pension	$-.4308^{***}$	$-.4205^{***}$
	(.1019)	(.1113)
θ_{iS_2}/Rival Outlets	$.1820^{**}$	$.3203^{**}$
	(.0795)	(.1391)
θ_{iS_3}/Own Outlets in Neighboring Markets	$.1139^{***}$	$-.0164$
	(.0409)	(.0162)
θ_{iS_4}/Rival Outlets in Neighboring Markets	$-.0503$	.0012
	(.0173)	(.0077)
Variable profits		
γ_{i_1}/Area	$-.4740$	$-.5269$
	(.3251)	(.7069)
γ_{i_2}/Wage	$.0474^{***}$	.0200
	(.0169)	(.0203)
θ_{iV_1}/Own Outlets	1.3980^{***}	1.8616^{***}
	(.4014)	(.6306)
θ_{iV_2}/Rival Outlets	.1360	.2526
	(.1711)	(.3124)
θ_{iV_3}/Own Outlets $*$ Rival Outlets	$-.0590^{***}$	$-.3257^{***}$
	(.0238)	(.1219)
θ_{iV_4}/Own Outlets in Neighboring Markets	$-.1091$	$-.3291$
	(.0902)	(.2431)
θ_{iV_5}/Rival Outlets in Neighboring Markets	$-.0381$	$.2653^{*}$
	(.0485)	(.1538)

Estimating an incomplete entry model (Tamer, 2003)

2-firm example

- Following e.g. Tamer (2003), let's specify the following static full information entry game:

Table: 1

Discrete game with stochastic payoff

	$y_2 = 0$	$y_2 = 1$
$y_1 = 0$	$0, 0$	$0, x_2\beta_2 + u_2$
$y_1 = 1$	$x_1\beta_1 + u_1, 0$	$x_1\beta_1 + \delta_1 + u_1, x_2\beta_2 + \delta_2 + u_2$

- Note what full information means in this context.
- x_j = firm-specific observable affecting profits.
- u_j = firm-specific unobservable ("shock") affecting profits.
- β_j, δ_j: parameters to be estimated.
- Let's assume $\beta_j > 0$, $\delta_j < 0$.

2-firm example

- This maps directly into the following econometric model:

$$y_1^* = x_1\beta_1 + y_2\delta_1 + u_1$$
$$y_2^* = x_2\beta_2 + y_1\delta_2 + u_2$$

$$y_j = \begin{cases} 1 \text{ if } y_j^* \geq 0 & \text{for } j = 1, 2 \\ 0 \text{ otherwise} \end{cases} \tag{1}$$

Coherent and complete econometric models

- Problem: multiple equilibria for a set of (u_1, u_2).
- **Coherency:** A model is coherent if it admits a *"well-defined reduced form"*.
- One can write a well-defined likelihood function for a coherent model.
- One can achieve coherency through theoretical and/or statistical assumptions.
- A **complete** model *"asserts that a random variable y is a function of a random pair (x, u), where x is observable and u is not"*.
- **Incompleteness:** the relationship between y and (x, u) is a correspondence, not a function.
- However, incomplete models (may) contain information!

How does an incomplete model solve the problem?

- BR solved the problem by concentrating on the number, not the identity of firms.
- Order of entry solves the problem by "brute force" imposing more structure.
- Tamer (2003) *circumvents* the problem.

How does an incomplete model solve the problem?

- In model defined by inequality restrictions one may identify the set of parameter values that satisfy those restrictions.
- One may or may not achieve point identification.

How does an incomplete model solve the problem?

- In Tamer's model the following hold.

$$P_1 = Pr[1,1|x] = \quad Pr(u_1 > -x_1\beta_1 - \delta_1 \; ; \; u_2 > -x_2\beta_2 - \delta_2)$$
$$P_2 = Pr[0,0|x] = \quad Pr(u_1 < -x_1\beta_1 \; ; \; u_2 < -x_2\beta_2)$$

- One can also write a condition with upper and lower bounds for $Pr[1,0|x]$ and $Pr[0,1|x]$.

Assumptions & Theorem

- **Assumption 1.** We have an i.i.d. sample such that $0 < Pr[(y_1, y_2)|(x_1, x_2)] < 1$ for all combinations of observables.
- **Assumption 2.** Let $U = (u_1, u_2)$ be a random vector independent of x with a known joint distribution function that is absolutely continous with mean 0 and unknown covariance matrix.
- **Assumption 2.** δ_1, δ_2 are negative.
- **Theorem 1.** Under the stated assumptions and as long as at least the one of the observables is continuous, the model parameters are identified.

Intuition

- Think of the region where (almost) surely firm 2 (1) does not enter.
- Think of the region where (almost) surely firm 2 (1) enters.
- Key insight: let x_j "take you" to each of these regions.
- This is a variant of the "identification at infinity" argument sometimes used in sample selection models for example.

Simple ML estimator

- One can then write down the likelihood fcn:

$$
\begin{aligned}
L_{ML} = \prod_{i=1}^{n} & P_1^{y_1 y_2} \\
& \times P_2^{(1-y_1)(1-y_2)} \\
& \times (1 - P_1 - P_2)^{[(1-y_1)y_2 + (1-y_2)y_1)]} \}
\end{aligned}
$$

- Notice three parts of the likelihood fcn.:
  1. Probability of duopoly
  2. Probability of no firm entering
  3. Probability of neither of the above happening.
- This likelihood function may be difficult to estimate in practice.

Incomplete information entry models

Approach

- This literature started with the seminal paper of Seim (2006).
- We will follow Bajari et al. (2010); see also Aguirregabiria (2021).
- We concentrate on a 2-firm game of incomplete information.

Theoretical model

- With incomplete information we often achieve uniqueness (of PBE).
- Assume profits are given by

$$\Pi_i m = x_{im}\beta_i - \delta_i a_{jm} - \epsilon_{im}$$

- where
  - Π_{im} = profits of firm i from market m, $i = 1, 2$.
  - x_{im} = (firm-) market level observables.
  - a_{jm} = indicator function for firm j having entered market m.
  - β_i, δ_i are parameters to be estimated.
  - ϵ_{im} is a market-firm specific productivity shock (from a known distribution) known to firm i but not to the econometrican nor firm j.

Theoretical model

- Let's denote the strategy function of firm i by $\alpha_i : S \times R \rightarrow \{0, 1\}$.
- Firm i's expected profits, given state s_m, are

$$\pi_i(s_m, \epsilon_{im}, \alpha_j) = s_{im}\beta_i - \epsilon_{im} - \delta_i \int 1\{\alpha_j(s_m, \epsilon_{jm} dG_j(\epsilon_{jm})\} \quad (2)$$

- The integral term is firm i's "guess" of what firm j is going to do, given the state S_m. We can rewrite eqn (2) as:

$$\pi_i(s_m, \epsilon_{im}, \alpha_j) = s_{im}\beta_i - \epsilon_{im} - \delta_i P_j(s_m) \quad (3)$$

Equilibrium

- Firm i's best response function is given by

$$\begin{aligned} b_i(s_m, \epsilon_{im}, \alpha_j) =& 1[\pi_i \geq 0] \\ =& 1[\epsilon_{im} \leq s_{im}\beta_i - \delta P_j(s_m)] \end{aligned}$$

- We can then define the *best response probability function* (by integration) as

$$\Psi(s_m, P_j) = G_i(s_{im}\beta_i - \delta P_j) \tag{4}$$

Equilibrium

- A BNE is a set of strategy functions such that for all players and any realization of $(s_m, \epsilon_{im}$.

$$\alpha_i^*(s_m, \epsilon_{im}) = G_i(s_{im}\beta_i - \delta P_j^*) \tag{5}$$

- For the set of (equilibrium) strategies α^* we can define a set of probability functions such that

$$P_i^*(s_m) = \Psi(s_m, P_j^*) = G_i(s_{im}\beta_i - \delta P_j^*) \tag{6}$$

- Note that is is convenient to think in terms of choice probabilities P^* instead of strategies α^*; there is a 1:1 relation between them.

Estimation

- Imagine you observe data from M independent markets $\{x_{im}, a_{im}\}$, $i = 1, 2,\ m = 1, ..., M$.
- You can identify the probability of firm j entering market m, $\hat{P}_j(x)$ (nonparametrically) from the data.
- What is the best response of firm i? enter iff

$$x_{im}\beta_i - \epsilon_{im} - \delta\hat{P}_j \geq 0 \qquad (7)$$

- which yields the probability of entering market m

$$G_i(x_{im}\beta_i - \delta\hat{P}_j(x)) \qquad (8)$$

Estimation & identification

- Estimation includes the following steps:
  1. Estimate a reduced form probability of entry equation for both firm.
  2. Generate $\hat{P}_i(x)$ for both firms.
  3. Plug the estimated responses to the rival's decision equation and estimate.
  4. Bootstrap s.e.'s.
- Notice that key to identification is a pair of exclusion restrictions: x_{im} does not directly enter the objective function of firm j for $i \neq j \ i, j = 1, 2$.

Discussion

- In the perfect information environment plugging the entry decision of the rival on the RHS was a problem. What is different now?
- In the perfect information environment we assumed all firms observe everything.
- In the imperfect information environment we assume that the rivals are as clueless as the econometrician. Does this make sense?
- What about multiple equilibria?
- The model generalizes to
  - more than 2 firms
  - opening more than 1 outlet at a time
  - more complicated informational environments. See Grieco (2014) and Aguirregabiria (2021).
  - In a 2-step approach (like above) the assumption is that the same equilibrium played. Also other solutions available (e.g. Bajari et al., 2010).