Functions of several variables

Load the following packages
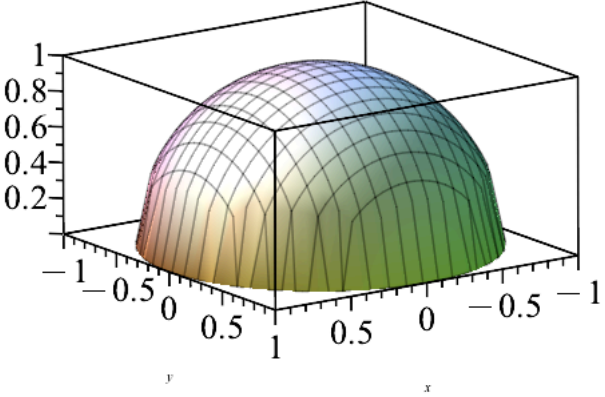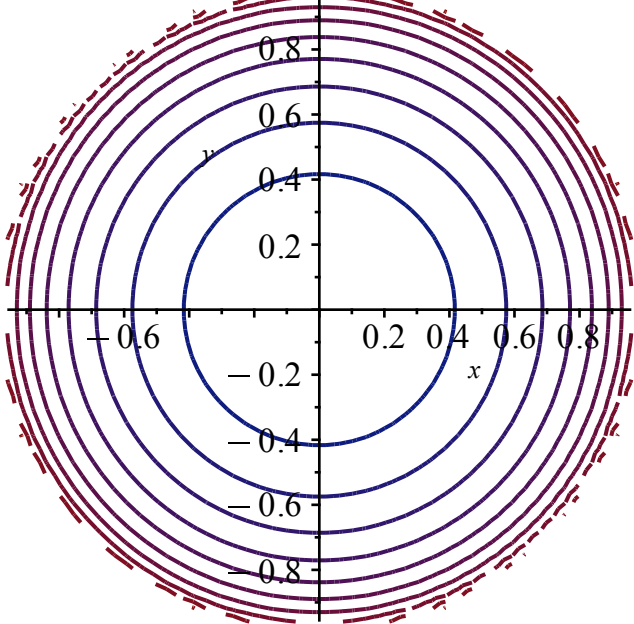*with*(*LinearAlgebra*) *with*(*VectorCalculus*) *with*(*Student*[*Calculus1*])*with*(*plots*) :
We consider functions of the form f: $\mathbb{R}$^n → $\mathbb{R}$. Here we set n=2. They can be used to represent surfaces in $\mathbb{R}$^3. That is, the graph z=f(x,y) is a surface in the 3D space.

Example 1: Hemisphere

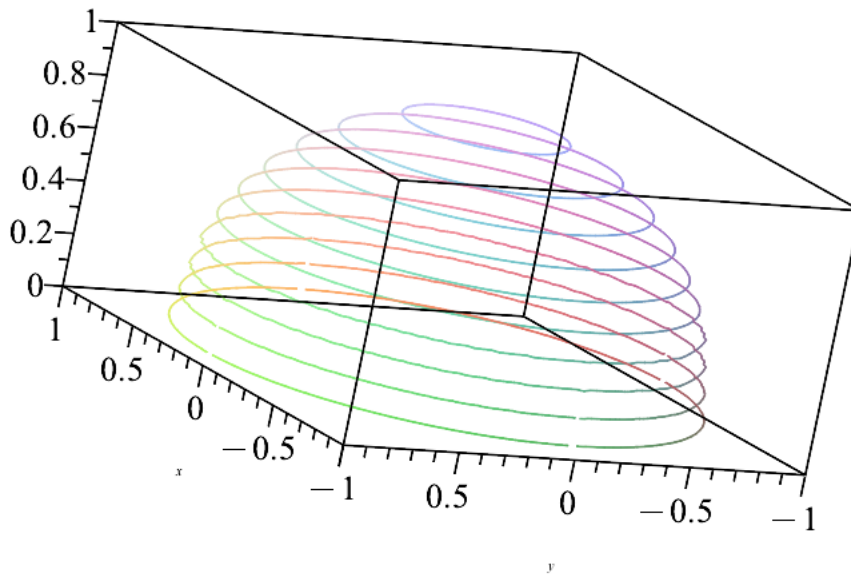$$f := \sqrt{1 - x^2 - y^2}$$

$$f := \sqrt{-x^2 - y^2 + 1} \tag{1}$$

Surface	Level curves
plot3d(*f*, *x* =−1 ..1, *y* =−1 ..1, *grid* = [40, 40], *view* = 0 ..1)	*contourplot*(*f*, *x* =−1 ..1, *y* =−1 ..1, *contours* = 10)

contourplot3d(*f*, *x* =−1 ..1, *y* =−1 ..1, *contours* = 10)

Example 2: Paraboloid

$$f := x^2 + y^2$$

$$f := x^2 + y^2 \qquad\qquad (2)$$

Surface	Level curves
$plot3d(f, x=-2\,..2, y=-2\,..2)$	$contourplot(f, x=-2\,..2, y=-2\,..2,$ $contours = [0, 1, 2, 4])$
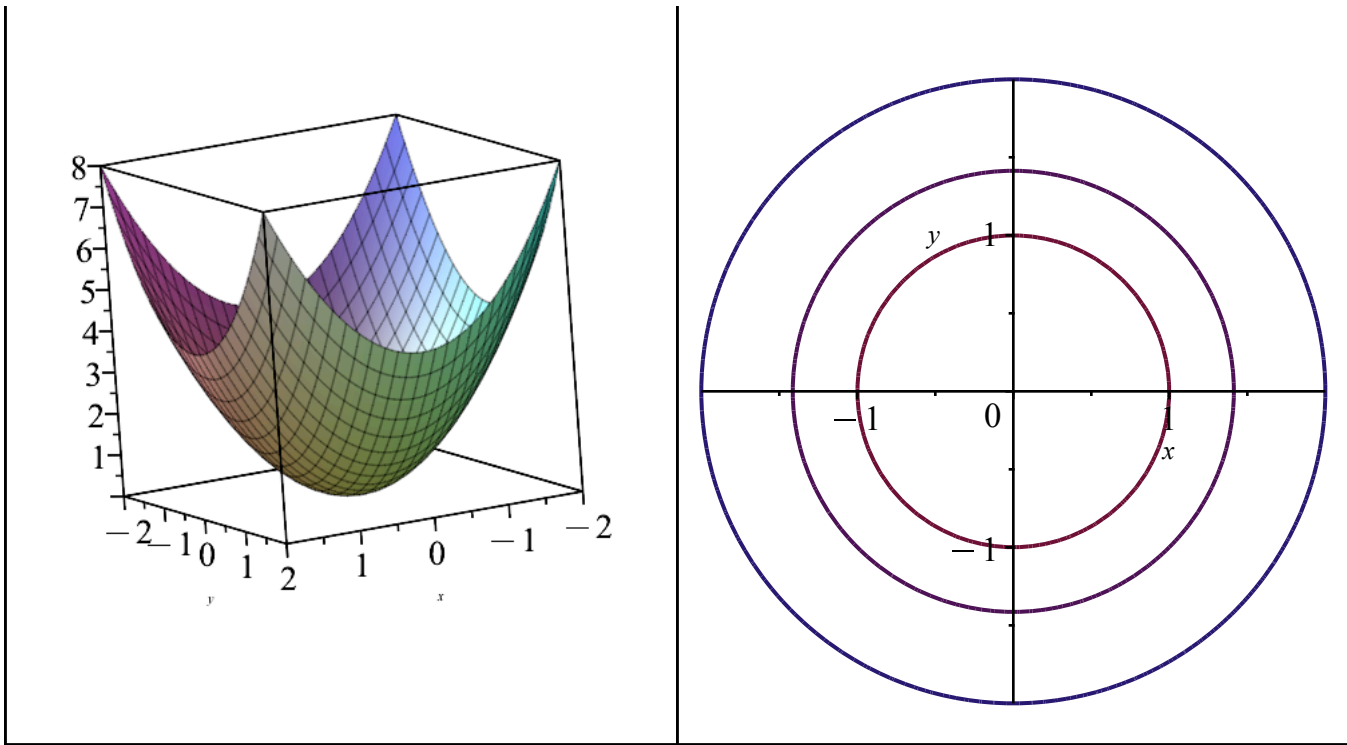
$contourplot3d(f, x=-2..2, y=-2..2)$
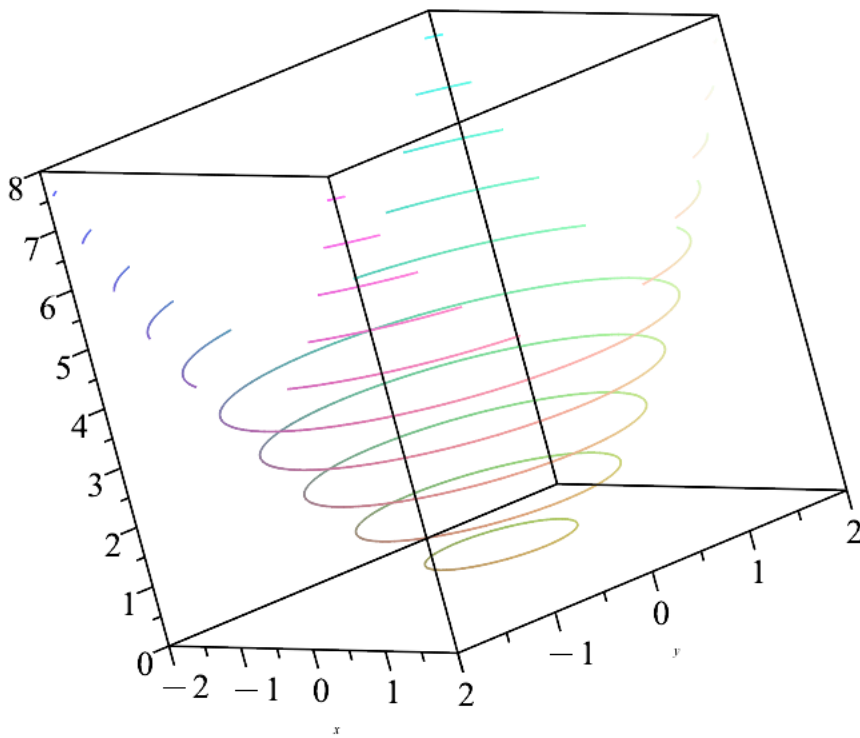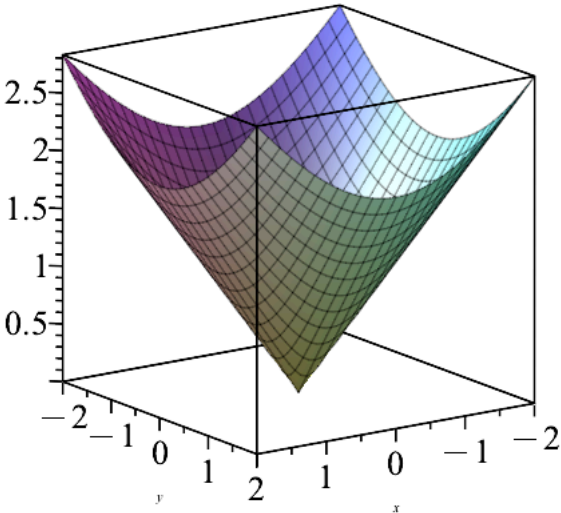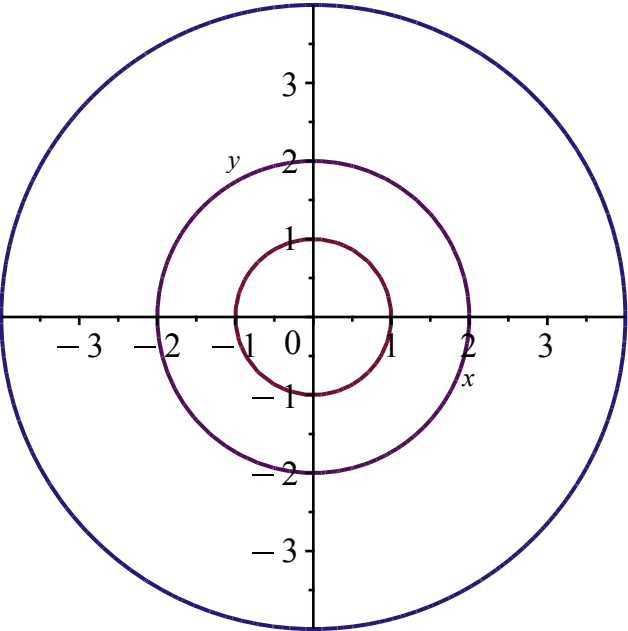
Example 3: Cone

$$f := \sqrt{x^2 + y^2}$$

$$f := \sqrt{x^2 + y^2} \tag{3}$$

Surface	Level curves
$plot3d(f, x = -2..2, y = -2..2)$	$contourplot(f, x = -5..5, y = -5..5, contours$ $= [0, 1, 2, 4])$

Example 4: Hyperboloid

$$f := x^2 - y^2$$

$$f := x^2 - y^2 \tag{4}$$
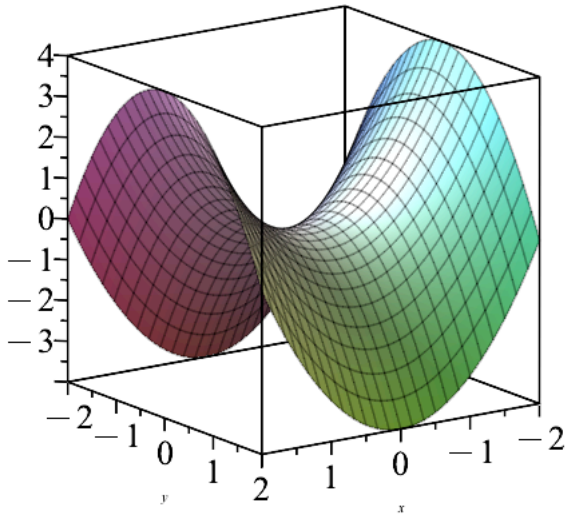
Surface	Level curves
$plot3d(f, x=-2 ..2, y=-2 ..2)$	$contourplot(f, x=-5 ..5, y=-5 ..5,$ $contours = [-4, -2, -1, 0, 1, 2, 4])$
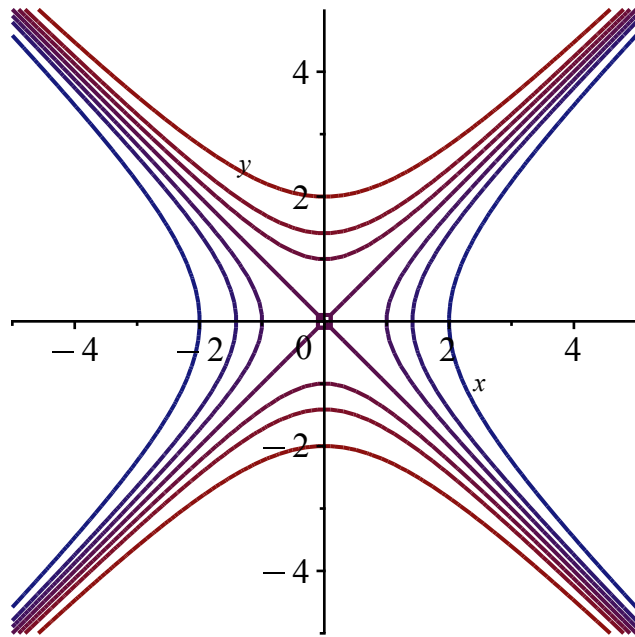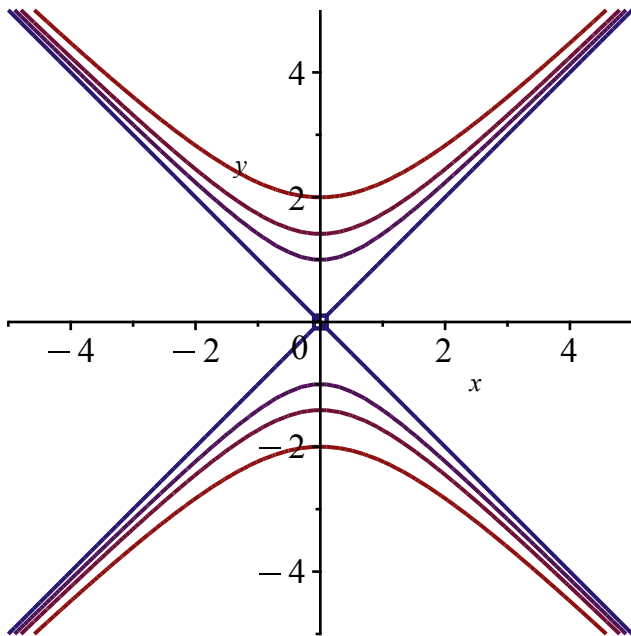
$contourplot(f, x=-5 ..5, y=-5 ..5, contours$ $= [-4, -2, -1, 0])$	$contourplot(f, x=-5 ..5, y=-5 ..5, contours$ $= [0, 1, 2, 4])$
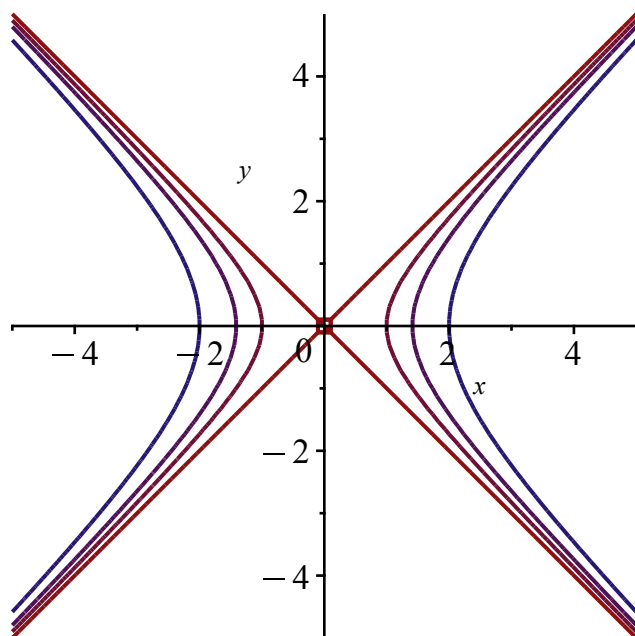
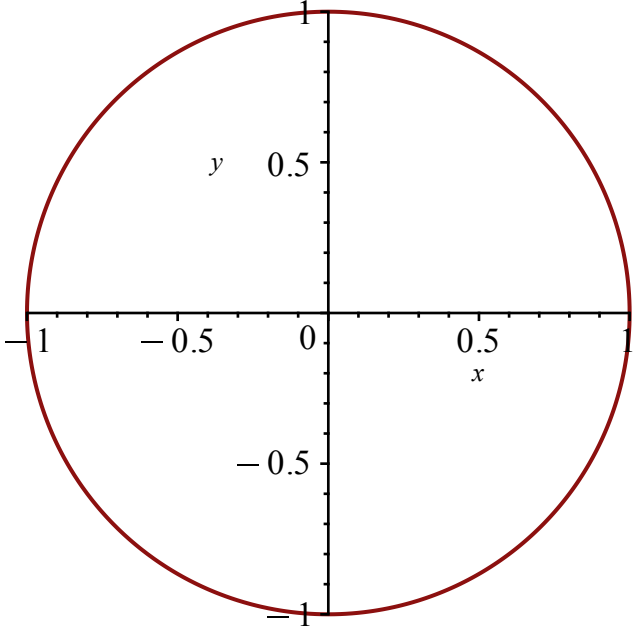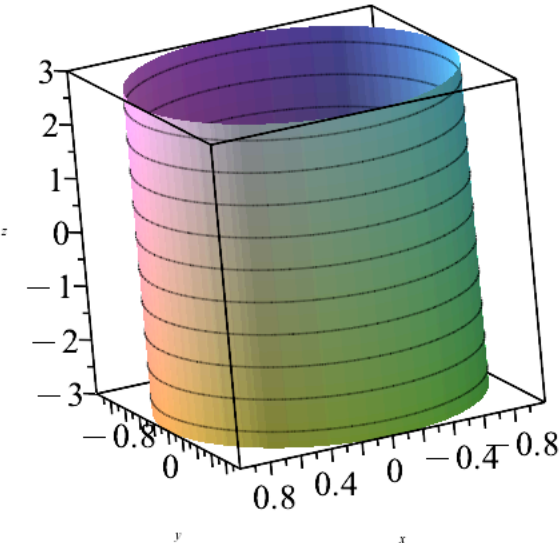# Graph of an equation f(x,y)=0 or f(x,y,z)=0

$$f := x^2 + y^2 - 1$$

$$f := x^2 + y^2 - 1 \qquad\qquad (5)$$

f(x,y)=0	f(x,y,z)=0
implicitplot(f, x = −1 ..1, y = −1 ..1)	*implicitplot3d*(f, x = −1 ..1, y = −1 ..1, z = −3 ..3)