

6A Tehtävä J1

with(VectorCalculus)

[&x, *, ^+, ^-, ^:, ^:, < >, <|>, About, AddCoordinates, ArcLength, BasisFormat, Binormal, ConvertVector, CrossProduct, Curl, Curvature, D, Del, DirectionalDiff, Divergence, DotProduct, Flux, GetCoordinateParameters, GetCoordinates, GetNames, GetPVDescription, GetRootPoint, GetSpace, Gradient, Hessian, IsPositionVector, IsRootedVector, IsVectorField, Jacobian, Laplacian, LineInt, MapToBasis, ∇, Norm, Normalize, PathInt, PlotPositionVector, PlotVector, PositionVector, PrincipalNormal, RadiusOfCurvature, RootedVector, ScalarPotential, SetCoordinateParameters, SetCoordinates, SpaceCurve, SurfaceInt, TNBFrame, TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorPotential, VectorSpace, Wronskian, diff, eval, evalVF, int, limit, series] (1)

with(LinearAlgebra)

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip] (2)

$x := ([1, 2, 3, 4, 6, 7])$

$x := [1, 2, 3, 4, 6, 7]$ (3)

$y := \text{Transpose}(\text{Matrix}([0.11, 1.62, 4.07, 7.55, 17.63, 24.20]))$

$y := \begin{bmatrix} 0.11 \\ 1.62 \\ 4.07 \\ 7.55 \\ 17.63 \\ 24.20 \end{bmatrix}$ (4)

$A := \text{Transpose}(\text{Matrix}([[1, 1, 1, 1, 1, 1], x.\sim x]))$

$$A := \begin{bmatrix} 1 & 1 \\ 1 & 4 \\ 1 & 9 \\ 1 & 16 \\ 1 & 36 \\ 1 & 49 \end{bmatrix} \quad (5)$$

$$pq := \text{Transpose}(\text{Matrix}([p, q]))$$

$$pq := \begin{bmatrix} p \\ q \end{bmatrix} \quad (6)$$

Saadaan yhtälö vektorille $[p, q]$:

$$A \cdot pq = y$$

$$\begin{bmatrix} p + q \\ p + 4q \\ p + 9q \\ p + 16q \\ p + 36q \\ p + 49q \end{bmatrix} = \begin{bmatrix} 0.11 \\ 1.62 \\ 4.07 \\ 7.55 \\ 17.63 \\ 24.20 \end{bmatrix} \quad (7)$$

(8)

Ratkaistaan yhtälöryhmä. Kerrotaan ensin molemmat puolet A:n transpoosilla:

$$\text{Transpose}(A) \cdot A \cdot pq$$

$$\begin{bmatrix} 6p + 115q \\ 115p + 4051q \end{bmatrix} \quad (9)$$

$$\text{Transpose}(A) \cdot y$$

$$\begin{bmatrix} 55.1800000000000 \\ 1984.50000000000 \end{bmatrix} \quad (10)$$

Nyt on enää kaksi yhtälöä, jotka on helppo ratkaista:

$$\begin{aligned} & \text{solve}(\{6 \cdot p + 115 \cdot q = 55.18, 115 \cdot p + 4051 \cdot q = 1984.5\}, \{p, q\}) \\ & \{p = -0.4226441657, q = 0.5018770869\} \end{aligned} \quad (11)$$

Eli saatiin paraabeli:

$$\begin{aligned} f & := -0.4226441657 + 0.5018770869 \cdot a^2 \\ f & := -0.4226441657 + 0.5018770869 a^2 \end{aligned} \quad (12)$$

Kun $a = 5$:

$$\begin{aligned} & \text{eval}(f, a = 5) \\ & 12.12428300 \end{aligned} \quad (13)$$