with(*LinearAlgebra*) *with*(*VectorCalculus*) *with*(*Student*[*Calculus1*])*with*(*plots*) :

Newton's method

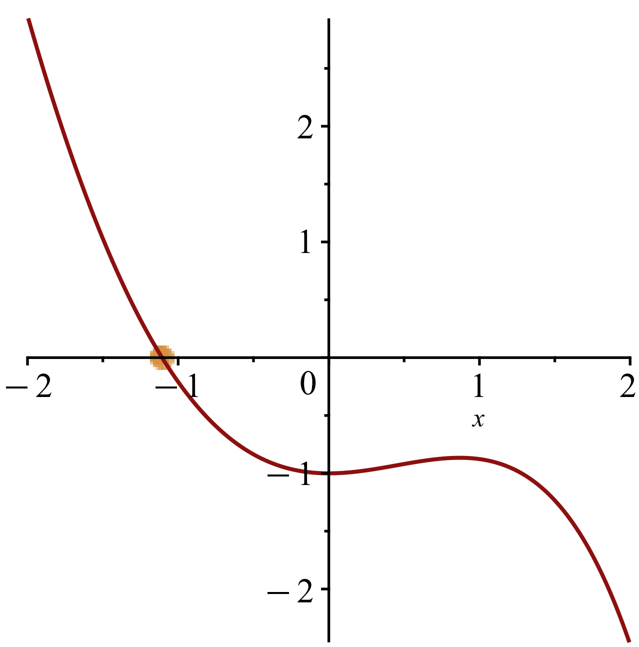
One variable

Consider the one variable function

$f := x^2 - \exp(x) + \sin(x)$

$$f := x^2 - e^x + \sin(x) \quad \textbf{(1)}$$

We want to find the solution(s) of f(x) = 0. Geometrically, this is the intersection of curve y=f(x) with the x-axis.

Solution with Maple:	**Approximation with Newton's method:**
$xsol := fsolve(f = 0, x)$ $xsol := -1.106742853$ **(2)** $pt := pointplot([xsol, 0], color = gold, symbolsize = 30, symbol = solidcircle)$: $curve := plot(f, x = -2..2)$: $display(pt, curve)$ 	Initial guess: $x[0] := 1$ $x_0 := 1$ **(3)** **for** i **from** 0 **to** 7 **do** $x[i+1] := evalf\left(x[i] - \frac{subs(x = x[i], f)}{subs(x = x[i], diff(f, x))}\right)$ **end** $x_1 := -3.926470376$ $x_2 := -2.049552680$ $x_3 := -1.370389794$ $x_4 := -1.140046777$ $x_5 := -1.107398620$ $x_6 := -1.106743115$ $x_7 := -1.106742853$ $x_8 := -1.106742853$ **(4)**

Two variables

Consider the two variables functions

$f := x \cdot (1 + y^2) - 1$

$$f := x\,(y^2 + 1) - 1 \qquad \textbf{(5)}$$

$g := y \cdot (1 + x^2) - 2$

$$g := y\,(x^2 + 1) - 2 \qquad \textbf{(6)}$$

We want to find the solution(s) of [f(x,y), g(x,y)] = [0, 0]. Geometrically, this is the intersection of the three surfaces z = 0, z=f(x,y) and z = g(x,y).

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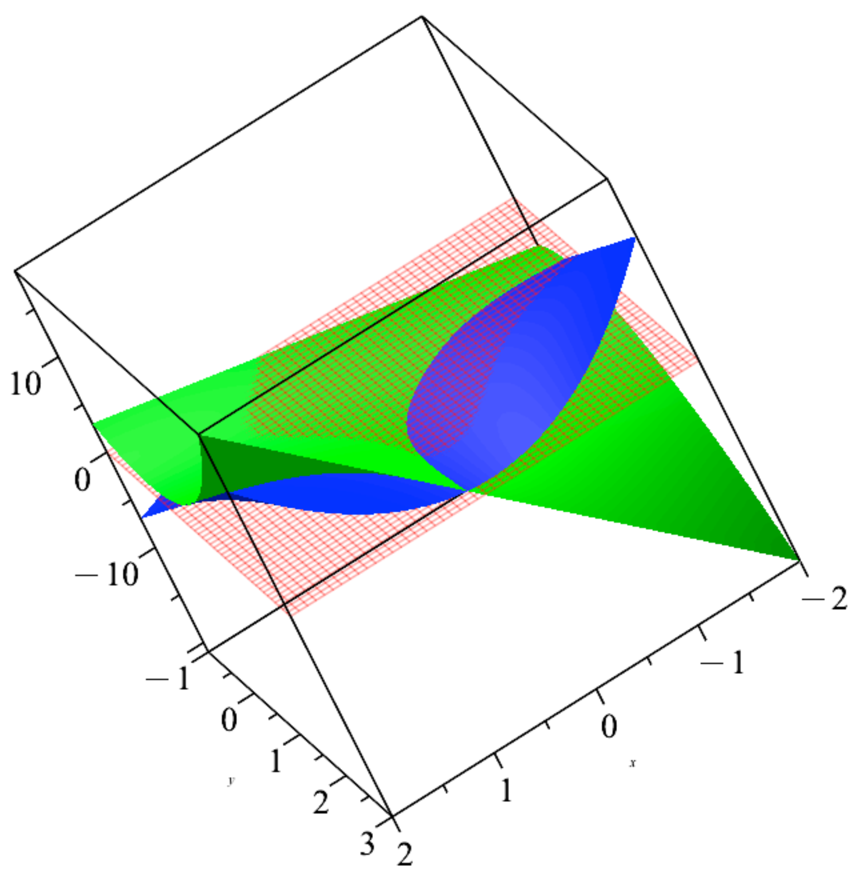
a := plot3d(f, x = -2..2, y = -1..3, color = green, style = surface ) :
b := plot3d(g, x = -2..2, y = -1..3, color = blue, style = surface) :
xyplane := plot3d(0, x = -2..2, y = -1..3, color = red, style = wireframe) :
display(a, b, xyplane)
```



Solution with Maple:

```
sol := fsolve({f, g}, {x, y}, {x = -1..3, y = -1..3})
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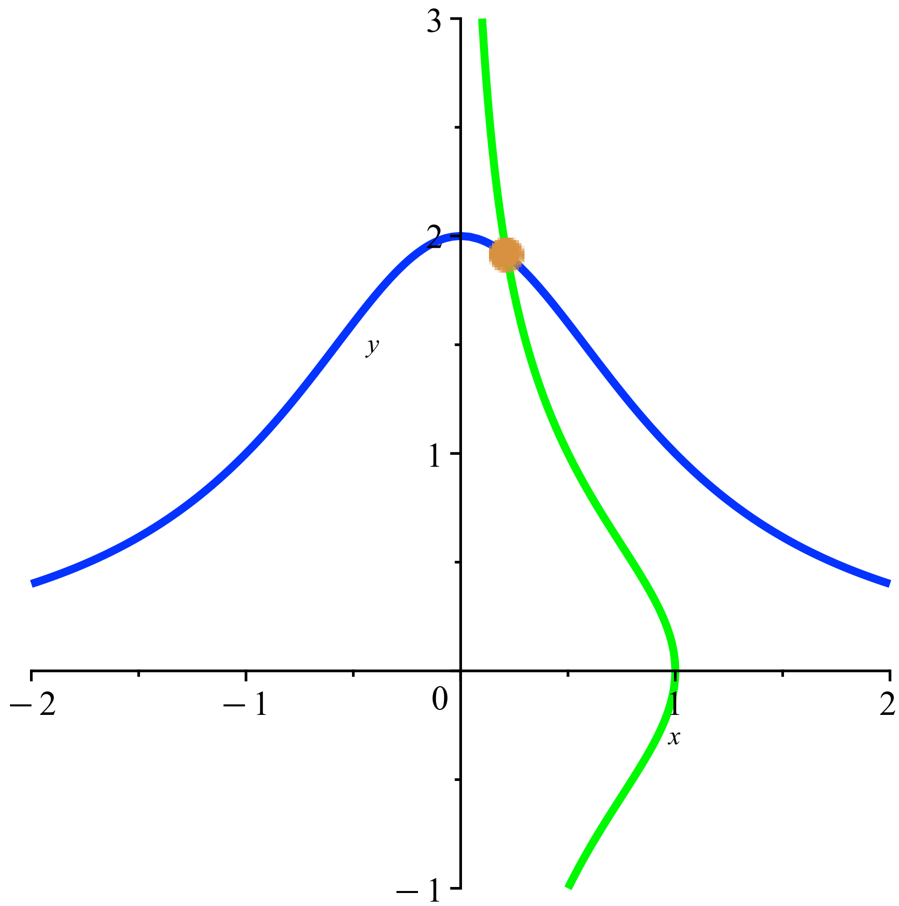
$$sol := \{x = 0.2148292327,\ y = 1.911768812\}$$

$xyinter := implicitplot([f = 0, g = 0], x = -2..2, y = -1..3, color = [green, blue], thickness = 3):$

$pt := pointplot([0.2148292327, 1.911768812], color = gold, symbolsize = 30, symbol = solidcircle):$

$display(xyinter, pt)$



Approximation with Newton's method:

$F := Matrix(2, 1, [f, g])$

$$F := \begin{bmatrix} x\left(y^2+1\right)-1 \\ y\left(x^2+1\right)-2 \end{bmatrix} \quad \textbf{(8)}$$

$J := Jacobian([f, g], [x, y])$

$$J := \begin{bmatrix} y^2+1 & 2\,x\,y \\ 2\,x\,y & x^2+1 \end{bmatrix} \quad \textbf{(9)}$$

$Jinv := MatrixInverse(J)$

$$Jinv := \begin{bmatrix} -\frac{x^2+1}{3x^2y^2-x^2-y^2-1} & \frac{2xy}{3x^2y^2-x^2-y^2-1} \\ \frac{2xy}{3x^2y^2-x^2-y^2-1} & -\frac{y^2+1}{3x^2y^2-x^2-y^2-1} \end{bmatrix} \quad (10)$$

Initial guess:

$x[0] := 3; y[0] := 2$

$$x_0 := 3$$

$$y_0 := 2 \quad (11)$$

for i **from** 0 **to** 7 **do** $A := Matrix(2, 1, [x[i], y[i]]) - subs(x = x[i], y = y[i], Jinv) \cdot subs(x = x[i], y = y[i], F) : x[i+1] := evalf(A[1, 1]); y[i+1] := evalf(A[2, 1]);$ **end**;

$$A := \begin{bmatrix} \frac{103}{47} \\ \frac{55}{47} \end{bmatrix}$$

$$x_1 := 2.191489362$$

$$y_1 := 1.170212766$$

$$A := \begin{bmatrix} 2.17221898774593 \\ 0.361704894993136 \end{bmatrix}$$

$$x_2 := 2.17221898774593$$

$$y_2 := 0.361704894993136$$

$$A := \begin{bmatrix} 0.115617630778300 \\ 0.914876325299808 \end{bmatrix}$$

$$x_3 := 0.115617630778300$$

$$y_3 := 0.914876325299808$$

$$A := \begin{bmatrix} 0.429997962360412 \\ 1.90798741626009 \end{bmatrix}$$

$$x_4 := 0.429997962360412$$

$$y_4 := 1.90798741626009$$

$$A := \begin{bmatrix} 0.162170472277894 \\ 2.05879939129760 \end{bmatrix}$$

$$x_5 := 0.162170472277894$$

$$y_5 := 2.05879939129760$$

$$A := \begin{bmatrix} 0.208782125988151 \\ 1.91842185216502 \end{bmatrix}$$

$$x_6 := 0.208782125988151$$

$$y_6 := 1.91842185216502$$

$$A := \begin{bmatrix} 0.214783574542434 \\ 1.91185475036423 \end{bmatrix}$$

$$x_7 := 0.214783574542434$$

$$y_7 := 1.91185475036423$$

$$A := \begin{bmatrix} 0.214829228884613 \\ 1.91176881717706 \end{bmatrix}$$

$$x_8 := 0.214829228884613$$

$$y_8 := 1.91176881717706 \quad \textbf{(12)}$$

pt2 := *pointplot*([*x*[8], *y*[8]], *color* = *gold*, *symbolsize* = 30, *symbol* = *solidcircle*) :
display(*xyinter*, *pt2*)