

General information

The exercise sessions will be held as blackboard sessions, where the participants will present their solutions to the group. As such, the problems should be set up and solved before the session. The focus of the exercises lies on analyzing and discussing the task at hand together with the group: thus, a perfect solution is not required to be awarded points. A point will be awarded for each question, and a person will be chosen to present their solution from the pool. *There are useful formulas at the bottom of this document!*

Exercise 1.

Energy confinement times and ignition

A central figure of merit within the field of fusion technology is the fusion triple product. The fusion triple product relates the fuel density, n_{DT} , fuel temperature, T , and energy confinement time, τ_E , to successful, self-sustained fusion burn. The triple product for deuterium-tritium fusion is

$$n_{DT}T\tau_E \geq 10^{22} \text{ keV s m}^{-3}.$$

Calculate the required confinement times for ignition in the following confinement concepts:

- a) **Magnetic confinement:** a tokamak with fuel density $n \sim 10^{20} \text{ m}^{-3}$ and temperature $T \sim 10 \text{ keV}$. Compare the value to the one foreseen for ITER.
- b) **Inertial confinement:** a laser fusion device with $n \sim 10^{32} \text{ m}^{-3}$ and fuel temperature of $T \sim 10 \text{ keV}$. Compare the obtained value to the ratio of the fuel sound speed ($c_s \sim 10^5 \text{ m/s}$) to the fuel capsule radius ($R \sim 100 \text{ }\mu\text{m}$).

Exercise 2.

Power densities and wall power loads in different fusion reactors

Estimate the power densities and wall power loads in the following fusion reactors:

- a) **Tokamak:** A tokamak operating with a 50-50 D-T fuel mix with major radius $R = 5.5 \text{ m}$ and minor radius $a = 1.8 \text{ m}$. Assume flat temperature and density profiles (= constant temperature and density) with $n_D = n_T = 10^{20} \text{ m}^{-3}$ and $T_e = T_i = 10 \text{ keV}$. Use the low temperature approximation from the "Fusion principles" lecture notes to determine the mean reaction rate.

- b) **Z-pinch:** A Z-pinch with $R = 10$ cm operating with a 50-50 DT fuel mix. The fusion reactions occur in a narrow rope with a diameter of $40 \mu\text{m}$. The DT density in the rope is $n_{DT} = 4.5 \times 10^{28} \text{ m}^{-3}$. The burn-up of the fuel is 30% and the pulse rate, f , of the device is 1 Hz. See figure 1 for illustration of the assumed geometry.

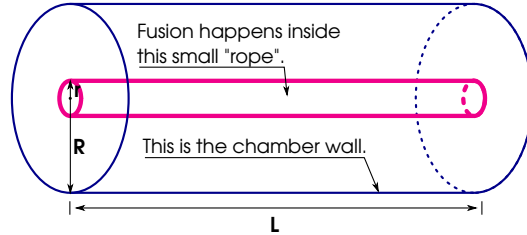
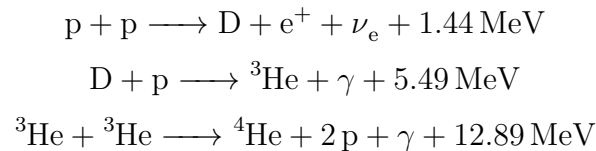


Figure 1: An illustration of the assumed z-pinch geometry in exercise 2b).

Exercise 3.

Sun and stars

In the sun, energy is released primarily through the following chain of reactions:



- a) Show that the energy released per proton fused in this reaction chain is approximately 6.68 MeV. What happens to the positrons and the neutrinos?
- b) Find reasonable estimates for the power output and mass of the sun and use them to calculate the reaction rate per proton per second in the sun. Assume that the power is produced in the core, which contains 10% of the total mass. How long does the average proton have to wait before it fuses in the sun?

Exercise 4.

Inertial confinement

In a laser fusion device, a spherical symmetric fusion target is compressed to 1000 times the initial density: $\rho = 1000\rho_0$, or $R = 0.1R_0$. The initial density of the fuel pellet, consisting of cooled, solid D-T, is $\rho_0 = 0.23 \text{ g/cm}^3$ and the radius is $R_0 = 0.2 \text{ cm}$.

- a) What is the minimum energy of the laser pulse E_{\min} required, if a temperature of 5 keV is required for the ignition? Assume that the fuel compresses adiabatically, i.e. $pV^\gamma = \text{const}$ in the process, with $\gamma = 5/3$.

- b) What is the fusion gain factor (the ratio of fusion energy released to the energy needed)? The burn-up B , the fraction of the fuel that fuses, is needed, and can be calculated according to:

$$B = \frac{\rho R}{\rho R + 6 \frac{\text{g}}{\text{cm}^2}},$$

where ρ is the mass density of the fuel.

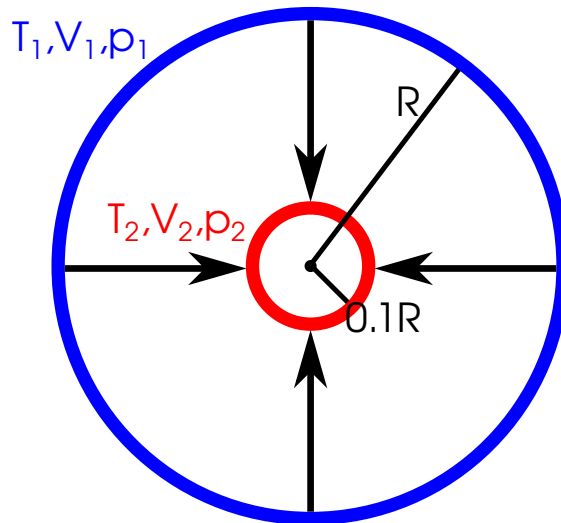


Figure 2: Illustration of the compression in laser fusion.

Constants:

$$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$$

$$m_p = 1.673 \times 10^{-27} \text{ kg}$$

$$m_n = 1.675 \times 10^{-27} \text{ kg}$$

$$N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$$

$$k_B = 1.381 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$$

Power equations assuming pure hydrogenic (Z=1) plasma:

$$\text{Fusion power density: } P_f = \alpha n_i n_j \langle \sigma v \rangle E_f,$$

where E_f represents the produced energy per a fusion reaction, n_i and n_j are the fuel isotope densities, n_e the electron density, T_S the surface temperature of the black body, and A the surface area of the black body. The α parameter in the fusion power density equation is 1 for D-T fusion, and 1/2 for D-D fusion. This parameter arises due to the fact that when calculating the fusion cross-section integral ($\langle \sigma v \rangle$) for like particle collisions (D-D), every collision is counted twice. This should not be confused with the 1/4-factor that arises in the D-T fusion cross-section with 50-50 % fuel mixture due to $n_D = n_T = n_e/2 \rightarrow n_D \times n_T = (1/4) \times n_e$. More information can be found in e.g. [1].

References:

[1] J. P. Freidberg, Plasma Physics and Fusion Energy, Cambridge University Press, 2007, p.44