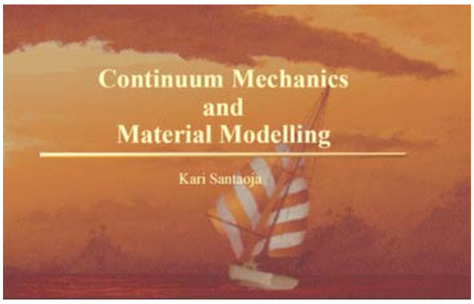
Continuum Mechanics and Material Modelling 2023

MEC-E8002 P (5 cr)

Letter to the students; February 10, 2023

The following comments are based on the observations I have made during the years I have lectured the predecessors of this course and the calculation hour we had on Fridays.



Please, avoid using notations which are not tensor notations but refer to other fields of mathematics. Notation $[\cdot]$ stands for matrix not for a tensor. Notation 1 is not a second-order identity tensor but it is a scalar "one".

Notation

$$\frac{\delta}{\delta x} \tag{1}$$

is not a partial derivative operator. The partial derivative operator reads

$$\frac{\partial}{\partial x}\,. \tag{2}$$

Boldface letters are difficult to write by hand. Therefore, when writing by hand, I use the following notations:

$$\text{second-order tensor} \quad \overset{\leftrightarrow}{1} \qquad \text{and} \qquad \text{fourth-order tensor} \quad \overset{\leftrightarrow}{\overset{\leftrightarrow}{I}}\,. \tag{3}$$

The idea of the tensor notations is that the mathematical derivation is carried out without substitution of the values 1,2,3 for the indices. Thus, do not write

$$\vec{\nabla}(\vec{x}) = \vec{i}_1 \frac{\partial}{\partial x_1} + \vec{i}_2 \frac{\partial}{\partial x_2} + \vec{i}_3 \frac{\partial}{\partial x_3}\,. \tag{4}$$

Values for the indices are sometimes substituted when the obtained results are interpreted. See the example below.

Please, remember that Einstein summation convention does not utilise summation symbol Σ. Matrices do not belong to tensor algebra.

There are some typos in my lecture notes. Usually they are fully harmless but some of them require corrections. I have prepare an errata document. It is available in MyCourses.

This example gives a hint how to avoid of giving numerical values for the indices.

Calculate the following:

$$N_s\, \sigma_{sj}\, \vec{i}_j - \lambda N_s\, \vec{i}_s = \vec{0} = 0_m\, \vec{i}_m \tag{1}$$

Using the Einstein summation convention in 2D case (I was lazy to write in 3D) the following is arrived at:

$$N_1\, \sigma_{11}\, \vec{i}_1 + N_1\, \sigma_{12}\, \vec{i}_2 + N_2\, \sigma_{21}\, \vec{i}_1 + N_2\, \sigma_{22}\, \vec{i}_2 - \lambda N_1\, \vec{i}_1 - \lambda N_2\, \vec{i}_2 = 0_1\, \vec{i}_1 + 0_2\, \vec{i}_2\,. \tag{2}$$

Let us collate the multipliers of the base vectors. It yields

$$(N_1\, \sigma_{11} + N_2\, \sigma_{21} - \lambda N_1)\, \vec{i}_1 + (N_1\, \sigma_{12} + N_2\, \sigma_{22} - \lambda N_1)\, \vec{i}_2 = 0_1\, \vec{i}_1 + 0_2\, \vec{i}_2\,. \tag{3}$$

Since the base vectors $\vec{i}_1$ and $\vec{i}_2$ are mutually independent, the Equation (3) gives two equations. They are

$$\begin{aligned} (N_1\, \sigma_{11} + N_2\, \sigma_{21} - \lambda N_1)\, \vec{i}_1 &= 0_1\, \vec{i}_1 \\ (N_1\, \sigma_{12} + N_2\, \sigma_{22} - \lambda N_1)\, \vec{i}_2 &= 0_2\, \vec{i}_2\,. \end{aligned} \tag{4}$$

The information of Equations (4) can be written in the form

$$N_s \, \sigma_{sj} \, \vec{i}_j - \lambda N_j \, \vec{i}_j = \vec{0} = 0_j \, \vec{i}_j \qquad \Rightarrow \qquad (N_s \, \sigma_{sj} - \lambda N_j) \, \vec{i}_j = 0_j \, \vec{i}_j \tag{5}$$

Equation $(5)_2$ yields

$$N_s \, \sigma_{sj} - \lambda N_j = 0_j = 0 \tag{6}$$

I propose you to change the indices already in Equation (1). Then you get Equation $(5)_1$ which leads to Equation (6). The step from Equations (4) to Equation $(5)_1$ is sometimes difficult to find. For me it was easy, since I see the result already in Equation (1).

Read the text in the book starting above Equation (2.35) and ending in the paragraph starting “Sometimes it is ...” on page 44.

Best Regards,

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