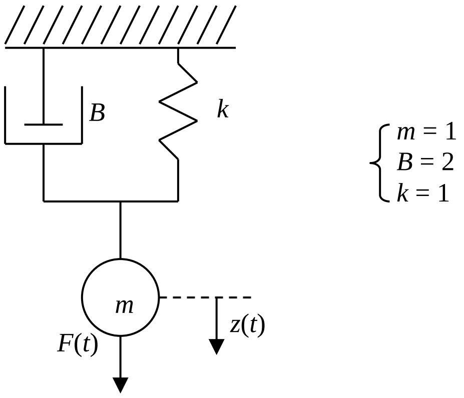
ELEC-C8201 Control and Automation
Exercise 1

Basics of control, differential equations, Laplace transformation

1. **a.** Create a block diagram which describes the closed loop control of a car. The parameter to be controlled is the direction of the car, the measurement method is visual observation and the controlling agent is the driver. Name the elements in the diagram in control theory terms.

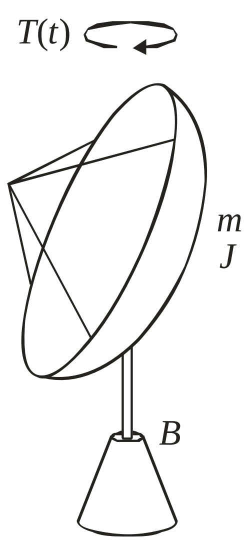
b. What are the disturbances to each part of the above system?

2. The control input in the figure given below is the force $F(t)$ and the output is $z(t)$



a. Construct a differential equation describing the system
b. Determine the state-space representation of the system with position and velocity as state vectors.

3. The radar antenna is rotated around the vertical axis with a torque $T(t)$. The angular deviation of the antenna from the reference position is $\theta(t)$, Moment of inertia is J and mass is m. The antenna is mounted on the base of the bearing, which gives resistance proportional to the angular velocity with constant of proportion, B. The antenna is not affected by any other forces.



a) Find the differential equation describing the system.

Note: The basic equation of the rotary motion is

$$\sum_i T_i = J\,\frac{d^2\theta(t)}{dt^2},$$

where Torques T_i are system-induced torques and J is the moment of inertia.

b) Write the differential equation obtained in part (a) in the state-space format. Select the angular deviation and angular velocity as the states and the angular deviation as output.

c) Let the required angle position be $\theta_{REF}(t)$. The required torque to go to reference angle is given by:

$$T(t) = k\left(\theta_{REF}(t) - \theta(t)\right),$$

where k is the control gain (standard). Find the state-space representation of this system. Select the angular deviation and angular velocity as the states and the angular deviation as output

4.

I. Derive $F(s)$ according to the definition of Laplace transformation, when

a. $f(t) = Au_s(t)$

b. $f(t) = e^{-at}u_s(t)$

Where function $u_s(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$ is the unit of the step function starting at 0. (sometimes not shown explicitly).

II. Derive $f(t)$, when $F(s)$ is:

a. $\dfrac{4}{s^2 + 6s + 9}$

b. $\dfrac{4}{s^2 + 4s + 3}$

c. $\dfrac{10s + 8}{s\left(s^2 + 3s + 2\right)}$

d. $\dfrac{10s + 50}{s^2 + 4s + 13}$

e. $\dfrac{4}{s^2 + 8}$

f. $\dfrac{16s + 16}{\left(s^2 + 16\right)\left(s^2 + 4s + 20\right)}$

III. Derive $F(s)$, when $f(t)$ is:

a. $\left(2 + 3t + 4t^2 - 2e^{-3t}\right)u_s(t)$

b. $\left(3te^{-4t} + 2 - 2e^{-4t}\right)u_S(t)$

c. $\left(4\sin 2t + 5\cos 2t\right)u_S(t)$

d. $\left[2(t-1)\right]u_S(t-1)$

e. $\left[4e^{-3t}\left(\sin 2t + \cos 2t\right)\right]u_s(t)$