**6A Tehtävä J2**

$with(LinearAlgebra)$

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column, ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix, CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy, CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm, FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic, GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix, HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition, LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply, NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis, SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip] **(1)**

$with(VectorCalculus)$

[&x, `*`, `+`, `-`, `.`, <,>, <|>, About, AddCoordinates, ArcLength, BasisFormat, Binormal, ConvertVector, CrossProduct, Curl, Curvature, D, Del, DirectionalDiff, Divergence, DotProduct, Flux, GetCoordinateParameters, GetCoordinates, GetNames, GetPVDescription, GetRootPoint, GetSpace, Gradient, Hessian, IsPositionVector, IsRootedVector, IsVectorField, Jacobian, Laplacian, LineInt, MapToBasis, ∇, Norm, Normalize, PathInt, PlotPositionVector, PlotVector, PositionVector, PrincipalNormal, RadiusOfCurvature, RootedVector, ScalarPotential, SetCoordinateParameters, SetCoordinates, SpaceCurve, SurfaceInt, TNBFrame, TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorPotential, VectorSpace, Wronskian, diff, eval, evalVF, int, limit, series] **(2)**

Käytetään Newtonin menetelmää, eli halutaan löytää pintojen

$f\_1 := x^4 + y^4 - 2\cdot x\cdot y^5$

$$f\_1 := -2\,x\,y^5 + x^4 + y^4 \qquad (3)$$

ja

$f\_2 := x^6 + x^2 + y^4 - 4$

$$f\_2 := x^6 + y^4 + x^2 - 4 \qquad (4)$$

leikkauspiste. Valitaan alkuarvoksi (1, 1).

$J := Jacobian([f\_1, f\_2], [x, y])$

$$J := \begin{bmatrix} -2\,y^5 + 4\,x^3 & -10\,x\,y^4 + 4\,y^3 \\ 6\,x^5 + 2\,x & 4\,y^3 \end{bmatrix} \qquad (5)$$

$f := Vector([f\_1, f\_2])$

$$f := \left(-2\,x\,y^5 + x^4 + y^4\right)e_x + \left(x^6 + y^4 + x^2 - 4\right)e_y \qquad (6)$$

Iteroidaan:

$$x\_1 := Vector([1, 1]) - MatrixInverse(eval(J, \{x = 1, y = 1\})) \cdot eval(f, \{x = 1, y = 1\})$$

$$x\_1 := \left(\frac{31}{28}\right)e_x + \left(\frac{29}{28}\right)e_y \tag{7}$$

$$x\_2 := evalf(x\_1 - MatrixInverse(eval(J, \{x = x\_1[1], y = x\_1[2]\})) \cdot eval(f, \{x = x\_1[1], y = x\_1[2]\}))$$

$$x\_2 := (1.090811962)e_x + (1.031435721)e_y \tag{8}$$

$$x\_3 := evalf(x\_2 - MatrixInverse(eval(J, \{x = x\_2[1], y = x\_2[2]\})) \cdot eval(f, \{x = x\_2[1], y = x\_2[2]\}))$$

$$x\_3 := (1.09029545139477)e_x + (1.03135254429896)e_y \tag{9}$$

$$x\_4 := evalf(x\_3 - MatrixInverse(eval(J, \{x = x\_3[1], y = x\_3[2]\})) \cdot eval(f, \{x = x\_3[1], y = x\_3[2]\}))$$

$$x\_4 := (1.09029493676617)e_x + (1.03135252270956)e_y \tag{10}$$

$$x\_5 := evalf(x\_4 - MatrixInverse(eval(J, \{x = x\_4[1], y = x\_4[2]\})) \cdot eval(f, \{x = x\_4[1], y = x\_4[2]\}))$$

$$x\_5 := (1.09029493676564)e_x + (1.03135252270959)e_y \tag{11}$$

Yksi yhtälöryhmän ratkaisu on menetelmän mukaan siis noin (1.09029, 1.03135).