

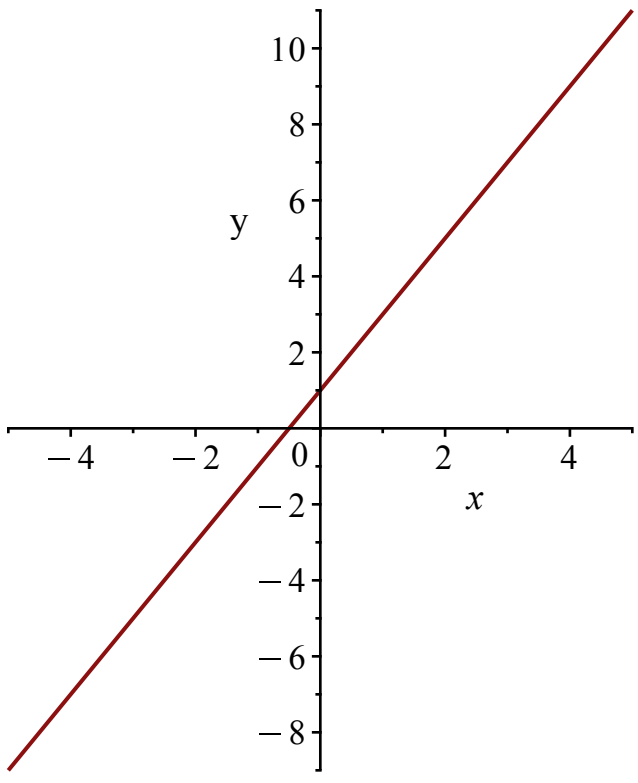
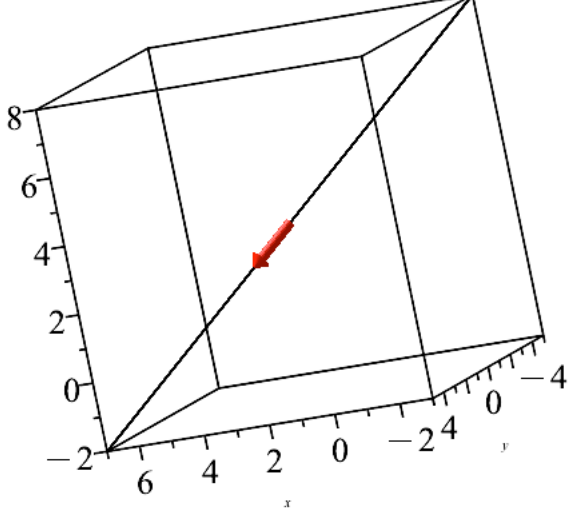
Vector - valued functions

Load the following packages

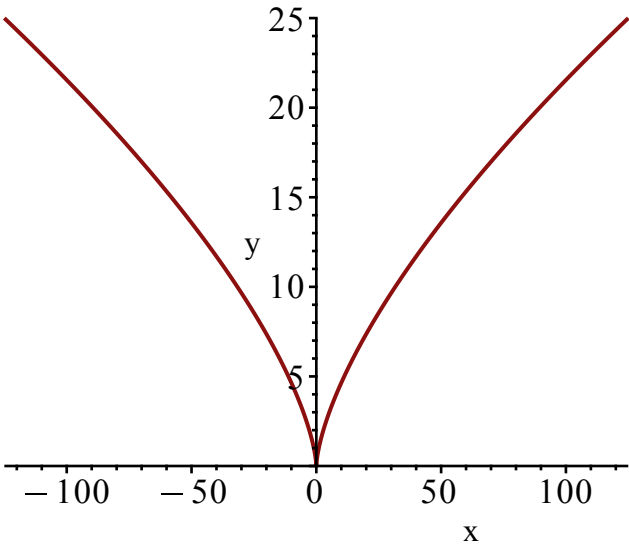
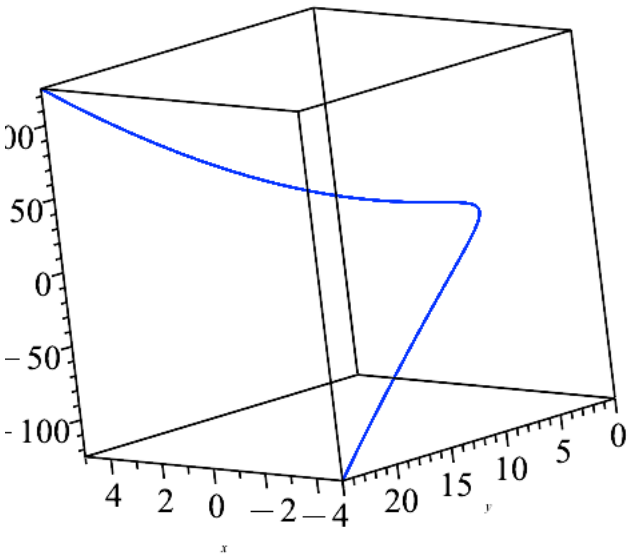
`with(LinearAlgebra) with(VectorCalculus) with(Student[Calculus1])with(plots) :`

We consider vector - valued functions of a single real variable of the form $r: \mathbb{R} \rightarrow \mathbb{R}^n$ with $n=2$ or $n=3$. They can be used to represent curves in \mathbb{R}^2 and \mathbb{R}^3 , respectively (we can interpret $r(t)$ as the position of a particle at time t).

Example 1: Lines

2D	3D
$r(t) = (t, 2t + 1)$ <pre>plot([t, 2t + 1, t=-5..5])</pre> 	$r(t) = (2, 0, 3) + t(1, 1, -1)$ <pre>s := plot3d([2 + t, t, 3 - t], t=-5..5) : v := arrow(<2, 0, 3>, <1, 1, -1>, color = red, width = 0.3) : display(v, s, labels = [x, y, z])</pre> 

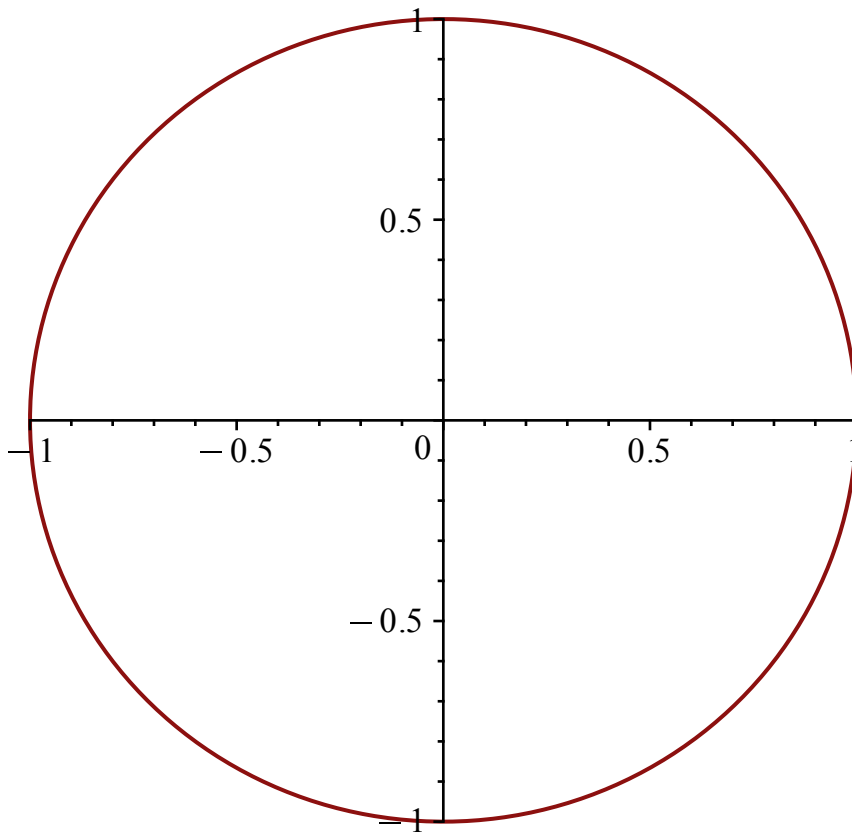
Example 2: Curves

2D	3D
$r(t) = (t^3, t^2)$ $\text{plot}([t^3, t^2, t=-5..5])$  <p>Remark: the curve is not smooth at (0,0).</p>	$r(t) = (t, t^2, t^3)$ $\text{plot3d}([t, t^2, t^3], t=-5..5)$ 

Example 3: Circle

As a parametric equation $r(t) = (\cos(t), \sin(t))$ for $0 \leq t < 2\pi$

$$\text{plot}([\cos(t), \sin(t), t=0..2\pi])$$



$$\left[\quad \quad \quad x^2 + y^2 = 1 \quad \quad \quad \right] \quad (1)$$

$$\left[\begin{array}{l} > \text{ArcLength}([\cos(t), \sin(t)], t = 0..2\pi) \\ \quad \quad \quad 2\pi \end{array} \right] \quad (2)$$