



EEN-E2001 Computational Fluid Dynamics

Lecture 3*: OpenFOAM code and structure

Taught by Prof. Ville Vuorinen
 Presenter: MEng. Ilya Morev

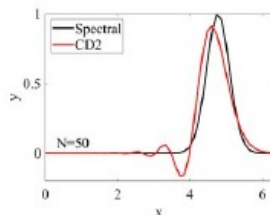
January 30th 2023

Aalto University, School of Engineering

Lecture 1: Linear PDEs and finite difference method

$$\frac{\partial T}{\partial t} + \nabla \cdot T \mathbf{u} = \alpha \nabla^2 T$$

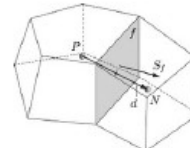
$$\frac{\partial T}{\partial x} \approx \frac{T_{i+1} - T_{i-1}}{2 \Delta x}$$



Lecture 2: Gauss' theorem and finite volume method

$$\int_{\Omega} \nabla \cdot (T \mathbf{u}) d\Omega = \int_{\partial\Omega} (T \mathbf{u}) \cdot \mathbf{n} dS$$

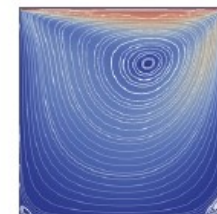
$$\int_{\partial\Omega} \mathbf{u} \cdot \mathbf{n} dS \approx \sum_f \mathbf{u}_f \cdot \mathbf{n}_f dS_f$$



Lecture 3: Navier-Stokes equation and pressure

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathbf{u} \mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u}$$

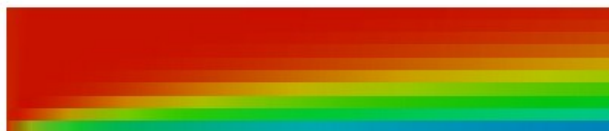
$$-\nabla^2 p = \nabla \cdot \nabla \cdot \mathbf{u} \mathbf{u}$$



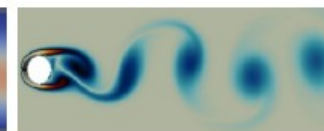
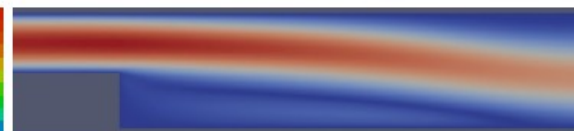
Lecture 4: OpenFOAM code and structure

```
fvVectorMatrix UEqn
(
    fvm::ddt(U)
    + fvm::div(phi, U)
    - fvm::laplacian(nu, U)
);
```

Lecture 5: Simulating fluid physical phenomena: part A



Lecture 6: Simulating fluid physical phenomena: part B



*Lectures 3 and 4 had to be swapped this year

OpenFOAM

Part 1

How to do simulations?

Part 2

How do solvers work?

Part 1

How to do simulations?

Terminology

<i>Solver</i>	Executable application designed to solve a specific problem in fluid or continuum mechanics (scalarTransportFoam , icoFoam ,...)
<i>Utility</i>	Executable application, designed to perform tasks that involve data manipulation (blockMesh , postProcess , foamListTimes ,...)
<i>Library</i>	Precompiled C++ libraries that are dynamically linked to the solvers and utilities. There are separate libraries containing e.g. turbulence models, post-processing functions, etc.
<i>Case folder</i>	Folder containing OpenFOAM dictionaries, describing particular problem (case)
<i>Dictionary</i>	A file/entity that contains data entries in the format understandable by OpenFOAM (0/U , system/controlDict , ...)
<i>Function objects</i>	Tools to ease workflow configurations and enhance workflows by producing additional user-requested data

OpenFOAM 10 – open-source CFD library

github.com/OpenFOAM/OpenFOAM-10

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OpenFOAM / OpenFOAM-10 Public Notifications Fork 18 Star 34

Code Issues 4 Pull requests Actions Projects Security Insights

master 1 branch 157 tags Go to file Code

Will Bainbridge HookFunctions: Corre... c4cf895 on Dec 7, 2022 6,060 commits

Name	Size	Modified
applications	4 items	4 Nov 2022
bin	55 items	4 Nov 2022
doc	4 items	4 Nov 2022
etc	12 items	4 Nov 2022
platforms	1 item	4 Nov 2022
src	46 items	4 Nov 2022
test	6 items	4 Nov 2022
tutorials	17 items	4 Nov 2022
wmake	20 items	4 Nov 2022
Allwmake	1.1 kB	31 Aug 2022
build-stamp	0 bytes	31 Aug 2022
COPYING	35.6 kB	31 Aug 2022
README.org	1.6 kB	31 Aug 2022

About
OpenFOAM Foundation repository for OpenFOAM version 10
Readme
View license
34 stars
3 watching
18 forks

Releases
157 tags

Packages
No packages published

Languages
C++ 97.9% Shell 1.6%

/opt/openfoam10/

Name	Size	Modified
applications	4 items	4 Nov 2022
bin	55 items	4 Nov 2022
doc	4 items	4 Nov 2022
etc	12 items	4 Nov 2022
platforms	1 item	4 Nov 2022
src	46 items	4 Nov 2022
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tutorials	17 items	4 Nov 2022
wmake	20 items	4 Nov 2022
Allwmake	1.1 kB	31 Aug 2022
build-stamp	0 bytes	31 Aug 2022
COPYING	35.6 kB	31 Aug 2022
README.org	1.6 kB	31 Aug 2022

OpenFOAM

```
graph TD; A[OpenFOAM] --> B[Pre-processing]; A --> C[Solving]; A --> D[Post-processing];
```

Pre-processing

1. Choose solver
2. Set up case dictionaries
3. Generate mesh
4. (optional) Adding extra terms and constraints
5. (optional) Set up function objects

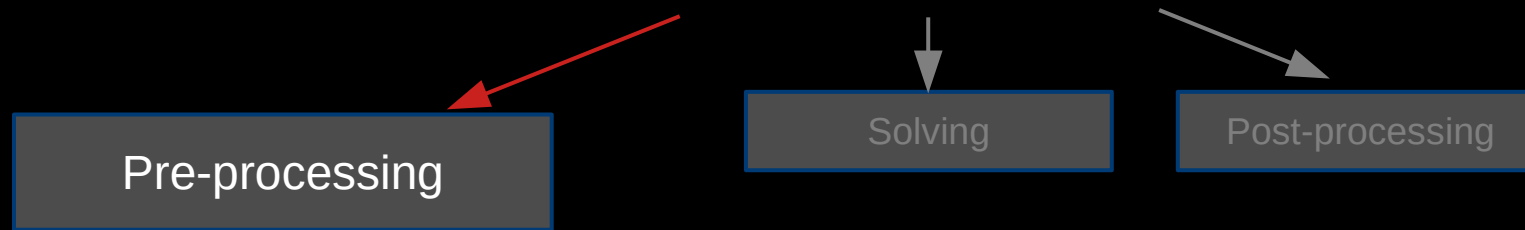
Solving

1. Running solvers
2. Run-time controls

Post-processing

1. Post-processing and sampling
2. Visualization

OpenFOAM

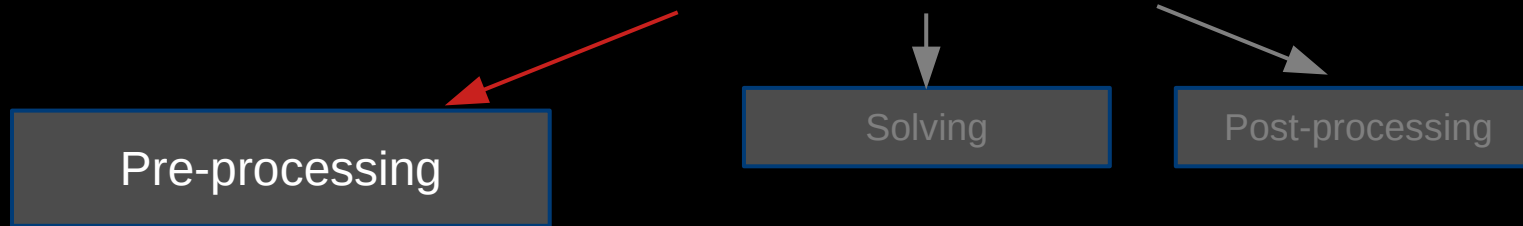


1. Choosing solver

Example list of solvers

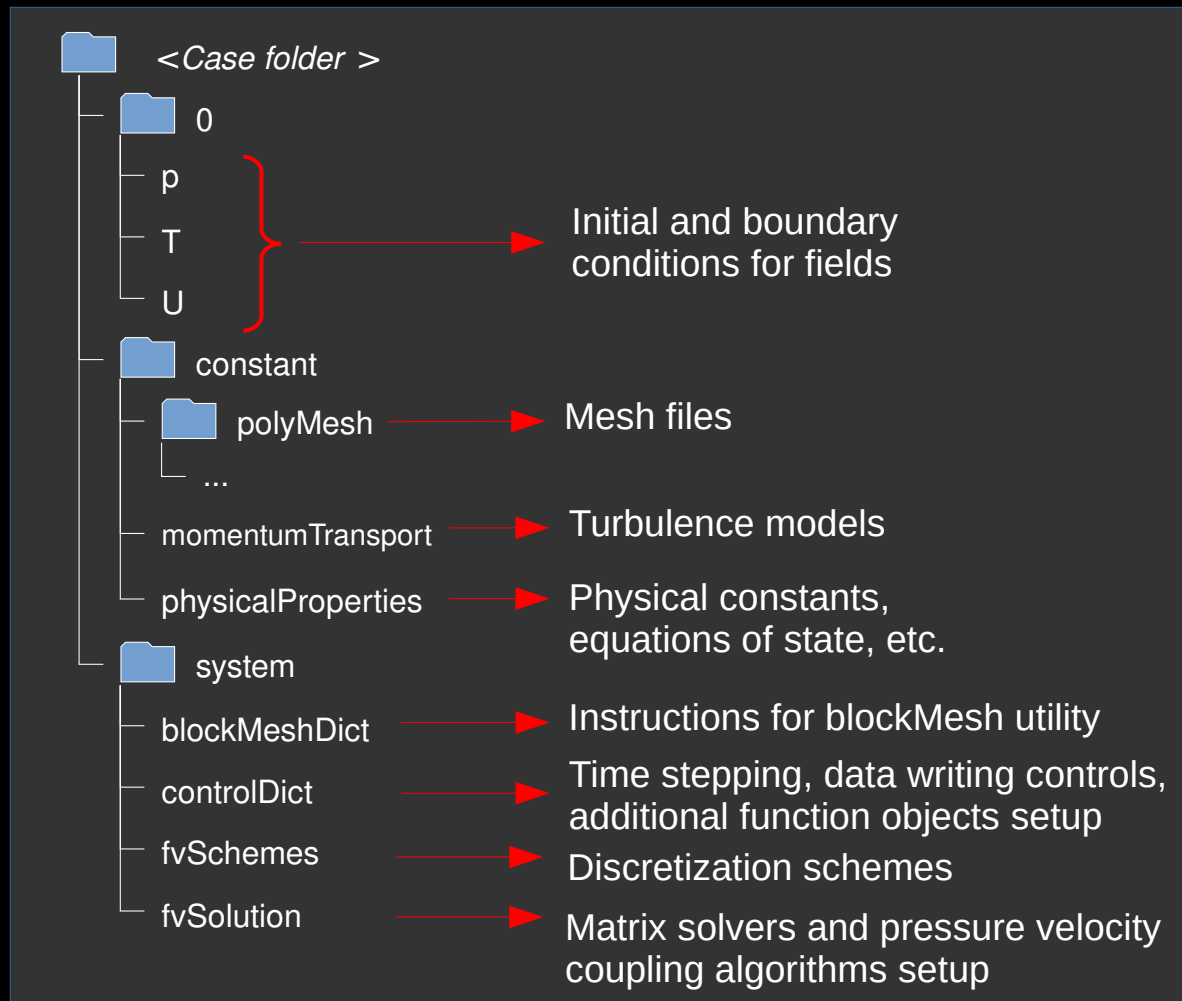
scalarTransportFoam	Convection-diffusion of a passive scalar
icoFoam	Simplified PISO solver for incompressible, laminar flow of Newtonian fluids
simpleFoam	Incompressible fluid flow solver, using SIMPLE algorithm. Used for steady-state flows
pisoFoam	Incompressible fluid flow solver, using PISO algorithm. Used for transient flows
pimpleFoam	Incompressible fluid flow solver, using PIMPLE algorithm. Used for transient flows
rhoPimpleFoam	Compressible fluid flow solver, using PIMPLE algorithm
buoyantPimpleFoam	Compressible fluid flow solver with buoyancy modeling, using PIMPLE algorithm
reactingFoam	Compressible reacting fluid flow solver with chemistry reactions , using PIMPLE algorithm
interFoam	Two incompressible fluids flow solver using “ volume of fluid ” model, using PIMPLE algorithm
interPhaseChangeFoam	Two incompressible fluids flow solvers using “ volume of fluid ” model with phase-change , using PIMPLE algorithm
multiphaseEulerFoam	System of any number of compressible fluid phases flow solver, using PIMPLE algorithm

OpenFOAM

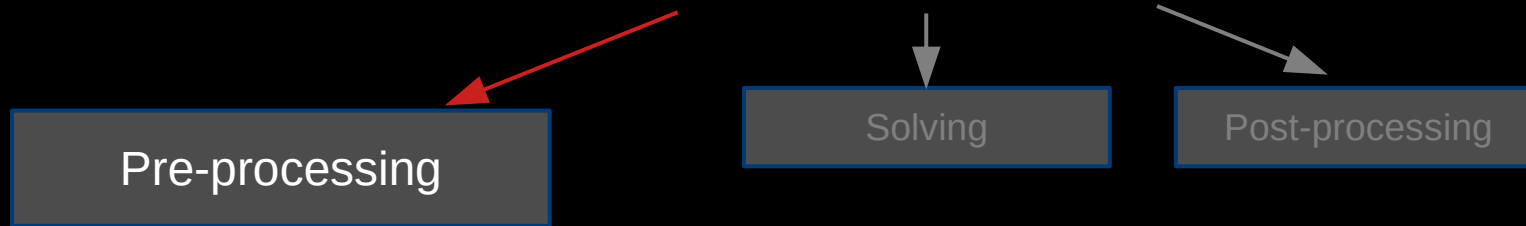


2. Setting up case dictionaries

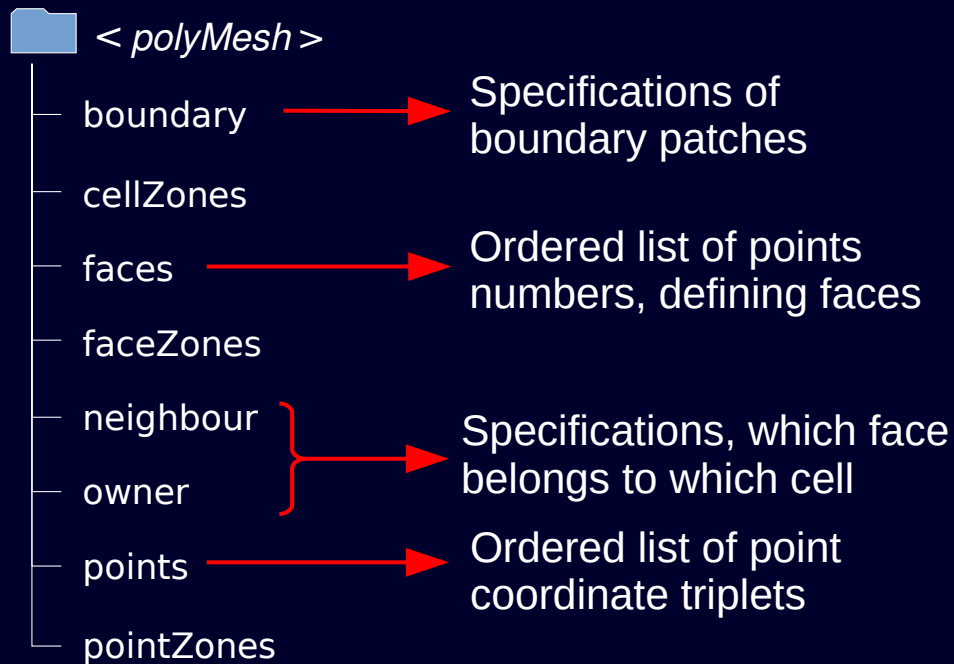
Usually starts with copying tutorial case



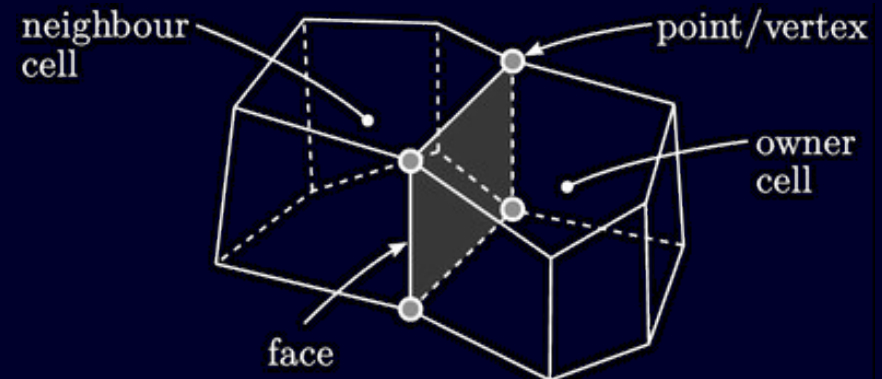
OpenFOAM



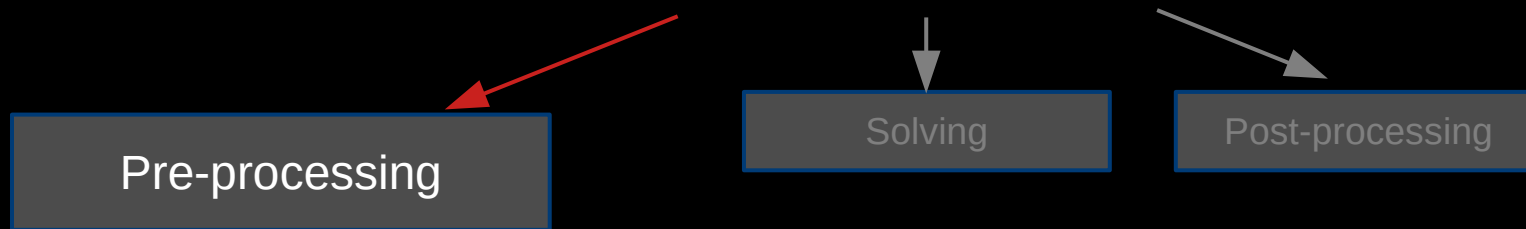
3. Mesh generation



Main mesh generation utilities	
blockMesh	for simple structured meshes
snappyHexMesh	for meshing complex geometries (using stl, obj, vtk, ...)
starToFoam fluentMeshToFoam gmshtoFoam ...	import 3rd party mesh formats



OpenFOAM



4. Adding extra terms and constraints

constant/fvModels

```
energySource
{
  type          heatSource;
  selectionMode all;
  q             1e7;
}
```

system/fvConstraints

```
limitp
{
  type          limitPressure;
  min           0.8e5;
  max           1.2e5;
}
```

Commands to list available options:

```
scalarTransportFoam -listFvModels
scalarTransportFoam -listFvConstraints
```

Command to show info and find usage examples:

```
foamInfo -a heatSource
foamInfo -a limitPressure
```

<https://github.com/OpenFOAM/OpenFOAM-10/tree/master/src/fvModels>

<https://github.com/OpenFOAM/OpenFOAM-10/tree/master/src/fvConstraints>

5. Set up function objects

system/controlDict

```
functions
{
  probes
  {
    type          probes;
    libs          ("libsampling.so");
    writeControl  timeStep;
    writeInterval 1;

    fields
    (
      p
    );

    probeLocations
    (
      (0.0254 0.0253 0)
      (0.0508 0.0253 0)
      (0.0762 0.0253 0)
      (0.1016 0.0253 0)
      (0.127 0.0253 0)
      (0.1524 0.0253 0)
      (0.1778 0.0253 0)
    );
  }

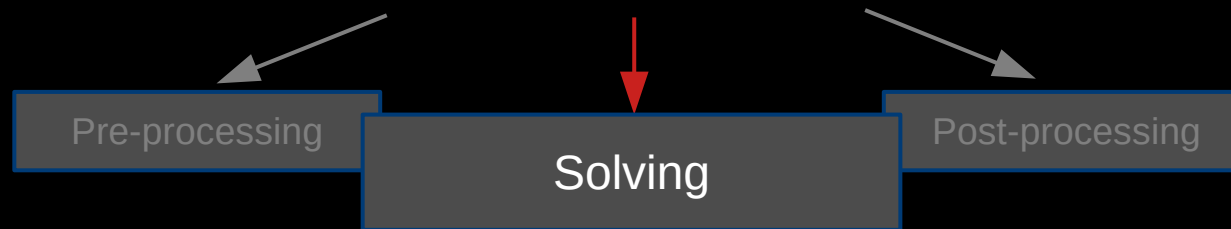
  #includeFunc fieldAverage(U, p, prime2Mean = yes)

  #includeFunc scalarTransport
}
```

```
scalarTransportFoam -listFunctionObjects
FoamInfo -a probes
```

<https://doc.cfd.direct/openfoam/user-guide-v10/post-processing-cli>

OpenFOAM



1. Running solver

a) On a single core

```
scalarTransportFoam
```

b) On multiple cores (requires *system/decomposeParDict*)

```
decomposePar  
mpirun -np 4 scalarTransportFoam -parallel  
reconstructPar
```

c) On supercomputing cluster: using schedulers

2. Run-time controls

If you set in *system/controlDict*:

```
runTimeModifiable true;
```

You can modify entries in dictionaries during run-time.

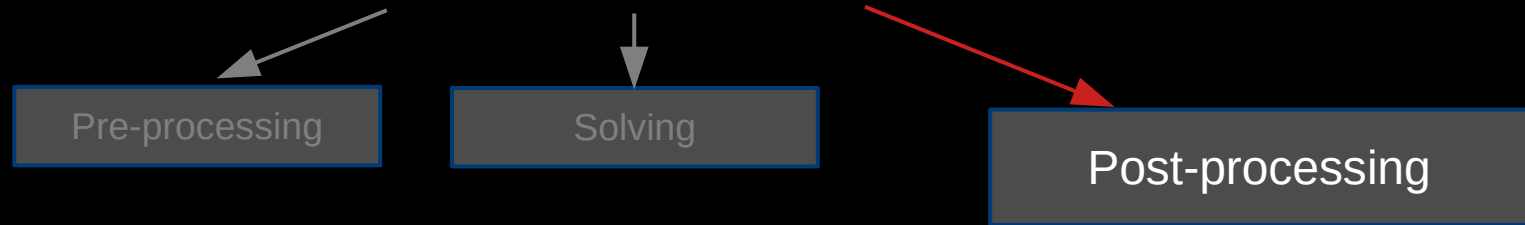
E.g. adjust tolerances, change write times, change time step etc.

You can also track residuals of your simulation using `foamMonitor` utility.

You can stop the simulation and write the fields immediately by setting in *system/controlDict*:

```
stopAt      writeNow;
```

OpenFOAM



1. Post-processing and sampling

Utility `postProcess` can execute `functionObjects` after simulation is finished.

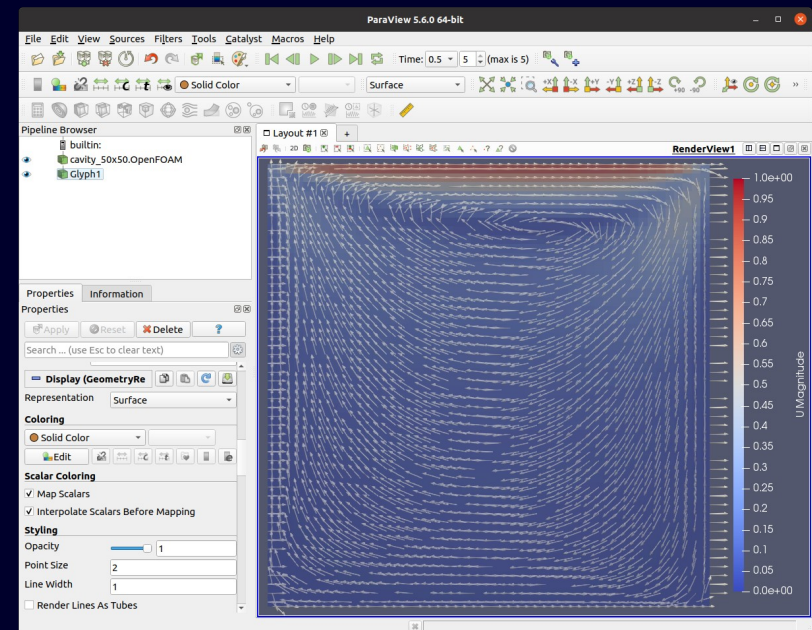
E.g. sample data over a line (requires file `system/singleGraph`):

```
postProcess -func singleGraph
```

```
line_T.xy
1 0.294375 0.0050457
2 0.490625 0.0219314
3 0.686875 0.025386
4 0.883125 0.00392596
5 1.07937 -0.037068
6 1.27562 -0.0748786
7 1.47187 -0.0791233
8 1.66812 -0.0279486
9 1.86437 0.082219
10 2.06062 0.233703
11 2.25687 0.395413
12 2.45312 0.534167
13 2.64937 0.625117
```

2. Visualization

Use `paraFoam` to visualize fields, make videos, renders, plots, etc.



Or export the data to 3rd part format, e.g. VTK:

```
foamToVTK
```

Part 2

How do solvers work?

Main question:

How does OpenFOAM solver proceed from one time step to the next one?

```
Time = 0.96
smoothSolver: Solving for T, Initial residual = 0.0190827, Final residual = 2.62384e-06, No Iterations 2
Time = 0.965
smoothSolver: Solving for T, Initial residual = 0.0190088, Final residual = 2.47331e-06, No Iterations 2
Time = 0.97
```

Matrix equation solver logs. But where does the matrix come from?

Time-marching

1. Equations are integrated over some time step.
2. New values of fields (u , p , T ...) are obtained and then used as initial values for next iteration.



Scalar transport equation

$$\frac{\partial c}{\partial t} + \vec{u} \cdot \nabla c - \alpha \nabla^2 c = \dot{\omega}_c$$

Temporal derivative

Convection

Diffusion

Source

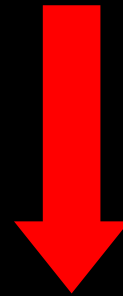
Initial condition

$$c(t=t_0) = c^{t_0}$$

Time step

$$\Delta t = t_1 - t_0$$

$$c^{t_1} = ?$$



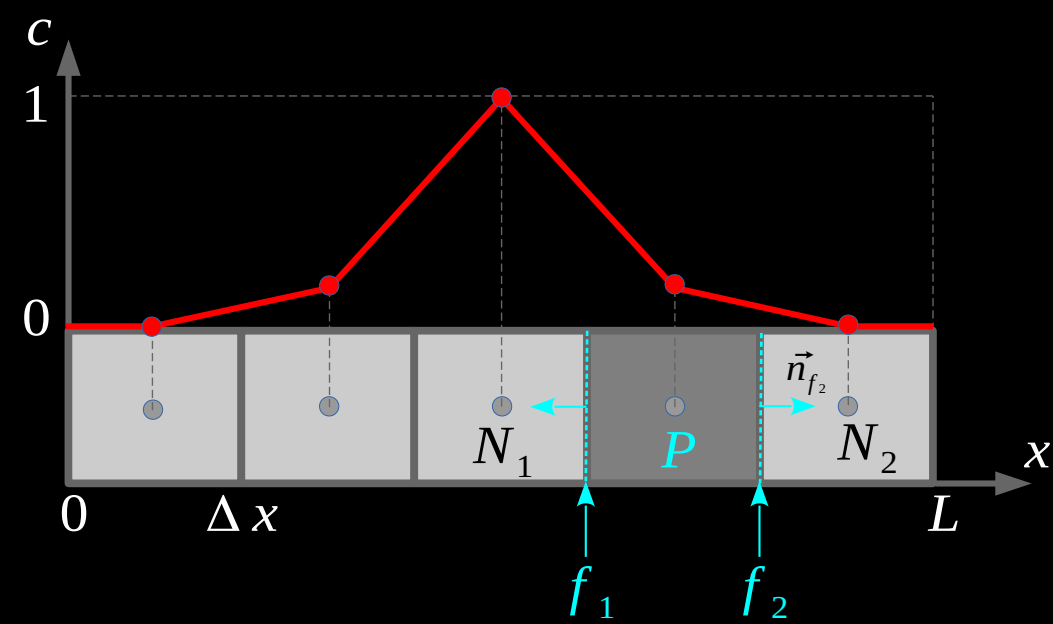
N – number of cells

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{pmatrix} \cdot \begin{pmatrix} c_1^{t_1} \\ c_2^{t_1} \\ \vdots \\ c_N^{t_1} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{pmatrix}$$

1d convection of a Gaussian

Goal: calculate field c after Δt

$u = \text{const} > 0$, $t_1 = t_0 + \Delta t$, 5 cells



$$\frac{\partial c}{\partial t} + \nabla \cdot (\vec{u}c) = 0$$

Integrate over cell P

$$\frac{1}{V_P} \int_{V_P} \nabla \cdot (\vec{u}c) dV = \frac{1}{V_P} \int_{A_P} (\vec{u}c) \cdot \vec{n} dA_P \approx \frac{1}{V_P} \sum_f (\vec{u}_f c_f) \cdot \vec{n}_f A_f = \frac{u_{f_2} c_{f_2} - u_{f_1} c_{f_1}}{\Delta x} = u \frac{c_{f_2} - c_{f_1}}{\Delta x}$$

Gauss theorem

Finite number of faces

1d uniform mesh

Constant velocity

Implicit Euler temporal discretization:

$$\frac{c_P^{t_1} - c_P^{t_0}}{\Delta t} + u \frac{c_{f_2}^{t_1} - c_{f_1}^{t_1}}{\Delta x} = 0$$

Putting constant coefficients to a_i

$$a_P c_P^{t_1} + \sum_{N_i} a_{N_i} c_{N_i}^{t_1} = b_P$$

Interpolating c_{f_1}, c_{f_2}

linear

$$c_{f_1} = \frac{c_{N_1} + c_P}{2}$$

upwind

$$c_{f_1} = c_P$$

We have 1 equation for each of 5 cells. Total:

- 5 equations
- 5 unknowns

$$a_P c_P^{t_1} + \sum_{N_i} a_{N_i} c_{N_i}^{t_1} = b$$

Diagonal Components
("owner")

Off-diagonal Components
("neighbor")



$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix} \cdot \begin{pmatrix} c_1^{t_1} \\ c_2^{t_1} \\ c_3^{t_1} \\ c_4^{t_1} \\ c_5^{t_1} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix}$$

Known coefficient matrix

Field vector to be solved

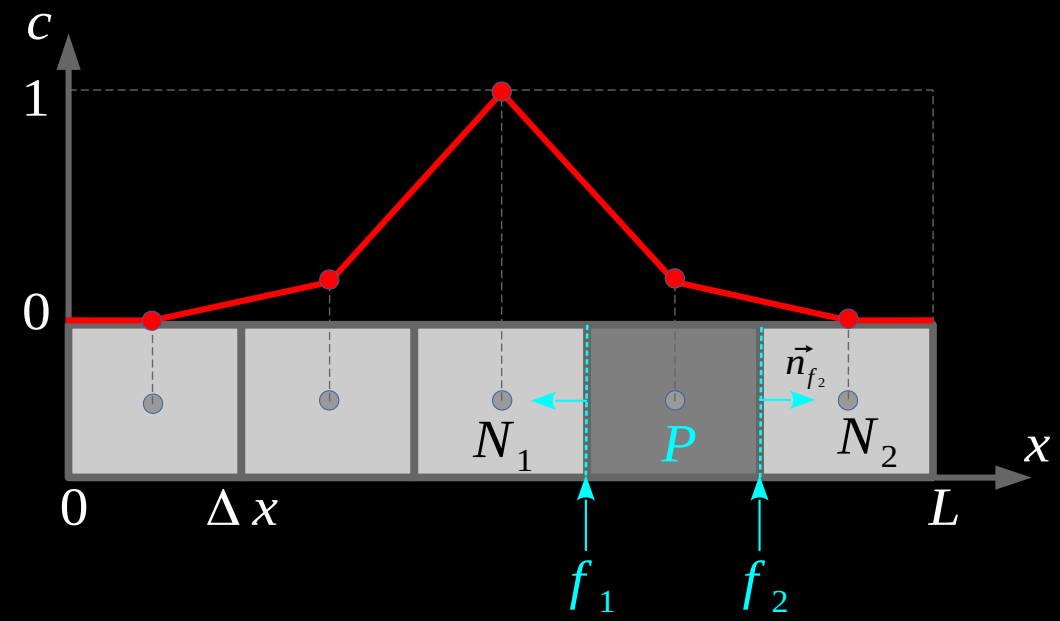
Known vector of explicit terms



$$A \vec{c}^{t_1} = \vec{b}$$



Use some matrix solver to obtain field c at time t_1



scalarTransportFoam source code

Located in `applications/solvers/basic/scalarTransportFoam/`

```
Info<< "\nCalculating scalar transport\n" << endl;

#include "CourantNo.H"

while (simple.loop(runTime))
{
    Info<< "Time = " << runTime.userTimeName() << nl << endl;

    fvModels.correct();

    while (simple.correctNonOrthogonal())
    {
        fvScalarMatrix TEqn
        (
            fvm::ddt(T)
            + fvm::div(phi, T)
            - fvm::laplacian(DT, T)
            ==
            fvModels.source(T)
        );

        TEqn.relax();
        fvConstraints.constrain(TEqn);
        TEqn.solve();
        fvConstraints.constrain(T);
    }

    runTime.write();
}

Info<< "End\n" << endl;
```

Loop over time steps, as defined in `system/controlDict`

$$\frac{\partial c}{\partial t} + \vec{u} \cdot \nabla c - \alpha \nabla^2 c = \dot{\omega}_c$$

Matrix equation is constructed here

$$\mathbf{A} \vec{c}^{t_1} = \vec{b}$$

Call matrix solver, defined in `system/fvSolution`

Write data, if current time step fits `writeTime` defined in `system/controlDict`

Time = 0.96

smoothSolver: Solving for T, Initial residual = 0.0190827, Final residual = 2.62384e-06, No Iterations 2
Time = 0.965

smoothSolver: Solving for T, Initial residual = 0.0190088, Final residual = 2.47331e-06, No Iterations 2
Time = 0.97

Discretization schemes

Located in *system/fvSchemes*

```
ddtSchemes
{
  default Euler;
}

gradSchemes
{
  default Gauss linear;
  grad(p) Gauss linear;
}

divSchemes
{
  default none;
  div(phi,T) Gauss linear;
}

laplacianSchemes
{
  default Gauss linear orthogonal;
}

interpolationSchemes
{
  default linear;
}

snGradSchemes
{
  default orthogonal;
}
```

$$\frac{\partial}{\partial t}$$

Temporal discretization schemes:
Euler (1st ord.), backward (2nd ord.), ...

$$\nabla$$

In the most cases, linear works perfectly well here. In our applications we discretize pressure gradient here.

$$\nabla \cdot$$

div(phi,...) are the most important schemes usually! Here we discretize convection term.
upwind (1st ord.), linear (2nd ord.), limitedLinear, Gamma and vanLeer are probably the most common choices

$$\nabla^2$$

The keyword "linear" refers to interpolation scheme, where linear is usually enough. The second keyword in surface normal gradient scheme, which usually is either orthogonal or corrected (for meshes with orthogonality)

$$C_f$$

Cell to face interpolations of values. Used in the interpolation of velocity to face centers for the calculation of flux

$$\nabla_n$$

Component of gradient normal to a cell face

Check what schemes are used in tutorials:

```
foamSearch -c $FOAM_TUTORIALS fvSchemes "divSchemes/div(phi,U)"
```

Flux limiting schemes

$$c_{f_1} = c_{f_1}^{low} - \phi(r_{N_1})(c_{f_1}^{low} - c_{f_1}^{high})$$

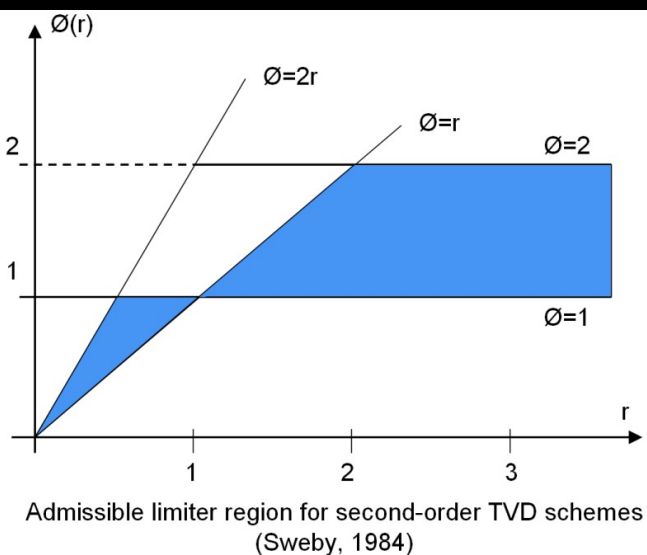
$$c_{f_2} = c_{f_2}^{low} - \phi(r_P)(c_{f_2}^{low} - c_{f_2}^{high})$$

$c_{f_1}^{low}$ - low resolution flux (upwind)

$c_{f_1}^{high}$ - high resolution flux (linear)

$\phi(r_P)$ - flux limiter function

$r_P = \frac{c_P - c_{N_1}}{c_{N_2} - c_P}$ - ratio of successive gradients



Recall:

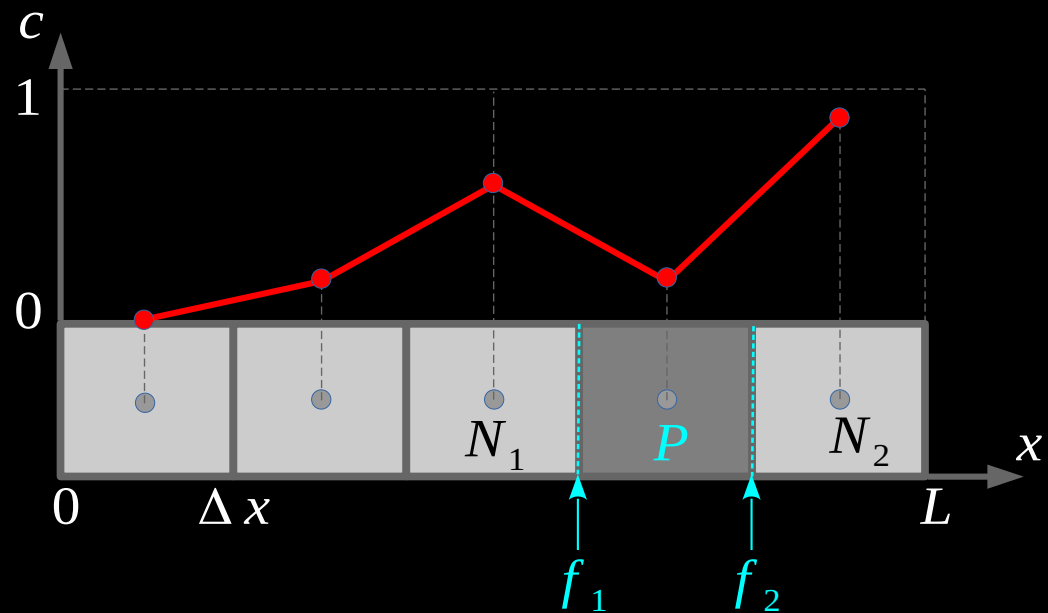
Interpolating c_f

linear

$$c_{f_1} = \frac{c_{N_1} + c_P}{2}$$

upwind

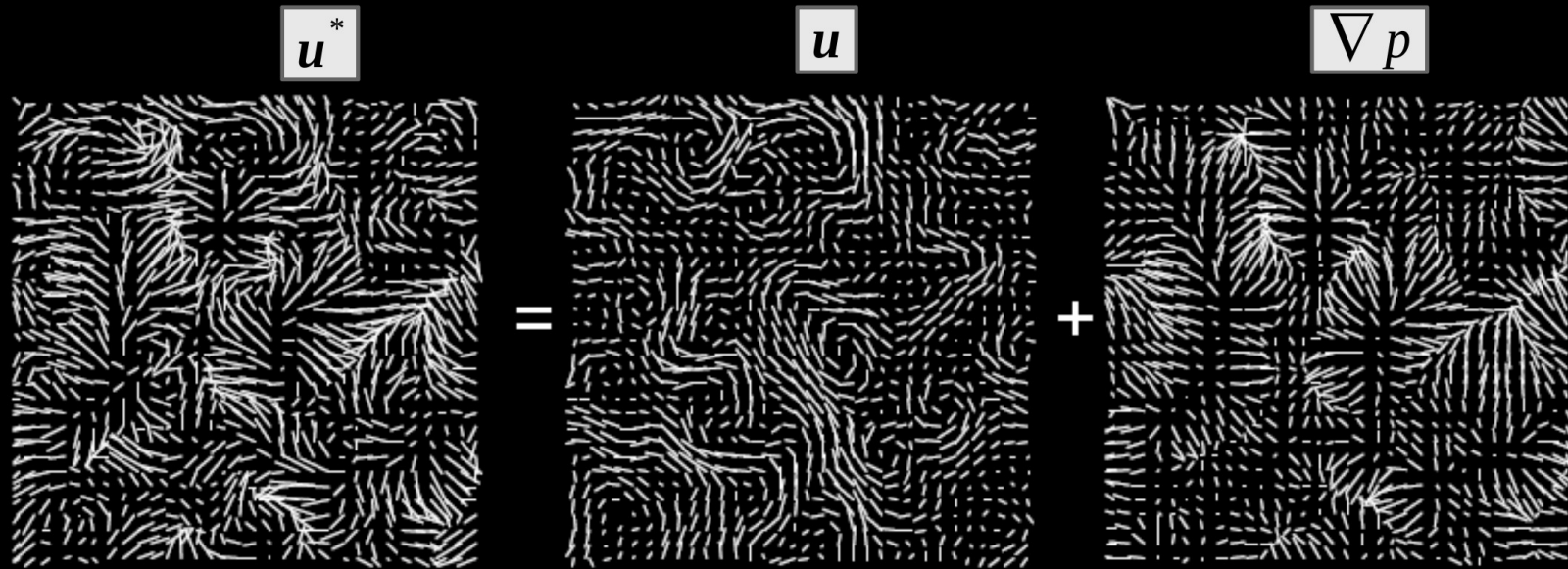
$$c_{f_1} = c_P$$



Vector fields can be divided into two parts via “Helmholtz-Hodge” decomposition

Recall lecture 3:
projection method

Conservation of Mass



Our field

=

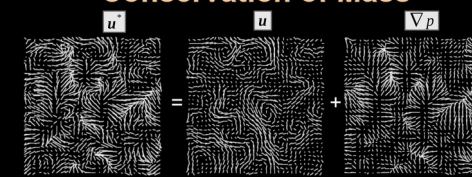
mass conserving +

gradient

Hodge decomposition

Pressure-velocity coupling algorithms

Conservation of Mass



Our field = mass conserving + gradient

Hodge decomposition

Start with Navier-Stokes equations:

$$\nabla \cdot \vec{u} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} - \nabla \cdot (\nu \nabla \vec{u}) = -\nabla p$$

Semi-discretize LHS
(velocity and pressure
gradient are unknown):

$$M \vec{u} = \vec{b} - \nabla p$$

A Diagonal
matrix

Off-diagonal part of M ,
evaluated with u from
prev. solution

$$H = A \vec{u} - M \vec{u} + \vec{b}$$

$$A \vec{u} - H = -\nabla p$$

$$\vec{u} = A^{-1} H - A^{-1} \nabla p$$

Poisson equation

Substitute to the
continuity equation

$$\nabla \cdot (A^{-1} \nabla p) = \nabla \cdot (A^{-1} H) \rightarrow p$$

Simplified solution scheme

Initial guess

1. Momentum predictor

Solve the momentum equation for the velocity field. This velocity field does not satisfy the continuity equation.

$$M \vec{u} = -\nabla p$$

2. Explicit part evaluation

Use the velocity to calculate explicit part H

$$H = A \vec{u} - M \vec{u} + \vec{b}$$

3. Pressure-corrector

Solve the Poisson equation for the pressure field.

$$\nabla \cdot (A^{-1} \nabla p) = \nabla \cdot (A^{-1} H)$$

4. Explicit velocity calculation

Use the pressure field to calculate new velocity field, satisfying the continuity equation. Pressure field is not corrected anymore

$$\vec{u} = A^{-1} H - A^{-1} \nabla p$$

Outer corrector (SIMPLE loop)

non-orthogonal corrector
Inner corrector (PISO loop)

OpenFOAM code (icoFoam)

```
fvVectorMatrix UEqn
(
    fvm::ddt(U)
    + fvm::div(phi, U)
    - fvm::laplacian(nu, U)
);
```

```
if (simple.momentumPredictor())
{
    solve(UEqn == -fvc::grad(p));

    fvOptions.correct(U);
}
```

```
volScalarField rAU(1.0/UEqn.A());
volVectorField HbyA(constrainHbyA(rAU*UEqn.H(), U, p));
surfaceScalarField phiHbyA
(
    "phiHbyA",
    fvc::flux(HbyA)
    + fvc::interpolate(rAU)*fvc::ddtCorr(U, phi)
);
```

```
// Non-orthogonal pressure corrector loop
while (simple.correctNonOrthogonal())
{
    fvScalarMatrix pEqn
    (
        fvm::laplacian(rAU*U(), p) == fvc::div(phiHbyA)
    );
    pEqn.setReference(pRefCell, pRefValue);
    pEqn.solve();

    if (simple.finalNonOrthogonalIter())
    {
        phi = phiHbyA - pEqn.flux();
    }
}
```

```
U = HbyA - rAU*fvc::grad(p);
```

Pressure-velocity coupling algorithms

Parameters are located in *system/fvSolution*

```
PIMPLE
```

```
{  
    momentumPredictor no;  
    nOuterCorrectors 1;  
    nCorrectors 2;  
    nNonOrthogonalCorrectors 0;  
}
```

momentumPredictor	switch controlling the momentum predictor. Can be set to “off” for some flows, including low Reynolds number and multiphase.
nOuterCorrectors	sets the number of outer correctors, number of loops over the entire system of equations within on time step, representing the total number of times the system is solved; must be ≥ 1 and is typically set to 1, replicating the PISO algorithm. If you experience pressure fluctuations, increasing this number can help.
nCorrectors	sets the number of inner correctors, i.e. times the algorithm solves the pressure equation and momentum corrector in each step; typically set to 2 or 3.
nNonOrthogonalCorrectors	specifies repeated solutions of the pressure equation, used to update the explicit non-orthogonal correction; typically set to 0 for orthogonal meshes and ≥ 1 for meshes with non-orthogonality

Further reading:

SIMPLE: https://openfoamwiki.net/index.php/The_SIMPLE_algorithm_in_OpenFOAM

PISO: https://openfoamwiki.net/index.php/OpenFOAM_guide/The_PISO_algorithm_in_OpenFOAM

PIMPLE: https://openfoamwiki.net/index.php/OpenFOAM_guide/The_PIMPLE_algorithm_in_OpenFOAM

Matrix solver setup

Located in *system/fvSolution*

$$A \vec{c} = \vec{b} \rightarrow \vec{c}$$

$$\text{residual}_i = A \vec{c}_{\text{guess}_i} - \vec{b}$$

$$\text{relTol}_i = \frac{\text{residual}_i}{\text{residual}_0}$$

```
solvers
{
  p
  {
    solver          PCG;
    preconditioner  DIC;
    tolerance       1e-06;
    relTol          0.05;
  }

  pFinal
  {
    $p;
    relTol          0;
  }

  U
  {
    solver          smoothSolver;
    smoother        symGaussSeidel;
    tolerance       1e-05;
    relTol          0;
  }
}
```

Usually:

- PCG with DIC preconditioner
- GAMG with GaussSeidel smoother

Tolerance for the final **inner corrector** step.
Usually tolerance is tightened here and relTol=0

Solver selection here depends on your grid parameters, which determines the filling of your matrix.
PBiCGStab with DILU preconditioner is quite robust

Time = 0.295

```
smoothSolver: Solving for Ux, Initial residual = 0.00336414, Final residual = 4.87212e-06, No Iterations 2
smoothSolver: Solving for Uy, Initial residual = 0.00395571, Final residual = 6.13208e-06, No Iterations 2
DICPCG: Solving for p, Initial residual = 0.00198918, Final residual = 9.51425e-05, No Iterations 27
time step continuity errors : sum local = 1.56381e-08, global = 5.72285e-20, cumulative = 2.48268e-20
DICPCG: Solving for p, Initial residual = 0.00061602, Final residual = 9.64995e-07, No Iterations 65
time step continuity errors : sum local = 1.49115e-10, global = 2.02111e-20, cumulative = 4.50379e-20
ExecutionTime = 0.31 s  ClockTime = 1 s
```

Further reading

User guide:

Online: <https://doc.cfd.direct/openfoam/user-guide-v10/index>

Offline: /opt/openfoam10/doc/Guides/OpenFOAMUserGuide-A4.pdf

Programmers Guide:

<https://sourceforge.net/projects/openfoam/files/v2112/ProgrammersGuide.pdf/download>

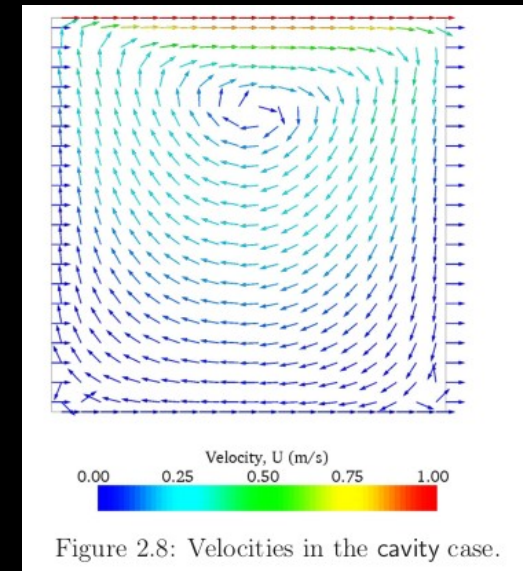
CFD textbook by authors of OpenFOAM (free web version):

<https://doc.cfd.direct/notes/cfd-general-principles/>

User guide
Tutorial relevant to HW2

Textbook

<p>Time, Sec. 3.17 Usually <i>1st-order</i> Euler scheme, Eq. (3.21)</p> <p><i>2nd-order</i> to preserve transient structures, e.g. large eddies, waves Crank-Nicolson, Eq. (3.29) backward implicit, Eq. (3.27)</p>	<p>Laplacian, Sec. 3.7 <i>surface normal gradient</i>, Sec. 3.8</p> <p>non-orthogonal correction Eq. (3.7) for $\theta_{no} \lesssim 75^\circ$</p> <p>limited correction for $\theta_{no} > 75^\circ$</p>
$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) - \nabla \cdot (\nu \nabla \mathbf{u}) = -\nabla p$	
<p>Advection, Sec. 3.9 linear interpolation, Eq. (3.4) creeping flows, large eddy simulation</p> <p>linear upwind, Sec. 3.14 with gradient limiting, Sec. 3.16</p> <p>limited linear, Eq. (3.13) minmod, Eq. (3.14)</p> <p>upwind interpolation, Sec. 3.10 fast-converging, approximate solution</p>	<p>Gradient, Eq. (3.18)</p> <p style="text-align: center;"><i>decreasing accuracy</i></p> <p style="text-align: center;"><i>increasing stability</i></p>



Programmer's guide

Operation	Comment	Mathematical Description	Description in OpenFOAM
Addition		$\mathbf{a} + \mathbf{b}$	$\mathbf{a} + \mathbf{b}$
Subtraction		$\mathbf{a} - \mathbf{b}$	$\mathbf{a} - \mathbf{b}$
Scalar multiplication		$s\mathbf{a}$	$\mathbf{s} * \mathbf{a}$
Scalar division		\mathbf{a}/s	\mathbf{a} / \mathbf{s}
Outer product	rank $\mathbf{a}, \mathbf{b} \geq 1$	$\mathbf{a}\mathbf{b}$	$\mathbf{a} * \mathbf{b}$
Inner product	rank $\mathbf{a}, \mathbf{b} \geq 1$	$\mathbf{a} \cdot \mathbf{b}$	$\mathbf{a} \& \mathbf{b}$