ELEC-C8201: Control Theory and Automation

Exercise 5

The problems marked with an asterisk (⋆) are not discussed during the exercise session. The solutions are given in MyCourses and these problems belong to the course material.

1. A feedback control system with a proportional gain 4 and a plant with transfer function

$$G(s) = \frac{s^2+1}{s(s+a)}$$

is shown in Figure 1.

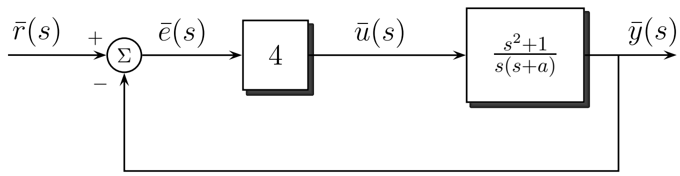

Figure 1: Feedback control system.

Sketch the root locus for $0 \leq a < \infty$.

2. A feedback control system with a plant transfer function

$$G(s) = \frac{1}{s(s-1)}$$

is shown in Figure 2.

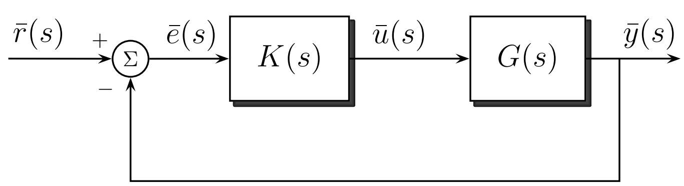

Figure 2: Feedback control system.

a) When $K(s) = k_p$, show that the system is always unstable by sketching the root locus.

b) When

$$K(s) = \frac{k_p(s+2)}{s+20},$$

sketch the root locus and determine the range of k_p for which the system is stable.

3. A feedback control system with a proportional gain k_p and a plant with transfer function

$$G(s) = \frac{s+10}{s(s+5)}$$

is shown in Figure 3.

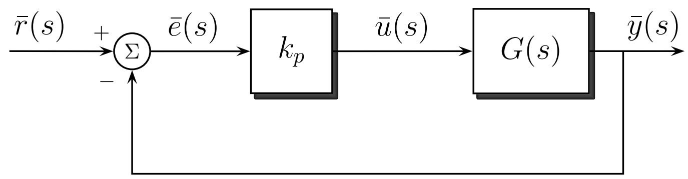

Figure 3: Feedback control system.

a) Determine the break-in and break-away points of the root locus and sketch the root locus for $k_p > 0$.

b) Determine the gain k_p when the two characteristic roots have a damping factor ζ of $1/\sqrt{2}$.

c) Calculate the roots.

4. A feedback control system with a proportional gain k_p and a plant with transfer function

$$G(s) = \frac{(s+2)^2}{s(s^2+1)(s+8)}$$

is shown in Figure 3.

a) First, sketch the root locus for $0 \leq k_p < \infty$ to indicate the significant features of the locus. Second, use MATLAB to plot the root locus and compare it with your sketch.

b) For what value of k_p do purely imaginary roots exist?

c) Determine the range of the gain k_p for which the system is stable.

d) Would the use of the dominant roots approximation for an estimate of the settling time be justified in this case for a large magnitude of gain ($k_p > 50$)?

⋆5. A magnetically levitated (MAGLEV) high-speed train "flies" on an air gap above its rail system (with up to 310mph!), as shown in Figure 4.

Figure 4: A MAGLEV train in China (Photo: Ren Long/China Features Photos).

The feedback control system is illustrated in Figure 5.

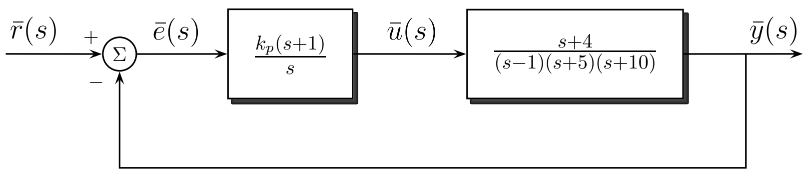

Figure 5: Feedback control system.

a) Sketch the root locus plot.

b) Select k_p so that all of the complex roots have a damping factor ζ greater than 0.6. Plot (in MATLAB) the actual response for the selected k_p.

c) Select k_p so that the response for a unit step input is reasonably damped and the settling time is less than 5 seconds. Plot (in MATLAB) the actual response for the selected k_p.