K1 $u(x,y) = \sin x + f(\sin y - \sin x)$

$$u_x = \frac{\partial u}{\partial x} = \cos x - \cos x \, f'(\sin y - \sin x)$$

$$u_y = \frac{\partial u}{\partial y} = \cos y \, f'(\sin y - \sin x)$$

$$u_y \cos x + u_x \cos y = \cos x \cos y$$

TEHTÄVÄ **Osoita, että funktio $z = \overline{\arctan}\,\frac{x}{y}$, missä $x = u + v$, $y = u - v$, toteuttaa osittaisdifferentiaaliyhtälön**

$$\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} = \frac{u - v}{u^2 + v^2}.$$

RATKAISU **Merkitään lisäksi $w(x, y) = x/y$, jolloin**

$$\begin{aligned}
\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} &= \left(\frac{\partial x}{\partial u}\frac{\partial z}{\partial x} + \frac{\partial y}{\partial u}\frac{\partial z}{\partial y}\right) + \left(\underbrace{\frac{\partial x}{\partial v}}_{=1}\frac{\partial z}{\partial x} + \frac{\partial y}{\partial v}\frac{\partial z}{\partial y}\right) \\
&= 1 \cdot \frac{\partial z}{\partial x} + 1 \cdot \frac{\partial z}{\partial y} + 1 \cdot \frac{\partial z}{\partial x} + (-1) \cdot \frac{\partial z}{\partial y} \\
&= 2\frac{\partial z}{\partial x} = 2\frac{\partial w}{\partial x}\frac{\partial z}{\partial w} = 2 \cdot \frac{1}{y} \cdot \frac{1}{1 + w^2} = \frac{2y}{x^2 + y^2} \\
&= \frac{2(u - v)}{(u + v)^2 + (u - v)^2} = \frac{u - v}{u^2 + v^2}.
\end{aligned}$$