1. Geometric optics
   A. The water acts as if it were a fiber-optic cable, and total internal reflection causes the light to seem like it is bending. In reality, the light is reflected inside the stream of water, propagating away from the laser. Not all of the light follows the water, only that which is at the exact critical angle. Light gradually leaks out, else we would not be able to see it in the water. This effect occurs for all wavelengths of light smaller than the diameter of the “water fiber”, provided that the critical angle can be achieved.
   B. Visible light and radiowaves have different wavelengths: visible light’s wavelength is less than 1 micro meter, and radiowaves’ wavelength is tens and hundreds of thousands times more than that. Reflection would appear when the surface irregularities are smaller than the wavelength, but sheet iron has irregularities larger than 1 micro meter and smaller than 1 mm.

2. Maxwell's equations and plane waves

a) Faraday's law: $\nabla \times E = -\mu \frac{\partial H}{\partial t}$

Ampère's law: $\nabla \times H = J + \varepsilon \frac{\partial E}{\partial t}$, $J = \sigma E$

Curl of Faraday's law

$$\nabla \times (\nabla \times E) = -\mu \frac{\partial}{\partial t}(\nabla \times H) \qquad \nabla \times (\nabla \times E) = \underbrace{\nabla(\nabla \cdot E)}_{=0} - \nabla^2 E$$

$$\nabla^2 = \mu \frac{\partial}{\partial t}\left(J + \varepsilon \frac{\partial E}{\partial t}\right)$$

$$\nabla^2 = \mu\sigma \frac{\partial E}{\partial t} + \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 - \mu\sigma \frac{\partial E}{\partial t} - \frac{1}{u^2}\frac{\partial^2 E}{\partial t^2} = 0 \quad , u = \frac{1}{\sqrt{\mu\varepsilon}}$$

Same approach for Ampère's law:

$$\nabla \times (\nabla \times H) = \nabla \times \left(J + \varepsilon \frac{\partial E}{\partial t}\right)$$

$$\nabla^2 = \sigma \nabla \times E + \varepsilon \frac{\partial}{\partial t}(\nabla \times E)$$

$$\nabla^2 = \sigma\mu \frac{\partial H}{\partial t} + \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 - \sigma\mu \frac{\partial H}{\partial t} - \frac{1}{u^2}\frac{\partial^2 H}{\partial t^2} \quad , u = \frac{1}{\sqrt{\mu\varepsilon}}$$

b) $k = \omega\sqrt{\mu\varepsilon} \Rightarrow \varepsilon = \frac{k^2}{\omega^2 \mu}$, $\mu = \mu_0$

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = \frac{k^2}{\omega^2 \mu_0 \varepsilon_0}$$

$$\varepsilon_r = \frac{(4{,}130 \cdot 10^{-5}\ \mathrm{rad/m})^2}{(6{,}283 \cdot 10^{3}\ \mathrm{rad/s})\mu_0\varepsilon_0} \approx 3{,}9 \Rightarrow \text{Silicon dioxide}$$

3. Wave-particle duality

A.

I. Explain how these results contradict the model of light acting only as a wave, and introduce the notion of a wave particle duality. Conclude with the formula E = hf, for a photon.

Answer:

If we assume light to only act as wave, then we would assume the measured current to depend completely or almost completely on the intensity of the light, with no known dependence on frequency. Additionally, if light interacted with the material like a wave, then with very low intensity light, we would expect the material to require some time to gather enough energy to be able to expel an electron, but in the experiment, the current appears immediately. Lastly, the intensity of the light should directly predict the kinetic energy of the electrons, the frequency should not impact it.

However, if in addition to wave properties, we give light particle properties, then all the experimental results can be explained by the energy in light being carried in small packets(photons) rather than a uniform wave. 2 for example is quite easily explained, because if the photons are carrying enough energy then, they will immediately transfer it to the electron causing it to eject from the material, rather than waiting a continuous interval until it absorbs enough energy from the wave. 1 and 3, can be fully explained by the energy of a photon being given by E = hf. This would explain the threshold frequency in 1, as photons without enough energy can not transfer it to electrons, therefore they do not eject. 3 is also explained similarly because the photons at a certain frequency will only transfer a certain amount of energy, with intensity dictating the amount of electrons ejected rather than their kinetic energy; a higher frequency will lead to a higher energy transfer, therefore the electrons will have higher kinetic energy.

II. For a given metal surface, the energy required to expel an electron is 2.5eV. Calculate the maximum wavelength of light capable of emitting photoelectrons.

Answer:

$E = 10\ eV$ photon energy $= hf = \frac{hc}{\lambda}$

$$10 \cdot \frac{1.6\times10^{-19}\ J}{1\ eV} = 16\times10^{-19}\ J$$

$$E = \frac{hc}{\lambda} \rightarrow \lambda = \frac{hc}{E} \approx \frac{6.625\times10^{-34}\ J\cdot s\ (3\times10^{8}\ m/s)}{16\times10^{-19}\ J} \approx 1.2423\times10^{-7}\ m$$

B.

propertie / experiment	wave	particle	both
spread of light	x		
energy absorption & consumption		x	
light diffraction	x		

single/double slit experiment			x
photo effect		x	
heat radiation		x	
interference of light	x		

4. Identifying elements with spectrometry

From every peak's distance we can calculate an angle by which the ray exits the grating:

$\theta = tan^{-1}(l/0.4m)$, *where l is the distance between the peak and the center of the screen*

We don't know the values of n for the peaks, but we know we have to sets of peaks that are consecutive, so we take n as the number for the first half of peaks, and n+1 for the second. We write down the given formula for both cases:

$$n\lambda_i = d\, sin(\theta_{i,1}),\ (n+1)\lambda_i = d\, sin(\theta_{i,2}),\ where\ i\ indexes\ the\ 6\ incoming\ wavelengths$$

Subtracting the first equation from the second, we get:

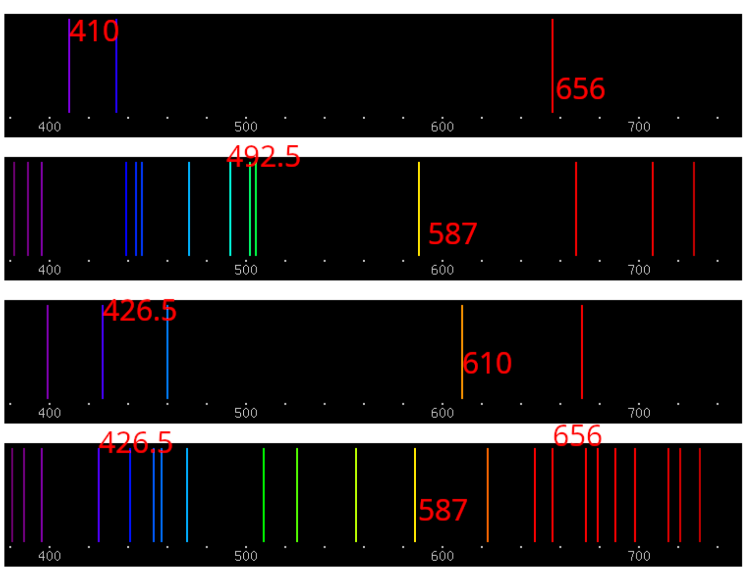
$$\lambda_i = d\,(\,sin(\theta_{i,2}) - sin(\theta_{i,1})\,)$$

Plugging in the formula for angles:

$$\lambda_i = d\,(\,sin(\,tan^{-1}(l_{i,2}/0.4m)\,) - sin(\,tan^{-1}(l_{i,1}/0.4m)\,)\,)$$

Without further simplification we calculate 6 incoming wavelengths:

410nm - from peaks at 18 cm and 31.2 cm,
426.5nm - from peaks at 18.9 cm and 33.3 cm,
492.5nm - from peaks at 22.6 cm and 43.8 cm,
587nm - from peaks at 29 cm and 74.3 cm,
610nm - from peaks at 30.8 cm and 90.7 cm,
656nm - from peaks at 34.8 cm and 220.9 cm,
We mark those peaks on the diagram:



And notice that there are peaks which are generated only by H (410nm), He (492.5nm), Li (610nm), so those are present in the object. As out of many spectral lines of Be we found only the ones which coincide with other elements, we conclude that Be is not present in the sample.
Answer: H, He, Li