ELEC-C8201: Control Theory and Automation

Exercise 8

The problems marked with an asterisk (⋆) are not discussed during the exercise session. The solutions are given in MyCourses and these problems belong to the course material.

1. Consider the *single-input, single-output* system described by

$$\begin{aligned}\dot{\mathbf{x}}(t) &= A\mathbf{x}(t) + Bu(t),\\ y(t) &= C\mathbf{x}(t)\end{aligned}$$

Determine whether the system is controllable and observable when:

a) $A = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 2 \end{bmatrix}.$

b) $A = \begin{bmatrix} -10 & 0 \\ 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$

c) $A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$

2. Consider the second-order system

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}u(t),\\ y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}(t)\end{aligned}$$

For what values of k_1 and k_2 is the system completely controllable?

3. Consider the third-order system

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -5 & -3 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}u(t),\\ y(t) &= \begin{bmatrix} 2 & -4 & 0 \end{bmatrix}\mathbf{x}(t)\end{aligned}$$

a) Verify that the system is observable.

b) Determine the observer gain matrix required to place the observer poles at $s_{1,2} = -1 \pm j$ and $s_3 = -5$.

4. Suppose that the vector differential equation describing the inverted pendulum is given by

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 9.8 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} u(t)$$

Assume that all state variables are available for measurement and use state variable feedback. Place the system characteristic roots at $s - -2 \pm j$,-5, and -5.

5. A system is represented by the differential equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = \frac{du}{dt} + u$$

where y = output and u = intput.

a) Develop a state variable representation and show that it is a controllable system.

b) Define the state variables as $x_1 = y$ and $x_2 = dy/dt - u$, and determine whether the system is controllable.

6. Consider the second-order system

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 1 & 2 \\ -6 & 12 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} -5 \\ 1 \end{bmatrix} u(t),$$
$$y(t) = \begin{bmatrix} 4 & -3 \end{bmatrix} \mathbf{x}(t)$$

a) Verify that the system is observable and controllable.

b) Design a full-state feedback law and an observer by placing the closed-loop system poles at $s_{1,2} = -1 \pm j$ and the observer poles at $s_{1,2} = 12$.