Auctions

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1 Introduction

Auction theory has been one of the most active and succesful fields in IO theory (and micro theory in general) for the last 30-40 years

Early work by e.g. Vickrey (1961). The field took off seriously with the help of (incomplete info) game theory developments (e.g. Harsanyi 1967, Selten 1975)

Lots of empirical work, mostly structural econometrics

A common subject in lab and field experiments

Analysis of auctions has spread into other fields of science, such as computer science and management science

Has been influential in practice, e.g. privatization of public assets, design of public procurement, emissions trading, government bonds, online auctions, internet ad auctions, interest rates

Continues to prosper partly due to this continuing practical importance

Despite its influence, many open questions remain:

endogenous entry, multi-dimensional bids, multi-unit/combinatorial auctions, commitment, various specifics of public procurement etc. not yet fully understood

Contrast to many other games (e.g. the market entry games studied before), the researcher actually knows the rules of the game played precisely

e.g. standard first price sealed bid auctions. We know that bidders submit real positive numbers as sealed bids, highest submitted bid wins and winner pays his own bid

Very specific predictions on equilibrium behavior, optimal mechanisms. Thus, fruitful area for structural empirical work

Even if still need to make many assumptions to model and for structural econometrics, there is less ambiguity than in many other cases

Analysis of auctions sometimes actually helps to improve the world, mechanism design aspect may actually be useful

Efficient allocation of scarce resources under asymmetric information (between strategic buyers and a seller)

Even if theory is decades old, practice is millenia old (e.g. cattle or slave auctions) e.g. Global warming might be even somewhat under control if governments had listened to economists 30 or so years ago instead of spoiling the emission auction design for political reasons

e.g. Paarsch 1992 succesful implementation of optimal reserve price in Canada forest auctions

e.g. eBay, Google, spectrum auctions, EU procurement law, stock market

Notorious failures as well, when applying theory designed for simpler setting in real life (e.g. New Zealand spectrum auction)

Auction transaction is similar to standard economics framework: Transfer of good from seller to buyer for monetary payment

Can be seen as subgroup of demand theory

Study mechanisms of price formation and what price represents

Window to understand competetive or oligopoly markets and test all kinds of theories

Most interesting when 2-8 bidders (or more if multi-unit). With many bidders, perfect competition and various mechanisms converge

2 Auctions and demand

How to sell an object to one among N potential buyers when little information on how much buyers value the object?

Take-it-or-leave-it (posted price)

Negotiations and beauty contests

Auctions

Many different auction mechanisms

In demand theory, demand structure is generated by well defined preference structure

In auction theory, "valuations" V are random numbers

Valuations v are drawn from some commonly know distribution(s) $F_V(v)$

At the simplest, values are iid (IPV paradigm)

In IPV, valuations can be ranked

$$v_{(1:N)} \geq v_{(2:N)} \geq \dots \geq v_{(N:N)}$$

With fixed N, one can obtain a step function of aggregate demand (draw)

$$Pr(V > v) = S_V(v) = [1 - F_V(v)] = [1 - Pr(V \leq v)]$$

Is the proportion of population having demand at price v

Expected demand curve is obtained by plotting $NS_V(v)$ on x-axis and p on y-axis

This can be estimated easily if we observe valuations (draw)

The goal of empirical work on auctions is to estimate expected demand curve $NS_V(v)$ based on bid data

Demand analysis with small number of consumers and endogenous price

Main concerns in estimation:

Heckman sample selection, we observe n but not N: Auctions with high n may be different from auctions with low n for unobserved reasons. Endogenous entry one of the main issues currently

True valuations often not bidded. How to infer valuations from bids?

What is N?

Some additional concerns in estimation in practice:

Asymptotics in N. Thus need data on many auctions. Are these independent?

How to control for observed and unobserved heterogeneity? (One option: Single index model: $V_{it} = g(z_t, \gamma) + U_{it}$)

Need some assumptions, such as risk attitude and information structure etc.

The structural econometrics of auction data uses the twin hypotheses of optimization and market equilibrium to identify $F_V(v)$!

i.e. bidders maximize expected utility and bid in equilibrium (dominance or Bayes-Nash)

Necessary ML regularity conditions met in the theory on auctions

Strategic behavior is the focus of analysis

Different auction formats generate differently informative bid data and thus require different estimation methods

3 Information structures

Symmetric independent private values: V_i iid

Independent private values: V_i independent

Affiliated private values: private values $(V_1, ..., V_N)$ affiliated (correlated/interdependent)

Pure common values: $V_i = V$. Signals S_i may be or may not be independent

Mineral rights: $V_i = V$. Signals S_i iid

Useful **definition** of private values and common values:

Bidders have private values if $E[V_i|S_1 = s_1, ..., S_N = s_N] = E[V_i|S_1 = s_1]$ for all $s_1, ..., s_N$ and all i. Bidders have common values if $E[V_i|S_1 = s_1, ..., S_N = s_N]$ is strictly increasing in s_j for all i,j and s_j.

Interpretation: In any common value setting, bidders would update their own valuation if they would observe other bidders information

This is the most useful way of thinking which object is a private value auction

Symmetric IPV

This is simplest for empiricists

Bidders draw iid valuations. Distribution(s) of valuations common knowledge

Ex ante bidders are identical, ex post different

Auction plays important role in efficient allocation

Example: buying art only for its subjective beauty

PCV

Bidders value the object exactly the same but have different information on the value

They receive unbiased signals about the value. Distribution(s) of signal common knowledge

e.g. oil exploration, buying art for resale

Auction plays no role in efficiency, only optimality of interest

May lead to winner's curse, because expected highest draw from unbiased F is an overestimate of V (elaborate on board)

This is accounted for in the equilibrium

Given value s, a potential bidder uses Bayes' rule to find out posterior probability density function of V

$$f_{V|S}(v|s) = \frac{f_{SV}(s,v)}{f_S(s)} = \frac{f_{S|V}(s|v) f_V(v)}{f_S(s)} = \frac{f_{S|V}(s|v) f_V(v)}{\int f_{S|V}(s|v) f_V(v) dv}$$

Can also combine to get closer to real world. e.g. buying art for own use only, but getting status utility or investment value from high resale value. Here bidders draw private value component and a signal for the common value component

We can also use general affiliated values model by Milgrom and Weber (1982) has IPV (zero correlation) and PCV (perfect correlation) as polar cases

4 Risk preferences

Typically bidders assumed to be risk-neutral:

$U = (v - p)Pr(\text{bid wins the auction})$

CARA:

$U(Y) = 1 - exp(-\alpha Y)$, $\alpha \geq 0$, $-\frac{U''(Y)}{U'(Y)} = \alpha$

Does not nest risk-neutrality easily

HARA:

$U(Y) = \eta Y^{1/\eta}$,$\alpha \geq 0, Y \geq 0, -\frac{U''(Y)}{U'(Y)} = \frac{\eta - 1}{\eta Y}$

When $\eta = 1$, preferences are linear, i.e. agents are risk-neutral

Useful in empirical analysis of auctions

5 Auction formats

The four standard auction formats are first-price sealed-bid, second-price sealed bid, open ascending and open descending auctions. We focus on these

Other formats exist in practise and/or in theory. e.g. third-price sealed-bid, Internet auction proxy format, all-pay auction, double auction

Focus on single-objects auctions. Multi-unit or multi-object auctions are also analyzed in a large literature

Some additional components of the mechanisms: reserve price (secret or public), bid increment, entry restrictions or fees, scoring rules, risk aversion

Analyze first under SIPV, risk-neutrality, and no reserve price

5.1 Second-price sealed-bid auctions

Vickrey auction

Bid submitted in sealed envelopes and opened simultaneosly

Highest bid wins, pays the second highest bid

Rare but used to sell e.g. stamps. Also eBay has some of these features

Estimation of bidder valuations in IPVP sealed bid second price auctions (Vickrey auction)

Auction theory tells us that bidding your own valuation is the dominant strategy in this game

$\beta_i = \beta(V_i) = V_i$

Proof:

If $\beta_i < V_i$, another bidder with lower valuation could win. Increasing bid has no effect on paid price if win, but increases the prob of win.

If $\beta_i > V_i$, Would make loss if paid more than V_i.

It is economically interesting to infer the the function $F(V)$ from which (symmetric) bidders draw their valuations

Knowing the valuation would allow us to calculate the optimal reservation price and simulate (any) counterfactual states of the world:

e.g. what is the efficiency and revenue under some other auction mechanism

(Non)parametric estimation of value functions in SPSB auctions is perhaps the simplest structural model

All bids B_{it} observed: $\hat{F}_V(v) = \frac{1}{TN} \sum_{t=1}^{T} \sum_{n=1}^{N} 1(B_{it} < v)$

Kernel smoothing $\hat{F}_V(v) = \frac{1}{TN} \sum_{t=1}^{T} \sum_{n=1}^{N} K(\frac{B_{it} < v}{h_F})$

5.2 Open ascending auction

Also known as English auction. Perhaps the most common auction format (in terms of nro of auctions held). Art auctions etc.

Think of clock model by Milgrom and Weber (1992): Auction begins at a low (zero or reserve price) level and the price rises continuously until only one bidder remains

Last bidder wins

In equilibrium, bidders stay in until clock reaches their valuation (valuation constant if IPVP, but updated if CVP)

All losing bids reveal valuation thrutfully

Winning bidder reveals only that valuation higher than second-highest valuation

In the case of English auction (or SPSB when only transaction price observed) we can estimate

$W = V_{(2:N)}$, $\hat{F}_W(v) = \frac{1}{T}\sum_{t=1}^{T} 1(W_t < v) = N(N-1)\int_0^{\hat{F}_V(v)} u^{N-2}(1-u)du$

Can estimate parametrically (e.g. Weibull) or nonparametrically using Kernel smoothing

Parametric (Weibull) code example in R (need to define stuff before and optimize after):

```
fct<-function(x) {

Fv<-1-exp(-(dataw/x1)^x2)

fv<-(x2/x1)*(dataw/x1)^(x2-1)*exp(-(dataw/x1)^x2)

-sum((dataN-2)*log(dataN*(dataN-1)*Fv)

+log(1-Fv)+log(fv))}
```

5.3 Open descending auction

Dutch auction, maybe due to flower sales in Netherlands

Initial price is very high and clock decreases the price

First one to press button/raise hand will win the object and pay the price denoted in the clock

Exactly equal to first-price sealed bid auction from theoretical perspective, because first to push button does not observe competitors' actions. Same as submitting sealed bids

For empirical purposes less information available because we only observe the winning bid

5.4 First-price sealed-bid auctions

Perhaps the most common auction format (in terms of monetary value of transactions) because most public procurement use this format

Each bidder submits a bid in a single sealed envelope which are all opened simultaneously

Highest bid wins and pays the own bid

In practice, public procurements are low-price tenders

Know own valuation but not that of competitors

$F_V(v)$ has support $[\overline{v}, \underline{v}]$ and continuous density $f_V(v)$

N common knowledge

Under these assumptions Dutch and FPSB are strategically equivalent

With symmetric bidders, can study bidder 1 without loss of generality

Profits $(v_1 - b_1)\,\mathsf{Pr}(\mathsf{win}|b_1)$

$$\mathsf{Pr}(\mathsf{win}|b_1) = \prod_{i=2}^{N} \mathsf{Pr}(S_i \leq s_1) = F_V[\hat{\beta}^{-1}(b_1)]^{(N-1)}$$

$\hat{\beta}(V)$ is the bidding rule and $\hat{\beta}^{-1}(b_1)$ its inverse

Bidder will trade-off probability of winning to profits conditional on winning

Maximising implies expected profits with respect to bid imply FOC:

$$-F_V[\hat{\beta}^{-1}(b_1)]^{(N-1)}+(v_1-b_1)(N-1)F_V[\hat{\beta}^{-1}(b_1)]^{(N-2)}f_V[\hat{\beta}^{-1}(b_1)]\frac{d\hat{\beta}^{-1}(b_1)}{db_1}=0$$

$b_1 = \hat{\beta}(v_1)$ by symmetry and $\frac{d\hat{\beta}^{-1}(b_1)}{db_1} = \frac{1}{\hat{\beta}'(v_1)}$ by monotonicity (inverse fct theorem)

We get $\hat{\beta}'(v) + \frac{(N-1)f_V[v]}{F_V[v]}\hat{\beta}(v) = \frac{(N-1)vf_V[v]}{F_V[v]}$

This differential equation has a closed form solution because of form $y\prime + p(x)y = q(x)$

equilibrium bidding rule $\beta(v) = v - \frac{\int_{\underline{v}}^{v} F_V[u]^{(N-1)}du}{F_V[v]^{(N-1)}}$

With HARA preferences $\beta(v;\eta,N) = v - \frac{\int_{\underline{v}}^{v} F_V[u]^{\eta(N-1)}du}{F_V[v]^{\eta(N-1)}}$

Because FPSB auction data is typically from public procurement, I will discuss identification and estimation in that context

Fist the symmetric case then the asymmetric case

To be more general, these discussed in the affiliated private values framework

5.4.1 The symmetric case

$n \geq 2$ (actual bidders in the data, assumed to equal N) symmetric risk-neutral bidders compete for a single procurement contract

In the AV model the cost of fulfilling a contract for bidder i is $C_i = c_i(S, v_i)$ where v_i denotes the private signal and S is a common component

In the APV model $c_i(S, v_i) = v_i$. Private costs v_i are affiliated

Symmetric Bayesian Nash equilibrium strategies $\beta_i(v_i)$ which are increasing and differentiable. Bidder i chooses bid b_i to maximize expected profits conditional on his own information v_i:

$$\max_{b_i}(b_i - v_i)\,\Pr(y_i \geq \beta^{-1}(b_i)|v_i),$$

where $y_i = \min_{j \neq i} v_j$. It can be shown that the first order condition for equilibrium is sufficient for estimation

GPV (2000) provide the identification result used in LPV (2002).The strict monotonicity of $s()$ allows a relation between the observed bid distributions and the latent costs distributions. FOC can be written as

$$v_i = b_i - \frac{\Pr(B_i \geq b, b_i = b)}{\Pr(B_i = b, b_i = b)}$$

This can be estimated using standard nonparametric techniques

The estimation is conducted separately for each n

h_G and h_g denote bandwidths and $K()$ a kernel function. b_{it} represents the bid made by bidder i in auction t, and B_{it} represents the lowest bid among i's opponents in auction t. The pseudo-costs $\hat{v}_{it}$ are estimated with the following equation:

$\hat{v}_{it} \equiv b_{it} - \frac{\hat{G}(b,b)}{\hat{g}(b,b)}$, where

$\hat{G}(b,b) = \frac{1}{T \times h_G \times n} \sum_{t=1}^{T} \sum_{i=1}^{n} K(\frac{b-b_{it}}{h_G}) 1\{B_{it} > b\}$ and

$\hat{g}(b,b) = \frac{1}{T \times h_g^2 \times n} \sum_{t=1}^{T} \sum_{i=1}^{n} K(\frac{b-b_{it}}{h_g}) K(\frac{b-B_{it}}{h_g})$

5.4.2 Asymmetric case

If bidders are asymmetric and their types are not observed, one cannot distinguish between changes in cost distributions resulting from different numbers of bidders and changes resulting from different sets/types of bidders

Can divide the bidders into (two) groups and treating them as symmetric within groups

G0 is low cost group and G1 is high cost group

G0 consists of n_0 bidders and G1 of n_1 bidders, with $n_0 + n_1 = n \geq 2$

If either n_0 or n_1 is zero, the estimation reduces to the symmetric case

The estimation equations become simpler if bidder i is the only bidder in either of the groups

The analysis must be performed separately for each given pair (n_0, n_1) because a bidder's strategy depends on both the number and types of his opponents

Let v_{1i} denote the costs of the bidders belonging to G1 and v_{0i} the costs of the bidders in G0

Bidders draw their costs from an n-dimensional cumulative distribution $F()$

Marginal distributions may vary across subgroups

I present the estimation strategy here only for group G1, as it is analogous for group G0

Let $y^*_{1i} = \min_{j \neq i, j \in G1} v_{1j}$ and $y_{0i} = \min_{j \in G0} v_{0j}$

The problem for any bidder i of type 1:

$\max_{b_{1i}} (b_{1i} - v_{1i}) \Pr(y^*_{1i} \geq s_1^{-1}(b_{1i}) \text{ and } y_{0i} \geq s_0^{-1}(b_{1i}) | v_{1i})$.

FOC is sufficient for estimation

$$v_{1i} = b_{1i} - \frac{\Pr(B^*_{1i} \geq b \text{ and } B_{0i} \geq b, b_{1i} = b)}{\Pr(B^*_{1i} = b \text{ and } B_{0i} \geq b, b_{1i} = b) + \Pr(B^*_{1i} \geq b \text{ and } B_{0i} = b, b_{1i} = b)},$$

where B^*_{1i} and B_{0i} denote the lowest bids of bidder i's opponents of a given type

The numerator can be estimated non-parametrically by $\hat{G}_1(b_1, b_1, b_1)$

The denominator is estimated as the sum of $\hat{D}_{11}(b_1, b_1, b_1)$ and $\hat{D}_{12}(b_1, b_1, b_1)$

The sum from t to T goes through the given pair (n_0, n_1) of the two bidder types

b_{1it} denotes the bid made by bidder i of type 1 in auction t

B^*_{1it} and B_{0it} denote the lowest bids of bidder i's opponents in auction t

Formally, $B^*_{1it} = \min_{j \neq i, j \in G1} b_{1jt}$ and $B_{0it} = \min_{j \in G0} b_{0jt}$

Pseudo-costs can then be estimated by

$$\hat{v}_{1it} = b_{1it} - \frac{\hat{G}_1(b_1,b_1,b_1)}{\hat{D}_{11}(b_1,b_1,b_1)+\hat{D}_{12}(b_1,b_1,b_1))}, \text{ where}$$

$$\hat{G}_1(b_1,b_1,b_1) = \frac{1}{T\times h_{G1}\times n_1}\sum_{t=1}^{T}\sum_{i=1}^{n_1} 1\{B_{1it}^* \geq b_1\}1\{B_{0it} \geq b_1\}K(\frac{b_1-b_{1it}}{h_{G1}}),$$

$$\hat{D}_{11}(b_1,b_1,b_1) = \frac{1}{T\times h_{g1}^2\times n_1}\sum_{t=1}^{T}\sum_{i=1}^{n_1} K(\frac{b_1-B_{1it}^*}{h_{g1}})1\{B_{0it} \geq b_1\}K(\frac{b_1-b_{1it}}{h_{g1}})$$

and

$$\hat{D}_{12}(b_1,b_1,b_1) = \frac{1}{T\times h_{g1}^2\times n_1}\sum_{t=1}^{T}\sum_{i=1}^{n_1} 1\{B_{1it} \geq b_1\}K(\frac{b_1-B_{0it}}{h_{g1}})K(\frac{b_1-b_{1it}}{h_{g1}})$$

Choosing the kernel and in particular the bandwidth is sort of an art

e.g. a triweight kernel is $K(u) = \frac{35}{32}(1 - u^2)^3 1\{| u | \leq 1\}$

For bandwidths, Silverman's rule of thumb is one option (Silverman 1986) $h = h_g = h_G = c_G(nT)^{-1/(1+2n)}$, where $c_g = c_G = 2,978 \times 1,06 \times$ (empirical std. deviation of bids)

The factor 2,978 follows from the use of triweight kernel instead of the Gaussian kernel

6 Classic theory results

Next, we discuss very briefly two classic results from auction theory:

The revenue equivalence theory

The optimal reserve price

Latter is is related to endogenous entry which will conclude this lecture

6.1 The revenue equivalence theory

Riley and Samuelsson (1981)

Under SIPVP, risk-neutrality and exogenous entry (reserve price is not binding and entry costs zero), all 4 standard mechanims generate same expected profits

In fact, any mechanism where highest bidder wins are revenue equivalent

Revelation principle: given strategy, choosing a bid is equal to choosing the level of private value to report, because auctioneer can calculate the bid function

Expected gain for bidder 1

$\pi(x, v_1) = v_1$ Pr(wins)$-$ Expected Payment

Payment can be separated from Pr, because of risk-neutrality (utility is linear)

1 reports x, then expected payment $P(x) = E$[Payment[$\beta(x), \beta(v_2), ..., \beta(v_N)$]]

$\pi(x, v_1) = v_1 F_V(x)^{N-1} - P(x)$

FOC under truth-telling:

$$\frac{\partial \pi(x^*, v_1)}{\partial x} = v_1(N-1)F_V(x^*)^{N-2} f_V(x^*) - P'(x^*) = 0$$

only when $x^* = v_1$. Optimal to report (not necessarily bid) v_1.

$$P'(v_1) = v_1(N-1)F_V(v_1)^{N-2} f_V(v_1)$$

From this we can derive the expected payment, which will not depend on the mechanism!

$$N \int_r^{\hat{v}} [u f_V(u) + F_V(u) - 1] F_V(u)^{N-1} du$$

This equals expected opportunity costs (second highest valuation)

6.2 Optimal reserve price

Riley and Samuelsson (1981), generalized by Myersson (1981)

Seller utility

$$v_o F_V(r)^N + N \int_r^{\hat{v}} [u f_V(u) + F_V(u) - 1] F_V(u)^{N-1} du$$

Utility from retaining object plus expected revenue

FOC wrt r:

$$N v_o F_V(r)^{N-1} f_V(r) - N[r f_V(r) + F_V(r) - 1] F_V(r)^{N-1} = 0$$

$$v_o f_V(r) - r f_V(r) - F_V(r) + 1 = 0$$

$$r^* = v_o + \frac{1 - F_V(r^*)}{f_V(r^*)}$$

Therefore, if we can estimate F from data (and ask v_o from the auctioneer), we can tell what the optimal reserve price is

Is optimal reserve price economically important

Jascisens (2017) studies Russian public procurement data on medicine purchases

DID with 2 shocs: a. Starting with 2014 buyers were not allowed to take regional markups into account when determining the reserve price of vital drugs. b. sudden depreciation of the ruble at the end of 2014. Producers of vital drugs are allowed to adjust regulated prices only once a year. Therefore, the unanticipated shock in the exchange rate at the end of 2014 led them to set too low reserve prices for 2015

Shock 1: 8% decrease in reserve prices and 9% in prices, no effect of zero bid propensity

Shock 2: 23% decrease in reserve prices and 20% in prices, 6% increase in no bids

Reserve prices at least 8% too high with same loss to tax payers

What would this mean in Finland with 31 billion in private sector PP, pathologically low competition and no serious consideration of optimal reserve price!

7 Endogenous entry

So far we have assumed exogenous (free) entry

Then $n = N$

Entry can be endogenous for various reasons. Then $n \neq N$ often.

Entry costs can be divided into **information acquisition** costs (Levin and Smith 1994) and **entry fees** or other direct costs of submitting bid (Samuelson 1985)

Entry can also be limited by binding **reservation price**

or by **bid increments** in ascending auctions or even in sealed bid auctions if entry is sequential

7.1 Reserve price

Paarsch and Hong (2006) devote much space to dealing with public reserve prices

Secret reserve prices can be modeled as one additional (asymmetric) bidder

With public reserve price, the distribution of bids represents a truncated distribution of valuations

This leads to Heckman sample selection

In second price auctions the density of the winning bid will have three parts, W equals zero when object is unsold, W equals reserve price when there is one bidder, and W equals second-highest valuation when more than one actual bidders

W is mixed discrete continuous random variable

Estimation is unchanged if all auctions in data have $n \geq 2$

If some auctions have $n < 2$, cannot simply omit those because that leads to selection/censoring bias

Data with censored cases can be analyzed by putting more structure on the model, i.e. parametric assumption about F_V.

$$F_W(w;\theta,N) = \{[F_V(r;\theta)]^N\}^{D_0}$$

$$\{N[F_V(r;\theta)]^{N-1}[1 - F_V(r;\theta)]\}^{D_1}$$

$$\{N(N-1)[F_V(r;\theta)]^{N-2}[1 - F_V(r;\theta)]f_V(r;\theta)\}^{1-D_0-D_1}$$

Then use ML to estimate θ

7.2 Bid increments

Here I will discuss Haile and Tamer (2003) who consider bid increments or jump bids in standard English auctions

Now winning bid may not be the second highest valuation

Important also study because one the first bound estimation papers

Idea is to get bound estimates on $F_V(v)$ under some reasonable assumptions:

A1: Bidders do not bid more than their valuation

A2: Bidders do not allow opponents to win at a price they are willing to beat

A1: bound from above

Then $b_i \leq v_i$ and thus $G_B(v) \geq F_V(v)$ (FOSD)

Then also order statistics must follow this inequality

Thus $F_V(v) \leq F_U(v) = \text{min}_i\, \varphi^{-1}[G_{(i:N)}(v); i, N]$

This has and the next bound has a sample analog

A2: bound from below

$\triangle$ is the bid increment

$$V_i \le Ui = \binom{\overline{v}\text{, when B}_\text{i}\text{=W}}{W+\triangle\text{, when B}_\text{i}\text{<W}}$$

$$V_{(2:N)} < W + \triangle$$

$$F_V(v) \ge F_L(v) = \varphi^{-1}[G_{W+\triangle}(v); 2, N]$$

7.3 Entry costs

We study a reduced form model by Li and Zhang (2010) and partly structural model by Athey, Levin and Seira (2010)

Important consideration are the timing on entry, bidders symmetry/asymmetry and how entry influences what information bids contain

7.4 Li and Zhang (2010)

Assume that an affiliated values (costs in procurement setting)

Bidders are asymmetric

Two pure strategy models of entry (submitting a bid)

Models differ on their assumptions on timing of information revelation

Model 1: Bidder $i = 1, ..., N$ learns private signal V_i of its costs of providing the contracted service or product, only after it pays an entry cost k_i

Bidders enter if their expected profits of entry exceed entry cost

This model generalizes Levin and Smith (1994) model by allowing asymmetric bidders

Unlike in the symmetric Levin and Smith (1994), this model has a pure strategy equilibrium

The second model is an similar asymmetric generalization of Samuelsson's (1985) model

Assume signals are known before the entry cost has to be paid

Bidders enter if their private signal is below a known screening level

The entry costs in the second model consists only of bid preparation costs, but the first model can also include information acquisition costs

In both the models, the equilibrium entry behavior is determined by a cut-off, which in the first model is in the entry cost and in the second model in the private signal

Therefore, a discrete choice model where the error term consists either of the entry cost or of the private signal provides a natural reduced form for these models.

Li and Zhang (2010) argue that in the Milgrom and Weber (1982) context, both of these different entry models generate the same reduced form binary choice model

$$y_{ti} = 1(x_t\beta + z_{ti}\gamma + \eta_t + e_{ti} > 0).$$

x_t is vector of observable auction characteristics

z_{ti} is vector of observable bidder specific characteristics

η_t denotes such auction level heterogeneity that is unobserved to the researcher but observed by all the players

e_{ti} is the idiosyncratic shock which is observed only by bidder i

In the language of literature on oligopoly entry, this is therefore a game of pure incomplete information

Li and Zhang (2010) propose to test for affiliation in either the private signals on costs or private entry costs by testing whether the private shocks e_{it} are correlated

Their test fails in the presence of unobservable bidder heterogeneity
that is correlated between bidders

They are able to account only for such competition effects that can be seen as contract characteristics, such as the number of potential bidders. They do not consider bidding

7.5 Athey, Levin and Seira (2011)

Asymmetric bidders (loggers and mills)

Bidding strategy and entry strategy. When bid, know who entered

Compare ascending to FPSB. Both cases in data

In FPSB equilibrium, Mills shade bids more than loggers

AA equilibrium is efficient

FOC FPSB:

$$\frac{1}{v_i - b_i} = \sum_{j \in n \setminus i} \frac{g_j(b_i, n)}{G_j(b_i, n)}$$

Estimate as in GPV

AA: Bid up to valuation

Ex ante profits:

$\Pi_i^\tau(p) = \sum_{n \subset N}^\tau \pi_i^\tau(n_L, n_M)\, \text{Pr}[n_L, n_M | i \text{ enters, rivals play } p_{-i}]$

p is the profile of entry probabilities

Type-symmetric entry equilibrium exists but is not necessarily unique

When AA rather than FPSB, loggers less likely to enter, mills are more likely to enter, less likely for logger to win

They estimate both reduced form treatment effect model (auction type is the treament) and a structural model

Three steps: 1. Account for observed and unobserved heterogeneity (parametric), 2. Estimate value functions (similar to GPV), 3. Estimate entry costs