

## Functions of several variables

Load the following packages

`with(LinearAlgebra) with(VectorCalculus) with(Student[Calculus1])with(plots) :`

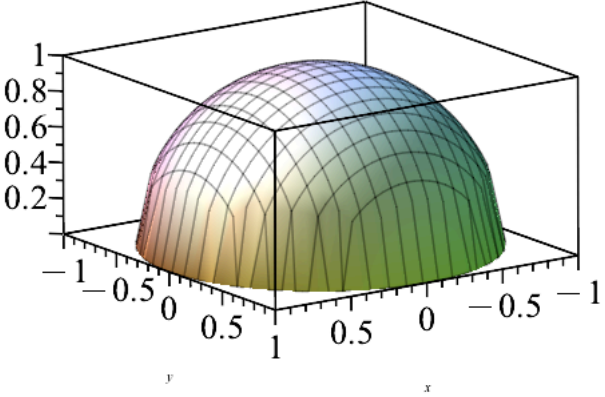
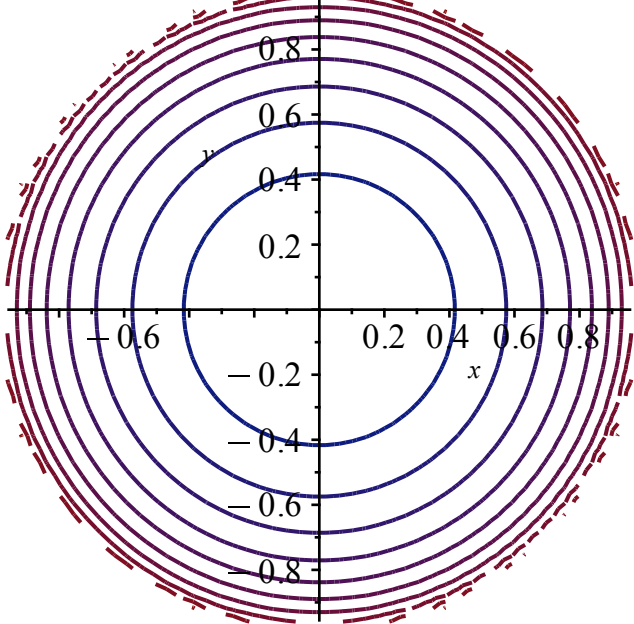
We consider functions of the form  $f: \mathbb{R}^n \rightarrow \mathbb{R}$ . Here we set  $n=2$ . They can be used to represent surfaces in  $\mathbb{R}^3$ . That is, the graph  $z=f(x,y)$  is a surface in the 3D space.

### Example 1: Hemisphere

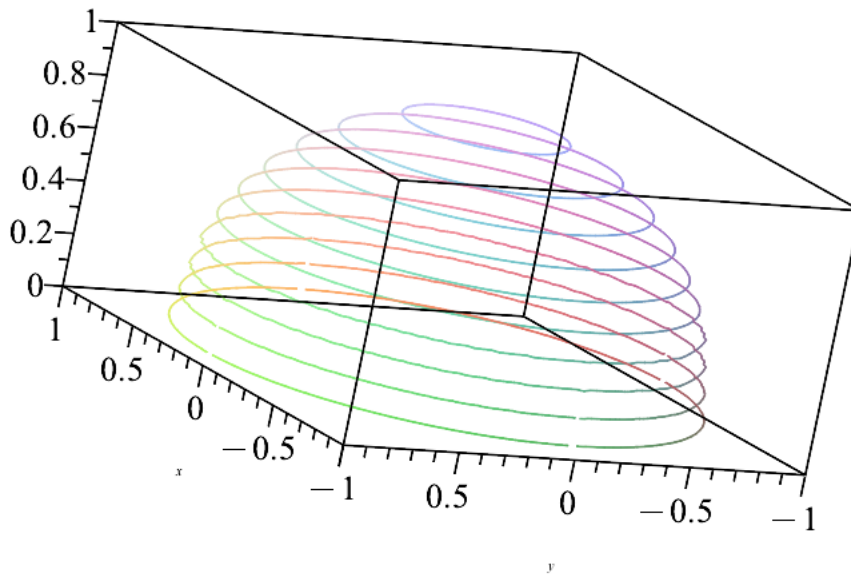
$$f := \sqrt{1 - x^2 - y^2}$$

$$f := \sqrt{-x^2 - y^2 + 1}$$

(1)

Surface	Level curves
<pre>plot3d(f, x=-1..1, y=-1..1, grid =[40, 40], view=0..1)</pre>  A 3D plot of a hemisphere surface. The x and y axes range from -1 to 1, and the z-axis ranges from 0 to 1. The surface is a smooth, dome-like shape with a grid pattern. The color transitions from green at the base to blue at the top.	<pre>contourplot(f, x=-1..1, y=-1..1, contours = 10)</pre>  A 2D contour plot of the hemisphere. The x and y axes range from -1 to 1. The plot shows concentric circular contours centered at the origin. The contours are colored in a gradient from blue (inner) to red (outer). The axes are labeled with values from -0.8 to 0.8.

`contourplot3d(f, x=-1..1, y=-1..1, contours = 10)`



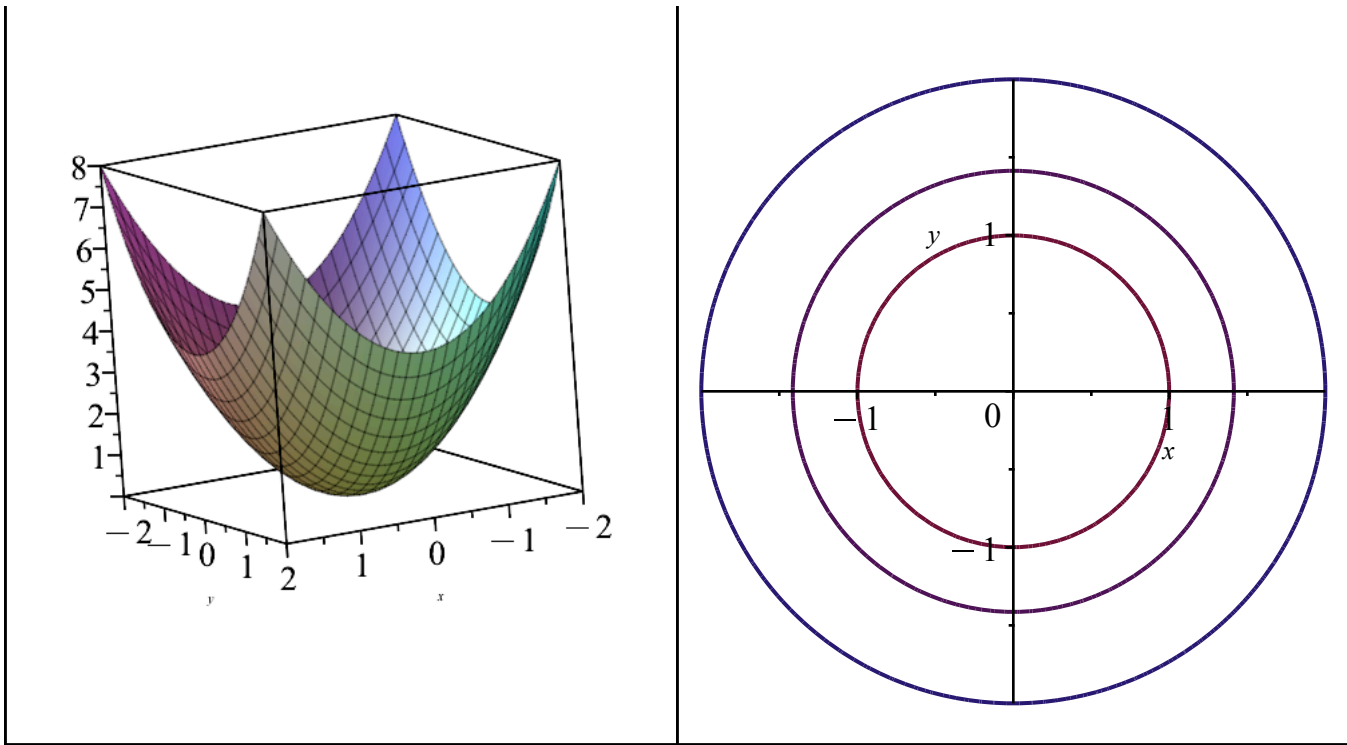
## Example 2: Paraboloid

$$f := x^2 + y^2$$

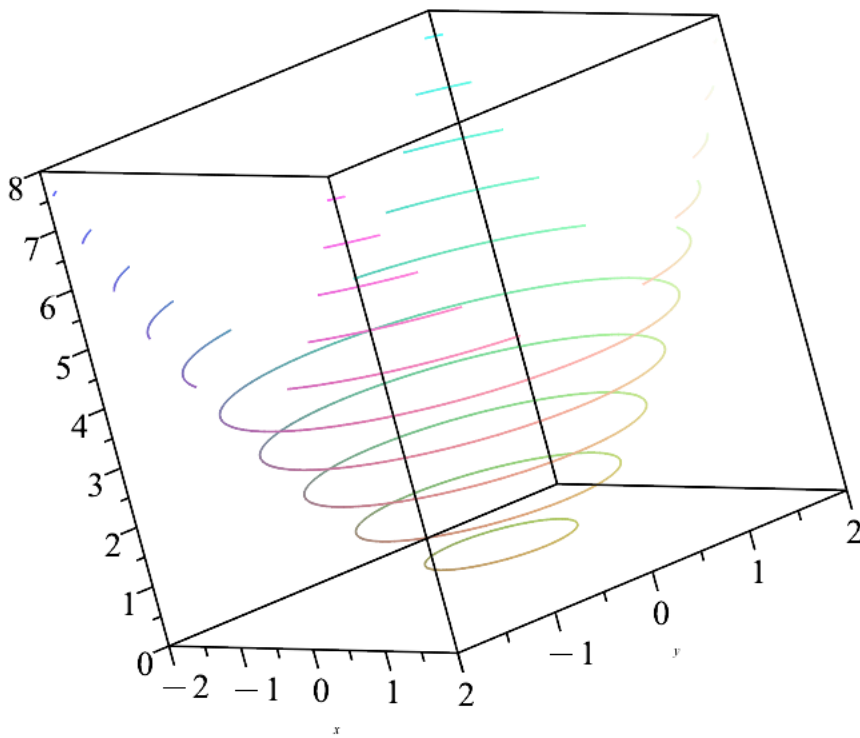
$$f := x^2 + y^2$$

(2)

Surface	Level curves
<code>plot3d(f, x=-2..2, y=-2..2)</code>	<code>contourplot(f, x=-2..2, y=-2..2, contours = [0, 1, 2, 4])</code>



*contourplot3d(f, x=-2..2, y=-2..2)*

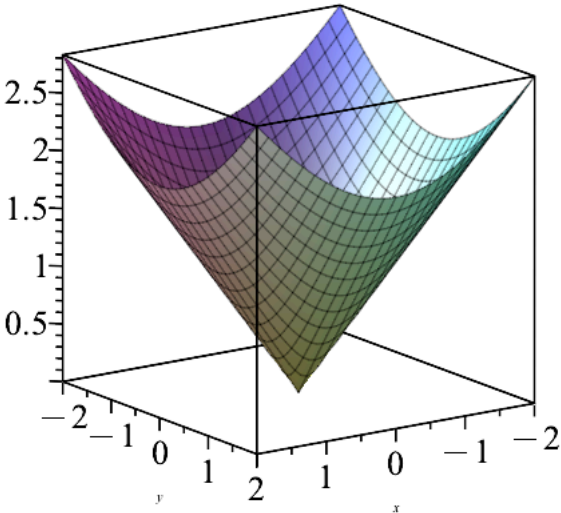
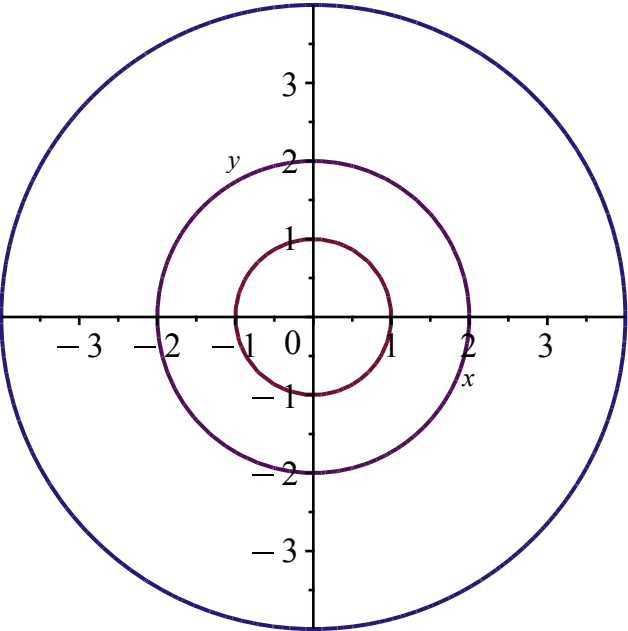


### Example 3: Cone

$$f := \sqrt{x^2 + y^2}$$

$$f := \sqrt{x^2 + y^2}$$

(3)

Surface	Level curves
<p data-bbox="94 615 586 657"><code>plot3d(f, x=-2..2, y=-2..2)</code></p>  <p>A 3D plot of a cone with its vertex at the origin (0,0,0) and its base as a circle in the xy-plane with radius 2. The surface is colored with a gradient from purple at the top to green at the bottom. The x and y axes range from -2 to 2, and the z-axis ranges from 0 to 2.5.</p>	<p data-bbox="748 615 1328 688"><code>contourplot(f, x=-5..5, y=-5..5, contours = [0, 1, 2, 4])</code></p>  <p>A 2D contour plot showing concentric circles centered at the origin in the xy-plane. The circles represent level curves of the function f. The x and y axes range from -5 to 5. The contours are labeled with values 0, 1, 2, and 4.</p>

### Example 4: Hyperboloid

$$f := x^2 - y^2$$

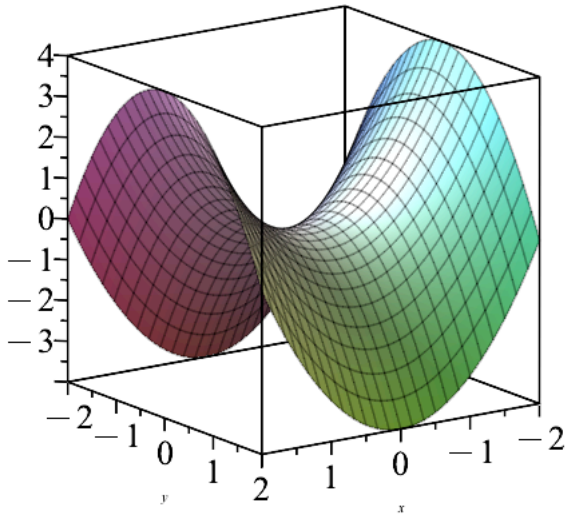
$$f := x^2 - y^2$$

(4)

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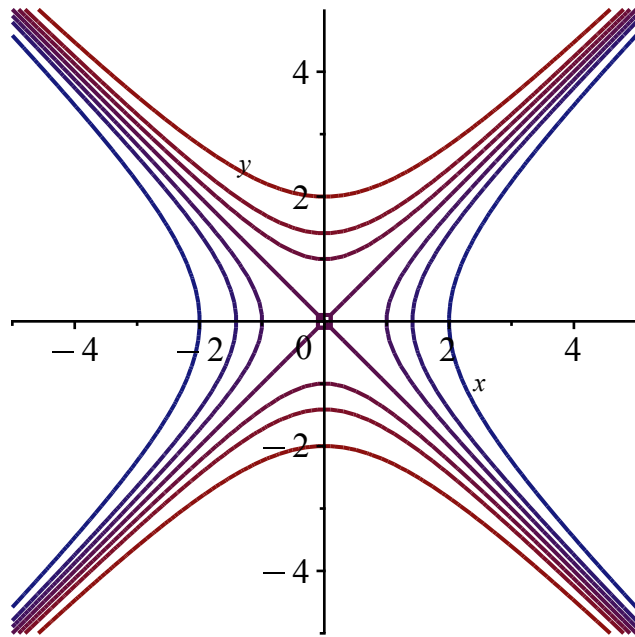
### Surface

```
plot3d(f, x=-2..2, y=-2..2)
```

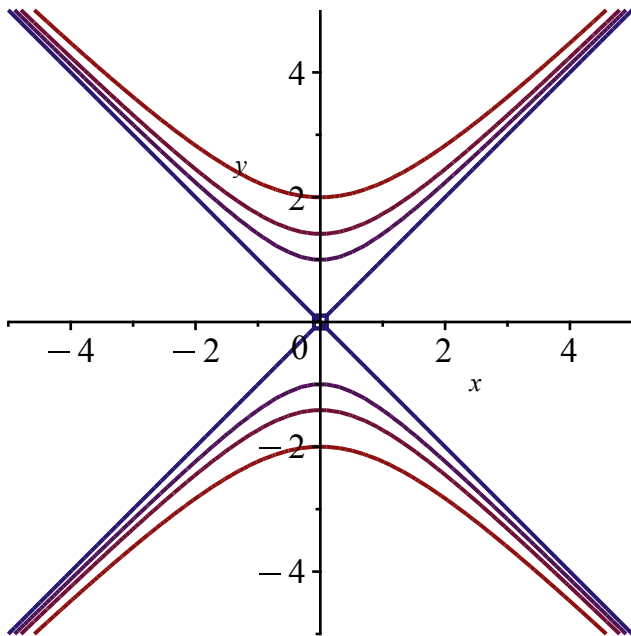


### Level curves

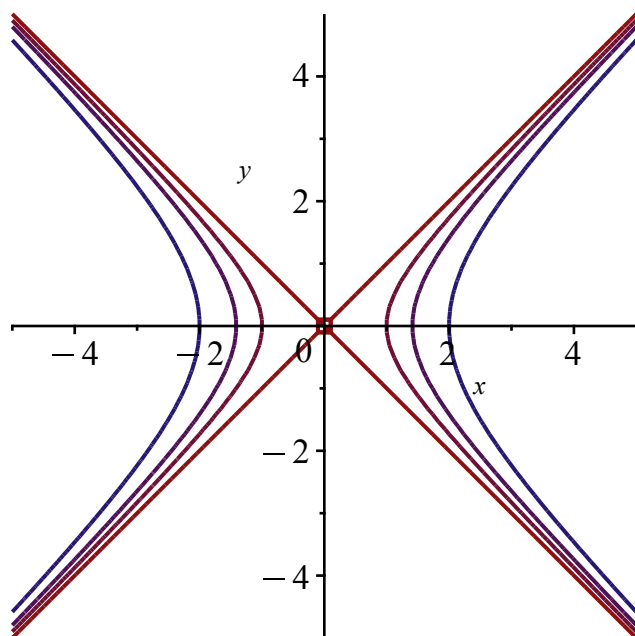
```
contourplot(f, x=-5..5, y=-5..5,  
contours = [-4, -2, -1, 0, 1, 2, 4])
```



```
contourplot(f, x=-5..5, y=-5..5, contours  
= [-4, -2, -1, 0])
```



```
contourplot(f, x=-5..5, y=-5..5, contours  
= [0, 1, 2, 4])
```

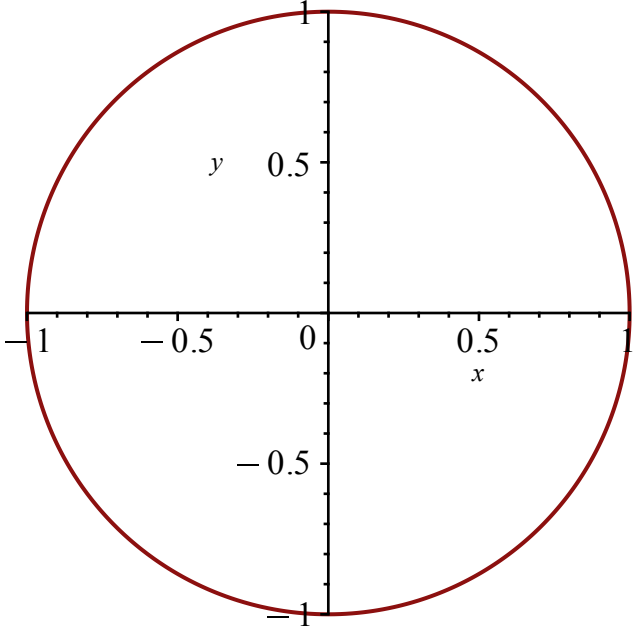
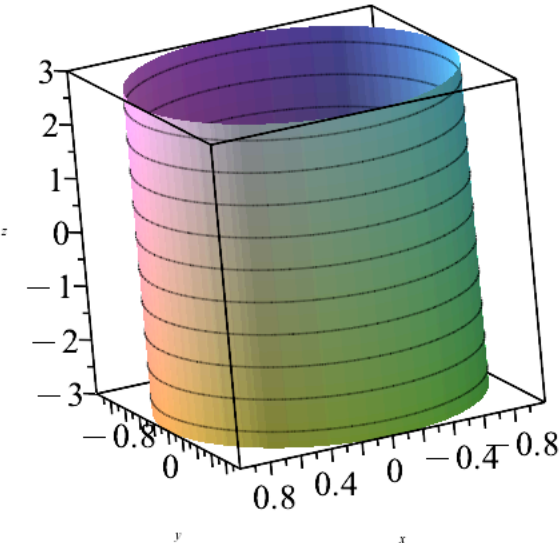


## Graph of an equation $f(x,y)=0$ or $f(x,y,z)=0$

$$f := x^2 + y^2 - 1$$

$$f := x^2 + y^2 - 1$$

(5)

$f(x,y)=0$	$f(x,y,z)=0$
<p data-bbox="87 863 651 905"><i>implicitplot(f, x=-1 ..1, y=-1 ..1)</i></p>  <p>A 2D plot showing a red circle centered at the origin (0,0) with a radius of 1. The x-axis is labeled 'x' and ranges from -1 to 1 with tick marks at -1, -0.5, 0, 0.5, and 1. The y-axis is labeled 'y' and ranges from -1 to 1 with tick marks at -1, -0.5, 0, 0.5, and 1.</p>	<p data-bbox="753 863 1365 947"><i>implicitplot3d(f, x=-1 ..1, y=-1 ..1, z=-3 ..3)</i></p>  <p>A 3D plot showing a cylinder centered at the origin of the xy-plane. The cylinder has a radius of 1 and extends along the z-axis from z = -3 to z = 3. The x and y axes range from -1 to 1, and the z-axis ranges from -3 to 3. The surface is colored with a gradient from purple at the top to green at the bottom.</p>