General information

The exercise sessions will be held as blackboard sessions, where the participants will present their solutions to the group. As such, the problems should be set up and solved before the session. The focus of the exercises lies on analyzing and discussing the task at hand together with the group: thus, a perfect solution is not required to be awarded points. A point will be awarded for each question, and a person will be chosen to present their solution from the pool.

Exercise 1.
Stellarator to tokamak comparison

(a) What is arguably the main difference between tokamaks and stellarators?

(b) Outline the implications of the difference on the reactor design and operation, such as the advantages and disadvantages of stellarators compared to tokamaks.

(c) Can you find out estimated major and minor radii and magnetic field strengths of reactor scale stellarators? Compare to the values estimated for reactor scale tokamaks.

Solution 1.

(a) Confinement of the plasma requires formation of nested magnetic flux surfaces, which support the thermal plasma pressure gradient according to $\mathbf{J} \times \mathbf{B} = \nabla p$

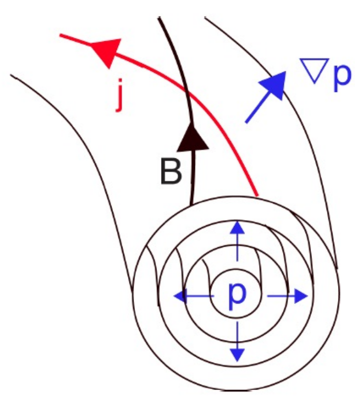


Figure 1: Nested flux surfaces are obtained by imposing a rotational transform to the magnetic field.

There are three ways of producing a rotational transform (see L. Spitzer, Phys. Fluids, 1958 and C. Mercier, Nucl. Fusion, 1964):

(1) **Driving a toroidal current:** This is the tokamak.

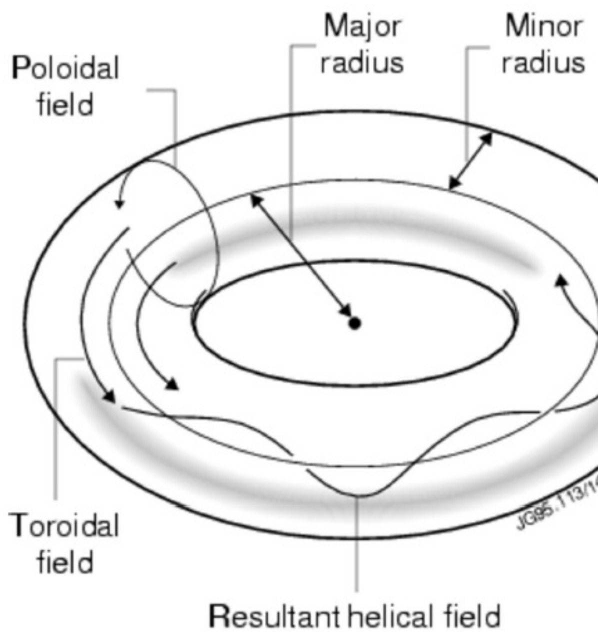


Figure 2: Producing rotational transform using a toroidal current.

(2) **Elongating the flux surfaces and making them rotate poloidally as one moves around the torus:** This is a standard helical system

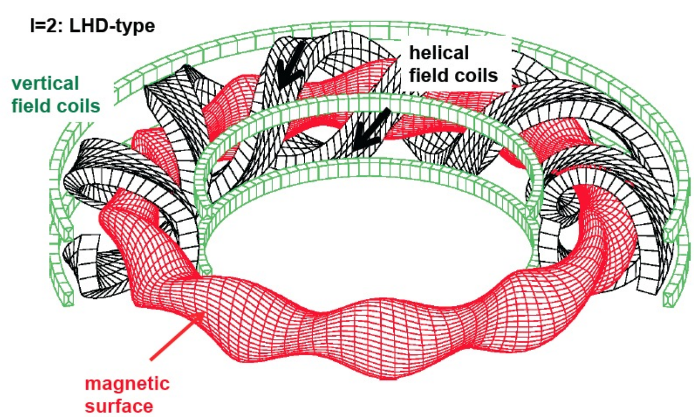


Figure 3: Nesting the flux surfaces by producing rotational elongations in the toroidal direction.

(3) **Making the magnetic axis non-planar:** This was used in the first "figure-8" stellarator of L. Spitzer.

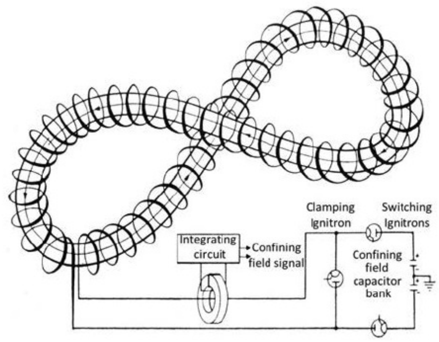


FIG. 8.3. Bending of a torus into "figure-8" geometry leading to rotational transform [8.11]. Reprinted from Ref. [8.11]. Copyright (2011), American Institute of Physics.

Figure 4: Nesting the flux surfaces by producing rotational elongations in the toroidal direction.

- The first method, tokamaks, has the benefit of being axisymmetric, but it requires current drive.
- The latter two methods (stellarators) are inherently 3D, steady-state, and do not have plasma current driven instabilities.
- The existence of flux surfaces is guaranteed in the axisymmetric concept, whereas one needs to be careful to avoid large magnetic islands and stochastic regions in the 3D systems.
- In tokamaks, the rotational transform decreases towards the edge (in baseline, inductive scenarios), while in stellarators it increases towards the edge (exercise 3).
- The absence of plasma current and, therefore, the absence of current driven MHD instabilities in the stellarator concept brings great advantages for plasma stability. In tokamaks, the toroidal current causes kink modes, sawteeth, and resistive and neoclassical tearing modes, which all limit the plasma performance and may cause disruptions, which can be detrimental for the plasma facing components in reactors.
- In stellarators, the plasma density is not limited by Greenwald limit but rather by radiative instabilities in the core plasma. Therefore, stellarators often operate at higher densities than tokamaks.

(b) Therefore, the advantages of stellarators compared to tokamaks are:

- Inherently steady-state – no need for current drive – recirculating power much lower than in tokamaks
- High density operation - the fusion power scales as n^2
- Absence of current driven instabilities and disruptions $\rightarrow$ higher performance, reactor component integrity – major disruptions are practically unacceptable in reactor scale devices

- No need for active control, such as positional feedback. If the active control fails, the system does not run into a disruption.

The main disadvantages of Stellarators compared to Tokamaks (all of which are occurring due to the 3D geometry) are:

- Complexity of the 3D geometry → challenging to optimize the confinement
- Stellarators have wider spectrum of trapped particles, which increase the neoclassical particle losses, especially in the low collisionality regimes (typical for hot central fusion plasma) → poor confinement and lower performance
- Helical windings lead to high aspect ratio devices. Large major radius due to need for support against the magnetic forces + neutron shielding of the superconductive magnetic coils. The equilibrium β is low for low aspect ratio, low rotational transform devices → poor utilization of the magnetic field → poor fusion power for a given magnetic field and volume
- The magnetic forces between the coils require supporting structure, while the space between the plasma and the coils is very limited. → increasing the radius helps, but the aspect ratio increase further and β is lowered more

The operational boundaries in stellarators and helical systems are determined by the available heating power, P_{in}, and confinement properties, τ_E, rather than by disruptive stability or density limits, as in tokamaks.

(c) For HELIAS 5-B, R = 22 m, a = 1.8 m and B = 5.9 T (on axis). For DEMO, R = 7.5 m, a = 2.9 m and B = 5.6 T (on axis).

Exercise 2.
Stellarator concepts Give a broad overview of the various stellarator types and their characteristics. What are the implications on reactor design and operation on a general level? Can you name actual experimental devices that are based on each concept?

Solution 2.

(1) **Classical stellarators**

- Magnetic field produced by toroidal and helical coils. Current flows into opposite directions in the helical coils.
- In the classical stellarator concept, the poloidal and toroidal fields can be varied independently → externally generated rotational transform can be varied in a single device
- Interaction forces between the toroidal and helical coils can be large → massive supporting structure needed → the device becomes packed with coils + supporting structure → hard to access the device
- Confinement comparable to equal sized tokamaks has been achieved in optimized configuration.

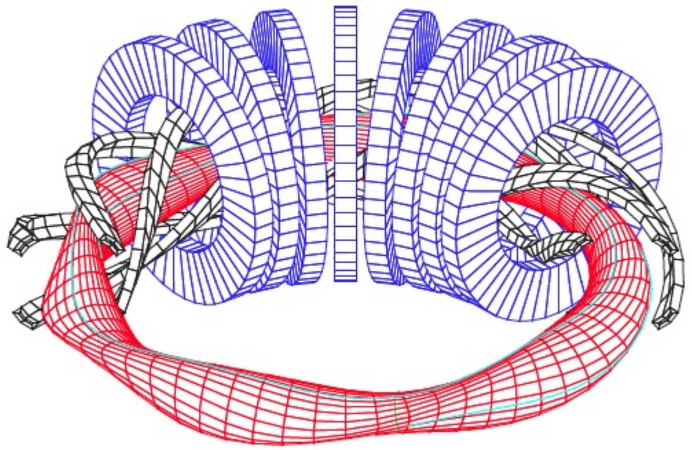


FIG. 8.4. Schematic of a classical $l = 2$ stellarator with toroidal field coils (blue) and helical windings (black). A magnetic surface is shown in red. Courtesy of J. Kisslinger, IPP, Garching.

Figure 5: A schematic of a classical stellarator.

(2) **Heliotron/Torsatron**

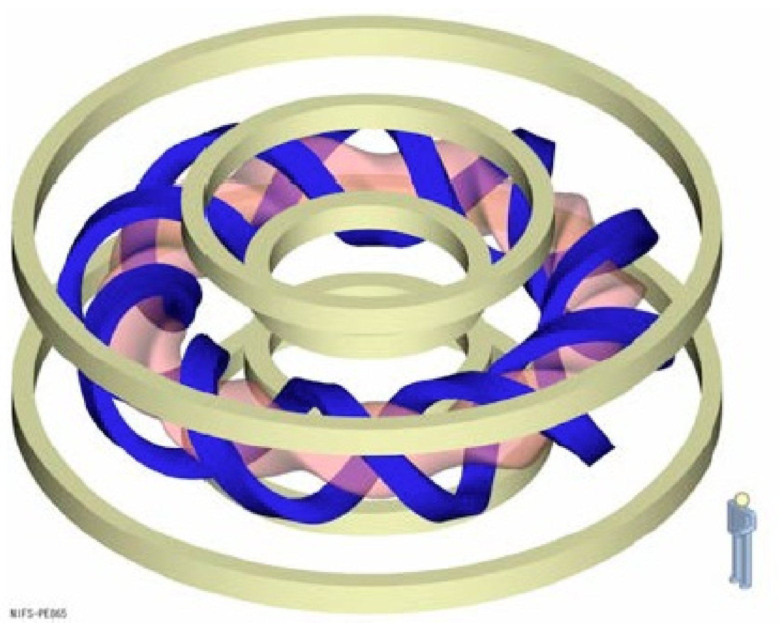


Figure 6: A schematic of a heliotron.

- The poloidal and toroidal magnetic fields are generated by helical windings carrying unidirectional current.
- Toroidal field coils can also be added to add flexibility to adjusting the rotation

transform. On the other hand, the magnetic forces go up if this is done $\rightarrow$ more support structure needed.

- The LHD (heliotron) is the second-largest helical device at present and the first one with superconducting coils (long pulse operation possible $\sim$ 1h)

(3) **Heliac**

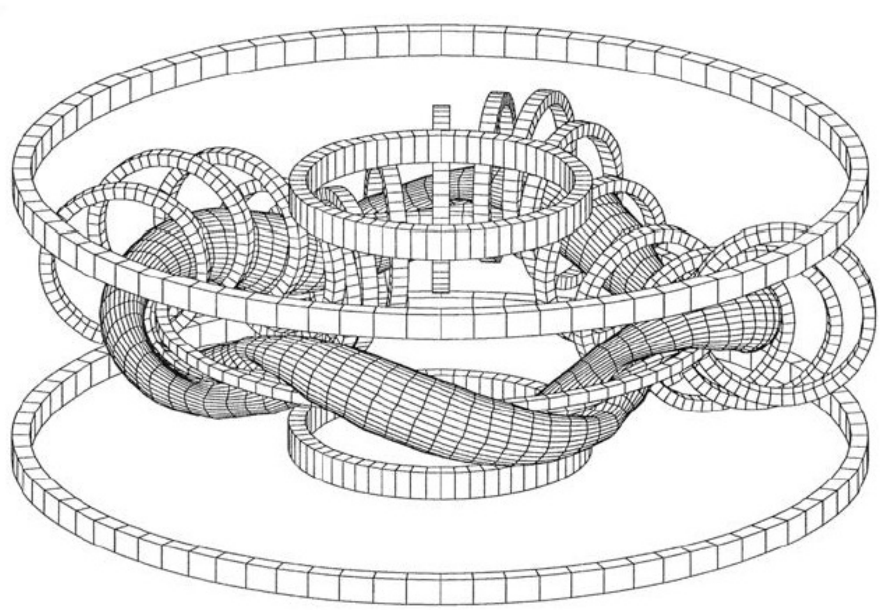


FIG. 8.6. Typical heliac coil set together with vertical field coils and a magnetic surface.

Figure 7: A schematic of a heliac.

- Rotational transform is generated by a helical magnetic axis.
- The heliac offers a high degree of flexibility such that important equilibrium, stability and transport issues can be addressed.
- However, it has an extremely complicated geometry and to reproduce on a reactor scale is likely to be an engineering nightmare.
- The first stellarator ("figure-8 of Spitzer") was a heliac.

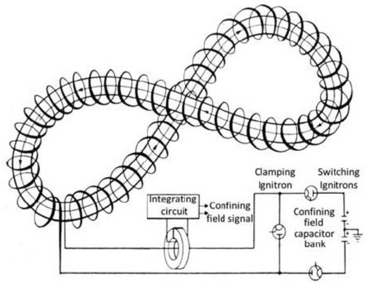


FIG. 8.3. Bending of a torus into "figure-8" geometry leading to rotational transform [8.11]. Reprinted from Ref. [8.11]. Copyright (2011), American Institute of Physics.

Figure 8: A schematic of the "Figure-8".

(4) **Modular stellarators**

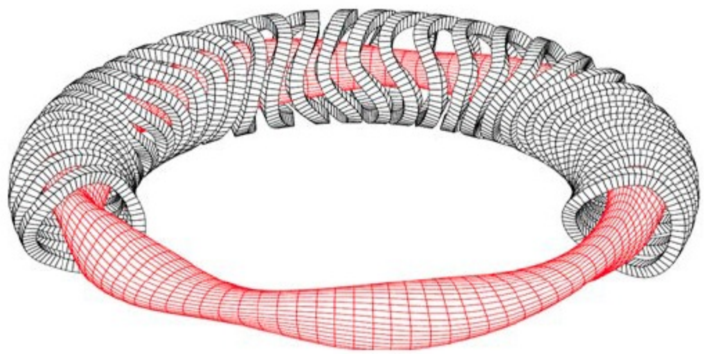
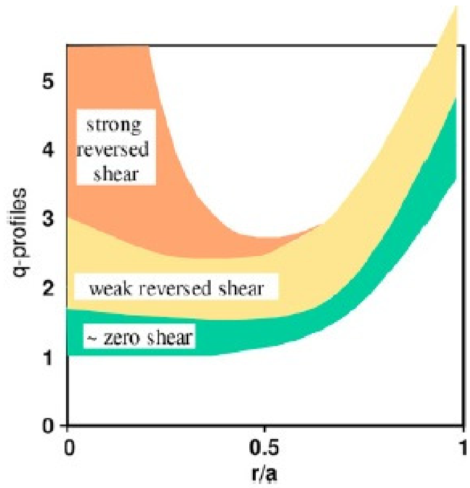


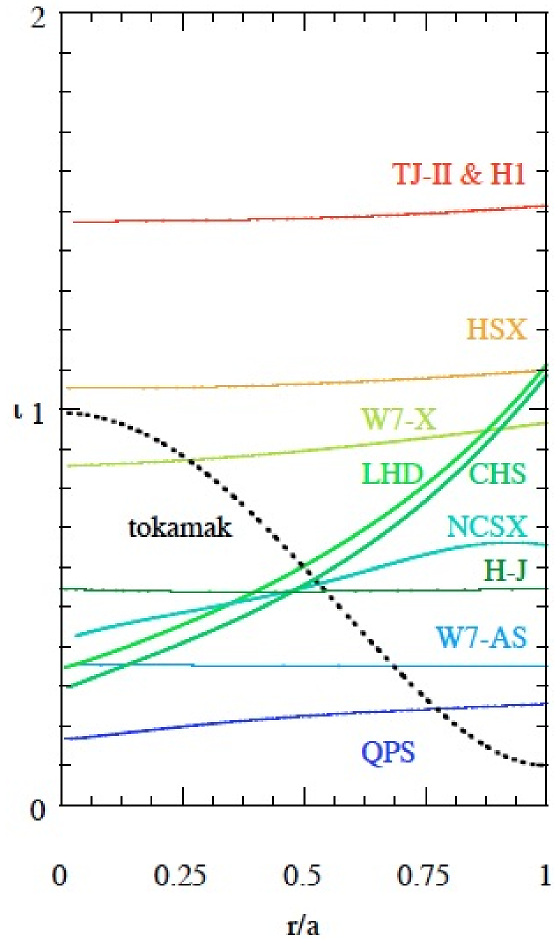
FIG. 8.8. Scheme of a classical l = 2 stellarator with five toroidal field periods with a modular coil system [8.33]. Reprinted from Ref. [8.33]. Copyright (2011), IOP Publishing Ltd.

Figure 9: A schematic of a modular stellarator.

- Modular stellarators do not separate between toroidal and poloidal field coils
- One can, in theory, reverse-engineer the problem and produce exactly the kind of magnetic field that is thought to be sufficient to the provide the needed confinement.
- Complicated field structures result.
- Since there is no toroidal current in the coils, vertical field coils are not needed $\rightarrow$ toroidally closed windings are not needed $\rightarrow$ confinement provided solely by the modular coils $\rightarrow$ this is a great advantage for maintainability of the reactor.
- Modular coil system produces a fixed configuration with fixed rotational transform $\rightarrow$ additional coils are needed if experimental flexibility is required.
- The currently largest stellarator, Wendelstein 7-X in Germany, is a **helical advanced stellarator** or **helias** for short, which is a type of modular stellarator with an optimized coil set designed to achieve good single-particle confinement, especially of trapped particles.



(a) Schematic q-profiles in tokamak plasmas with zero, low, and strong magnetic shear, as a function of the normalized minor radius of the device [1]. $r/a = 0$ stands for the magnetic axis and $r/a = 1$ for the edge of the plasma.



(b) Rotational transform in a baseline tokamak plasma, and in the various stellarator concepts [2].

Exercise 3.
Rotational transform and q-profiles in tokamaks and stellarators

(a) In typical tokamak baseline, inductive plasmas, the q-profile increases (rotational transform decreases) towards the edge, as is seen in the figures 1 and 2. The safety factor profile as a function of the radius r is approximately

$$q_s \propto \frac{r^2 B_T}{R_0 I_{MA}} \propto \frac{B_T}{R_0 J},$$

where R_0 is the major radius, B_T is the strength of the toroidal magnetic field at the magnetic axis, I_{MA} is the current in MA, and J is the current density. Sketch the current and current density profile required to obtain the regular zero-shear q_s profile of figure 1(a). If the resistivity of the plasma scales approximately as $\eta \propto T_e^{-3/2}$ (Spitzer resistivity), which part of the tokamak plasma is the most conductive?

(b) For stellarators, the rotational transform $\iota = 1/q_s$ is a common concept. The stellarator rotational transform generally increases (safety factor decreases) towards the edge, as seen in figure 1(b) for a variety of experimental devices, which is opposite to what we see for tokamaks. Explain this observation based on the fundamental differences between stellarators and tokamaks.

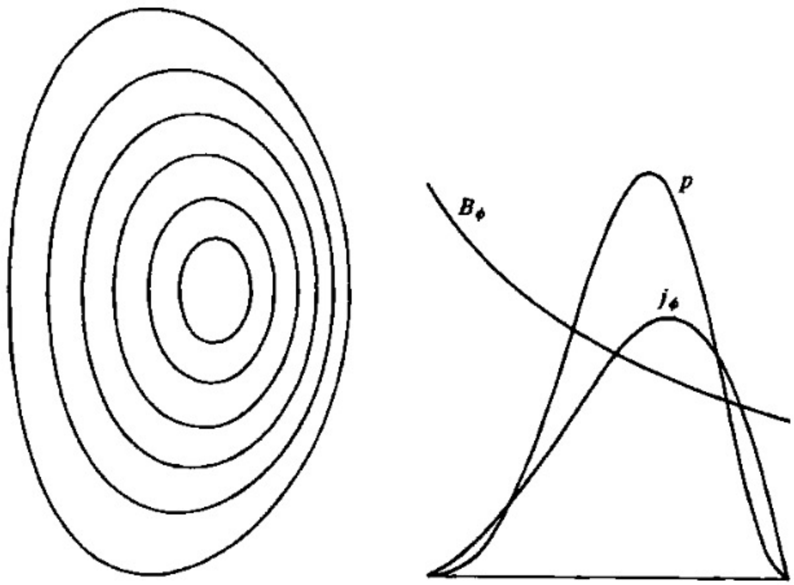


Figure 11: Schematic tokamak equilibrium surfaces and profiles of toroidal magnetic field, pressure and toroidal current.

Solution 3.

(a) Due to the temperature dependence of the plasma resistivity, the ohmically driven plasma current peaks at the magnetic axis (in the case of inductively driven plasma current that is fully diffused). The safety factor at radius r is given by

$$q \approx \frac{rB_T}{RB_\theta} = \frac{2\pi r^2 B_T}{\mu_0 R I_{MA}} = \frac{r^2 B_T}{R\langle J\rangle \pi r^2} = \frac{B_T}{R\pi\langle J\rangle}.$$

Since $\langle J\rangle$ is a decreasing function of the minor radius, the q-profile increases towards the edge.

(b) In stellarators, the rotational transform is driven by the external coil system, which produces most of the magnetic field in the system. The magnetic field in stellarators is very close to the vacuum field.

There is not any significant radially varying plasma current modifying the rotational transform. In stellarator concepts there are, however, confinement related Pfirsch-Schlüter currents and Bootstrap currents, which can influence the rotational transform towards the edge.

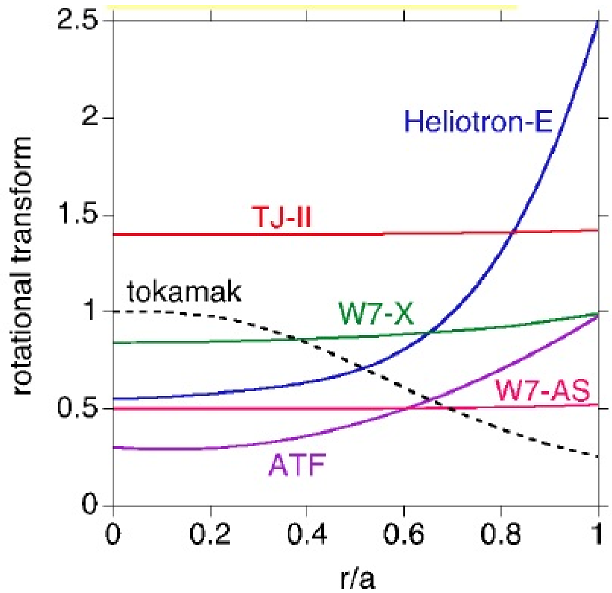


Figure 12: Rotational transform in a baseline tokamak plasma, and in the various stellarator concepts.

Exercise 4.
Neoclassical transport in stellarators and tokamaks
The transport in toroidal magnetic confinement devices is driven by classical collisional, neoclassical collisional, and turbulent transport. In the earlier exercises, the classical and neoclassical collisional transport coefficients in tokamaks were investigated, with the conclusion that the experimentally observed (turbulent) transport exceeds these classical and neoclassical levels by more than 2 orders of magnitude. On the other hand, in stellarators the neoclassical contribution, originating from the inhomogeneity of the magnetic field, can be the dominating cross-field transport mechanism, and, therefore, the principal factor limiting energy confinement.

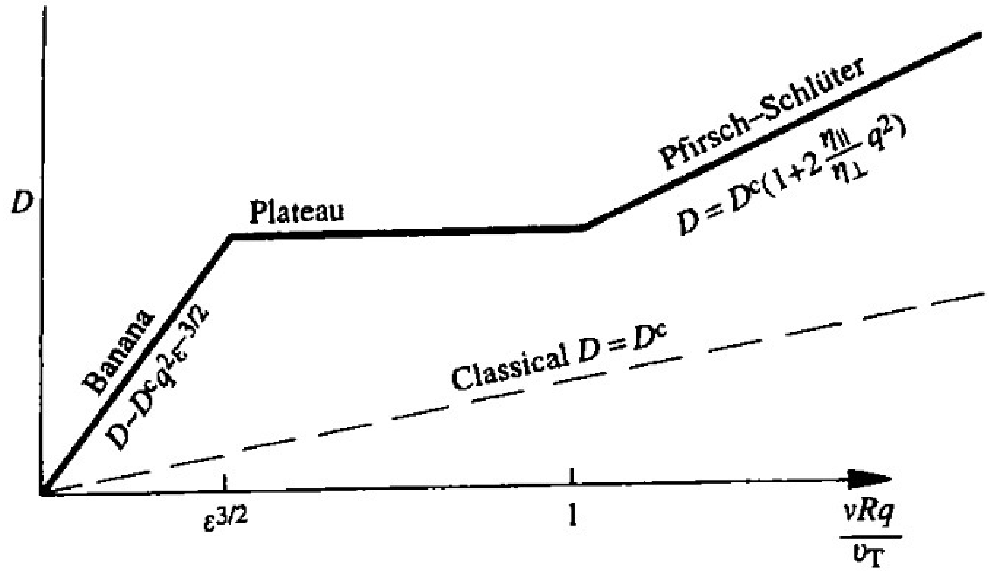


Figure 13: Neoclassical transport regimes as a function of plasma collisionality in axisymmetric toroidal magnetic confinement system [3].

(a) Figure 2 shows the neoclassical transport as a function of the plasma collisionality (collision frequency normalized to banana orbit bounce frequency) in an axisymmetric toroidal magnetic confinement system (such as an ideal tokamak). The transport can be roughly divided into three regimes: Banana, Plateau, and Pfirsch-Schlüter. Explain what these regimes mean and why they exist? Chapters 4.6 – 4.7 in John Wesson's Tokamaks can come in handy, or alternatively Chapter 3.3 in the PhD thesis "*Turbulent and neoclassical transport in tokamak plasmas*" by Istvan Pusztai [4].

(b) Figure 3 illustrates the neoclassical transport in a stellarator as a function of collisionality. At low collisionalities, the neoclassical transport is greatly enhanced in stellarators. Explain the reasons for this, and how this fact fits together with the ideal temperatures for fusion. How does radial electric field change the situation and why?

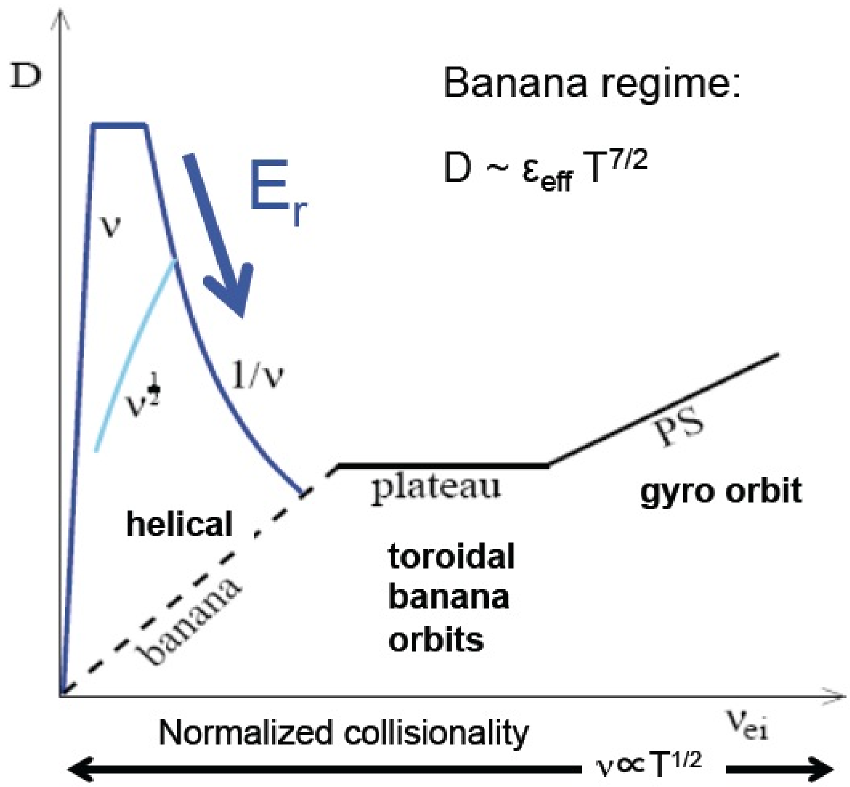
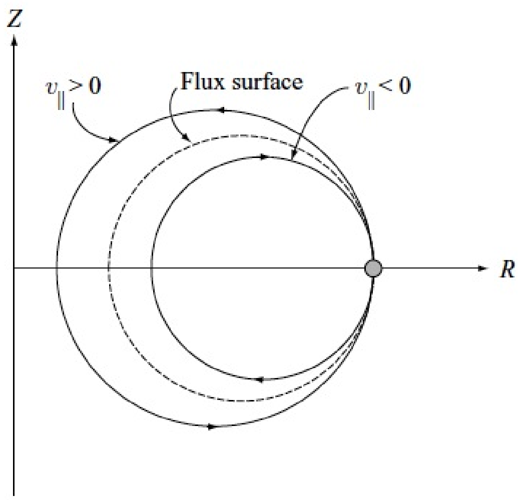


Figure 14: Identical to figure 2, with the observed diffusion in stellarators included. [5]

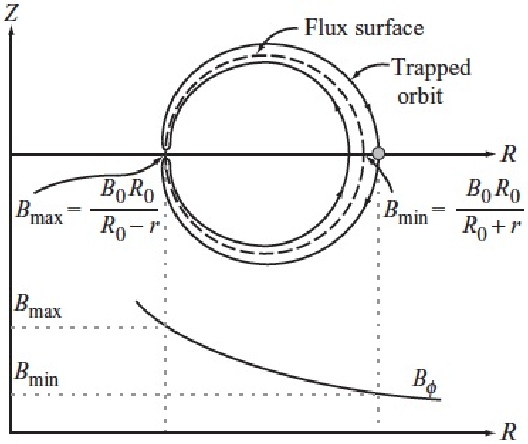
Solution 4.

(a) In the *perfectly collisionless situation* ($\nu = 0$), the particles with $v_\parallel < \sqrt{r/R}v_\perp$ are trapped on banana-orbits. In axisymmetric conditions, *these orbits are confined.*

If *collisionality is non-zero and smaller than the banana bounce frequency* ($\nu < (\epsilon^{3/2}v_T)/(qR)$), the collisional transport of these banana trapped particles (fraction of $\sim \sqrt{\epsilon}$) dominates the overall collisional transport, and, hence, the name "banana regime". Collisions, by interrupting and de-trapping particles at different points of their banana orbits can therefore cause a radial displacement of particles with a step length comparable to banana orbit width and in the long term they lead to a diffusive transport.



(a) Passing particles orbit



(b) Trapped particles orbits [J. Freidberg, Plasma physics and fusion energy]

In the high collisionality regime, the collisionality is too high for the banana-trapped particles to complete their orbits. Therefore, the step-size for the collisional cross-field transport becomes smaller. In this regime, the collisional cross-field transport is dominated by passing particles. This is called the Pfirsch-Schlüter regime.

The transition region between the Banana and Pfirsch-Schlüter regimes is called the plateau regime, where the collisional cross-field diffusion is observed to be independent of the collisionality of the plasma. Here the transport is caused by a resonance between collisions and slowly passing particles.

The clear distinction between these regimes is a consequence of the large aspect ratio approximation. In practice, in tokamaks, **the domains merge smoothly**.

(b) As we observed, the low collisionality region of the neo-classical collisional transport was dominated by the trapped particles. The collisionality of the plasma was low enough for these trapped particles to complete full bounce orbits. Due to the 3D geometry of the stellarators, there are a wider variety of trapped particles in stellarators than in tokamaks. The 3D features can therefore greatly enhance the particle transport in low collisionality, which is exactly the regime of the fusion relevant core plasma. Therefore, the optimization of the 3D fields in stellarators is of fundamental importance if an igniting stellarator concept is about to be designed!

To be more precise, additional ripple, the helical ripple, arises from the 3D coil structure in stellarators. These ripples are localized with an extension of the order of the coil spacings (in tokamaks the finite number of coils is only a weak perturbation of axial symmetry). For particles trapped in such a local ripple the vertical drift is not averaged as it is for passing or banana particles and they are rapidly lost unless they are de-trapped by collisions. Therefore the diffusivity strongly increases with decreasing collisionality ($1/\nu$-regime). Since a reactor has to operate at high temperature, i.e. low collisionality, the stellarator specific $1/\nu$-regime is detrimental unless transport can be reduced by other means.

The radial electric field and the resulting ExB-drift can greatly reduce the cross-field transport of the trapped particles via de-trapping.

[1] Progress in the ITER physics basis – Chapter 1, Nucl. Fusion, (2007), 47, S1
[2] E. Ascasíbar, et al., 17th International Toki Conference on Physics of Flows and Turbulence in Plasmas and 16th International Stellarator/Heliotron Workshop 2007, Invited Talk 01.
[3] J. Wesson, Tokamaks, 3rd edition, p.168
[4] https://research.chalmers.se/publication/147852/file/147852_Fulltext.pdf
[5] Lecture slides, Stellarators, p.26