Chapter 10

Switching DC Power Supplies

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• One of the most important applications of power electronics

Linear Power Supplies

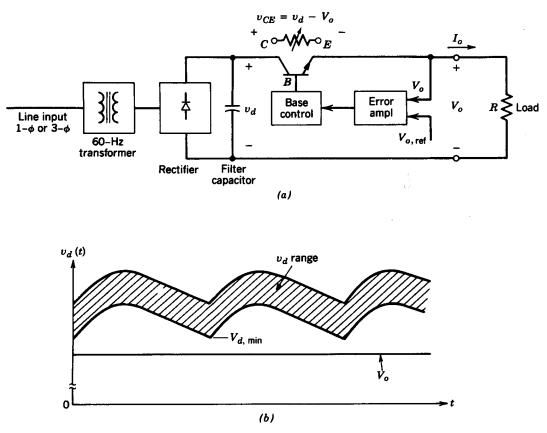


Figure 10-1 Linear power supply: (a) schematic; (b) selection of transformer turns ratio so that $V_{d,\min} > V_o$ by a small margin.

• Very poor efficiency and large weight and size

Switching DC Power Supply: Block Diagram

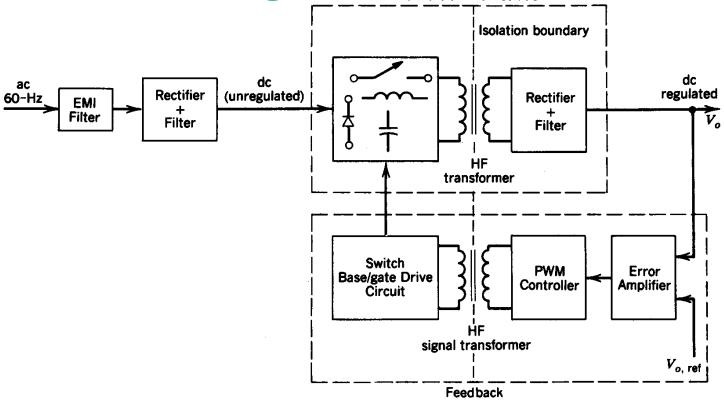


Figure 10-2 Schematic of a switch-mode dc power supply.

• High efficiency and small weight and size

Switching DC Power Supply: Multiple Outputs

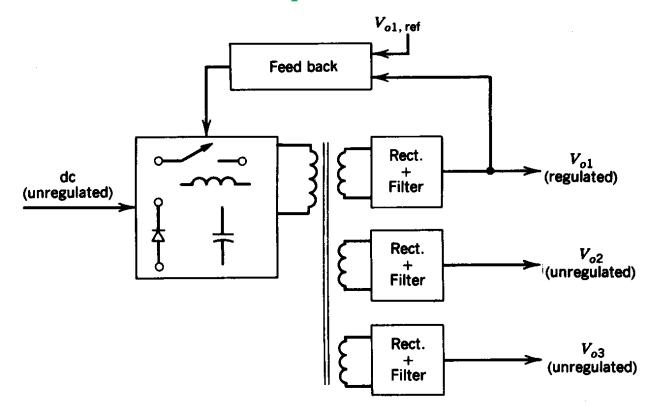


Figure 10-3 Multiple outputs.

• In most applications, several dc voltages are required, possibly electrically isolated from each other

Transformer Analysis

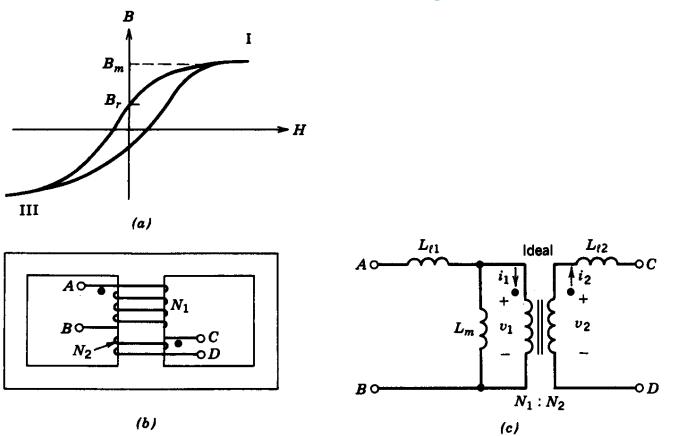


Figure 10-4 Transformer representation: (a) typical B-H loop of transformer core; (b) two-winding transformer; (c) equivalent circuit.

Needed to discuss high-frequency isolated supplies

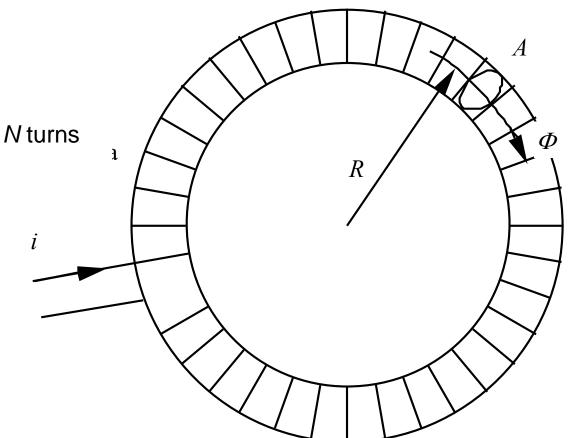
Magnetic circuits

- Toroid with ferrite core
- Ampères law

$$\oint_{s} H \cdot ds = \sum_{j=1}^{n} i_{j}$$

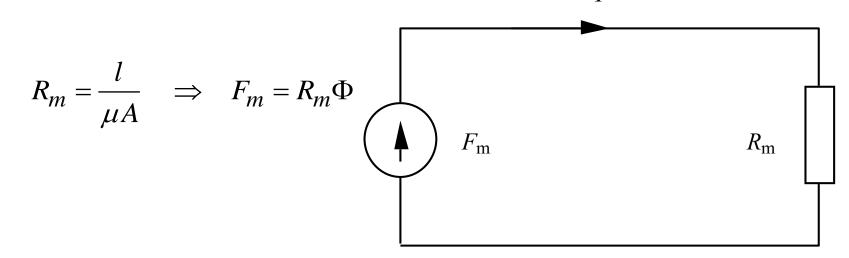
$$Hl = Ni = F_{m}$$

- Magnetomotive force F_m
 - Similar to voltage source



Analogy with circuit theory

- Magnetic field $\Phi = BA = \mu HA = \mu \frac{Ni}{l}A = \Lambda Ni = \Lambda F_m$
- Magnetic conductance, permeance $\Lambda = \frac{\mu A}{l}$
- Often easier to use the inverse, i.e_reluctance



Flux and inductance

- Inductance of a winding is defined with the flux so that $\Psi = N\Phi = Li \implies L = N^2 \Lambda = \frac{N^2}{R}$
- Faraday's law for induction
- Induced voltage tries to prevent the change in the flux
 - When resistive voltage drops are taken into account

$$u = Ri + \frac{d\Psi}{dt} = Ri + L\frac{di}{dt}$$

Equivalent circuit of transformer

- Power systems
 - Power flow in the system
 - Load are reduced either to primary or secondary
- Electric machines
 - Ofthen quantities are reduced to stator side
 - Rotor quantities are often impossible to be measured
 analysis done in stator side
- SMPS
 - Interested both on primary and secondary quantities, reduced values are not used
 - Quantities are not reduced and equivalent circuit is a bit different

Ideal transformer

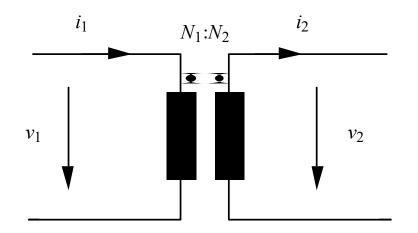
- Independent on load or frequency $\frac{u_1}{u_2} = \frac{N_1}{N_2}$
- No losses

$$p_1 = u_1 i_1 = u_2 i_2 \Longrightarrow N_1 i_1 = N_2 i_2$$

- Load resistor R in primary side $p_2 = u_2 i_2 = \frac{u_2^2}{R} = \frac{u_1^2}{R'} \Rightarrow R' = \left(\frac{u_1}{u_2}\right)^2 R = \left(\frac{N_1}{N_2}\right)^2 R$
- Permeability μ of the core material is infinite
 - Reluctance is zero and transformer is not needing any magnetizing current

Direction of the magnetization (1/2)

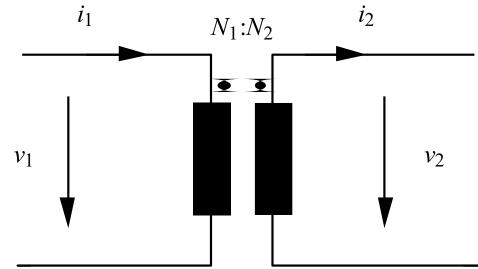
- Winding direction is marked with a dot
- Ampère's and Faraday's laws:
 - Voltage induced by flux changes has same the polarity of the dot
 - When current flows in from the dotted side
 - Fluxes are adding, i.e. currents are magnetiing in the same direction
 - If other winding is open ja the current of the oth increases in the direction of the dot, a voltage induces in the second winding tries to cause a current cancelling the flux created by the first current



Direction of the magnetization (1/2)

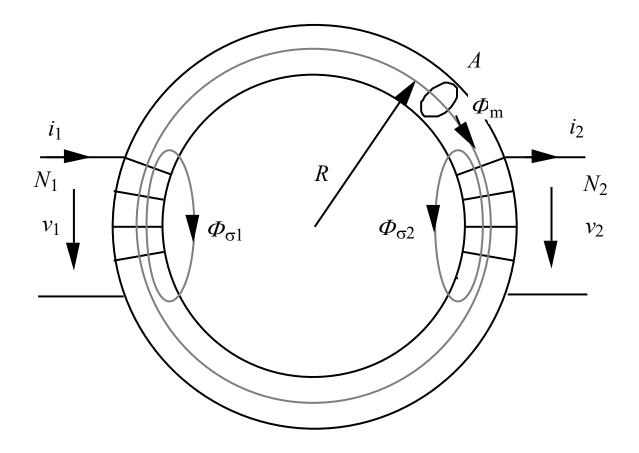
- Positive current below is defined to flow outwards from the transformer
- Rule to remember:

- From dot in and dot out



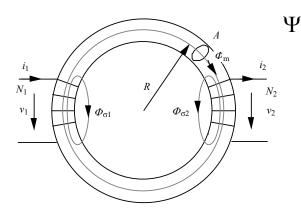
Real transformer

• Two winding toroid is used as an example



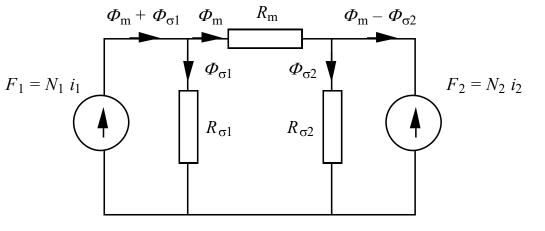
Equivalent circuit

• Primary winding flux



- Magnetizing inductance
- Leakage in primary

$$L_m = \frac{N_1^2}{R_m}, L_{\sigma 1} = \frac{N_1^2}{R_{\sigma 1}}$$



Primary flux and inductances

• Equivalent circuit for primary flux

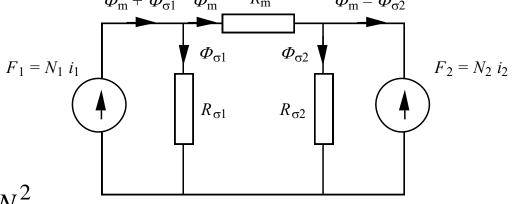
Ideal transformer used

 $\Psi_1 = i_1 \left(L_m + L_{\sigma 1} \right) - L_m \frac{N_2}{N_1} i_2$ $\frac{N_2}{N_1} i_2$ $L_{\sigma 1}$ \dot{l}_1 *l*₂ $N_1:N_2$ • • ψ_1 $L_{\rm m}$ Ideal

Secondary

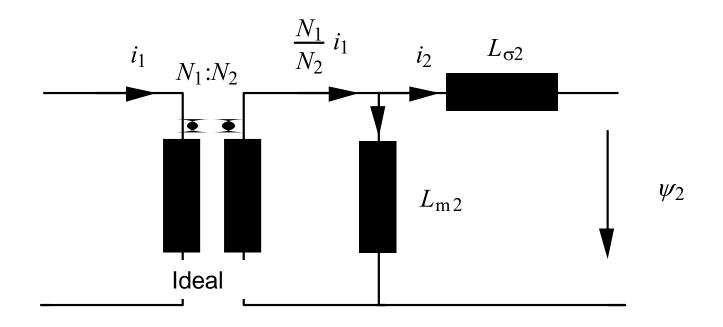
- Magnetizing inductance seen from the secondary
- Leakage inductance

$$L_{m2} = \frac{N_2^2}{R_m} = \frac{N_2^2}{N_1^2} L_m , \ L_{\sigma 2} = \frac{N_2^2}{R_{\sigma 2}}$$



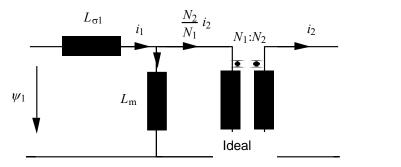
Secondary flux and inductances

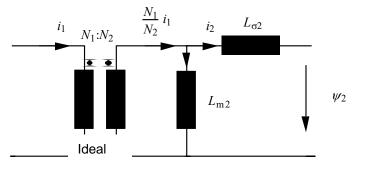
$$\Psi_2 = -i_2 \left(L_{m2} + L_{\sigma 2} \right) - L_{m2} \frac{N_1}{N_2} i_1$$

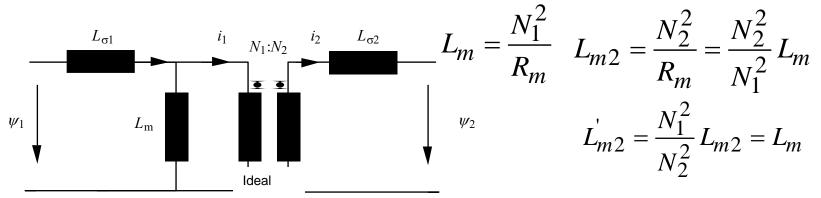


Equivalent circuit of transformer

- Previous equations for primary and secondary are connected
 - Magnetizing inductance in primary as the switch is normally in primary too

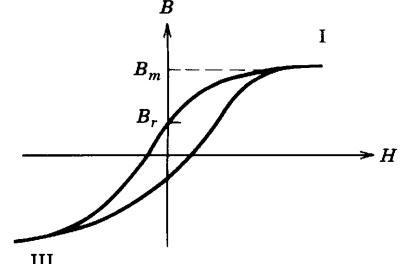






Magnetizing curve

- B_m saturation
- *B*_r remanence flux density
- Depending on the converter topology, core is magnetized
 - 1) in one direction (quadrant 1)
 - 2) two directions (quadrants 1 and 3)



Transformer as part of SMPS

- Magnetization in one quadrant
 - Changing previous dc-dc topologies and adding transformer galvanically isolated version are obtained
 - Flyback (derived from buck-boost)
 - Forward (derived from buck)
- Magnetization in two quadrants
 - Single-phase dc-ac inverter produces high frequency ac (square wave) and it is fed through high frequency transformer and rectified
 - Push-pull
 - Half-bridge
 - Full-bridge

Transformer inductances (1/2)

- *L*_m is magnetizing inductance seen in primary
- Should be as high as possible so that effect of in switches small
- $L_{\sigma 1}$, $L_{\sigma 2}$, leakage inductances
 - Should be small, are adding voltage stresses of switches in hard switching
 - Need to be considered when selecting switches and dimensioning snubber circuits

Transformer inductances (2/2)

- Flyback
- Magnetic circuit acts
 - Isolation
 - and as energy storage (inductance)
- L_m cannot be large
 - Resonant converters
 - Leakage inductances can be used for ZCS/ZVS

PWM to Regulate Output

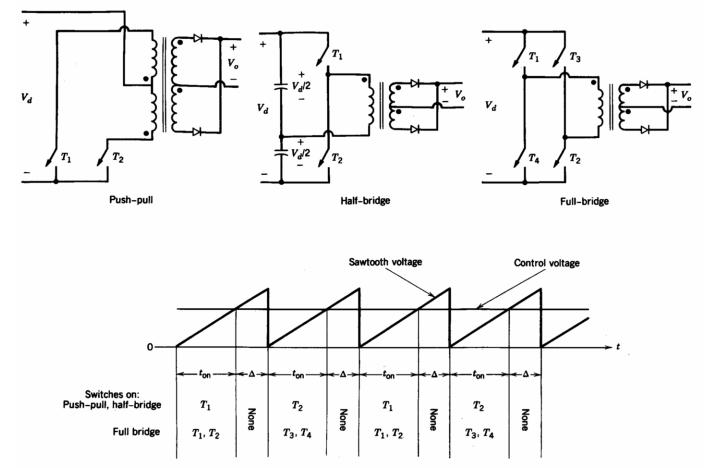
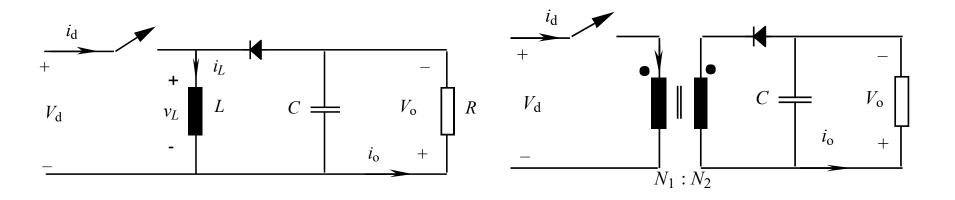


Figure 10-5 PWM Scheme used in dc-dc converters, where the converter output is rectified to produce a dc output.

• Basic principle is the same as discussed in Chapter 8

Flyback derived from buck-boost

- Galvanic isolation
 - Second winding added to buck-boost inductance



Flyback Converter

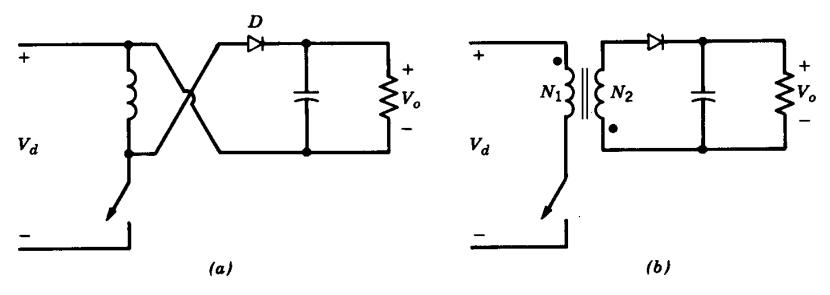


Figure 10-6 Flyback converter.

• Derived from buck-boost; very popular at small power (< 50 W) power levels

Flyback Converter

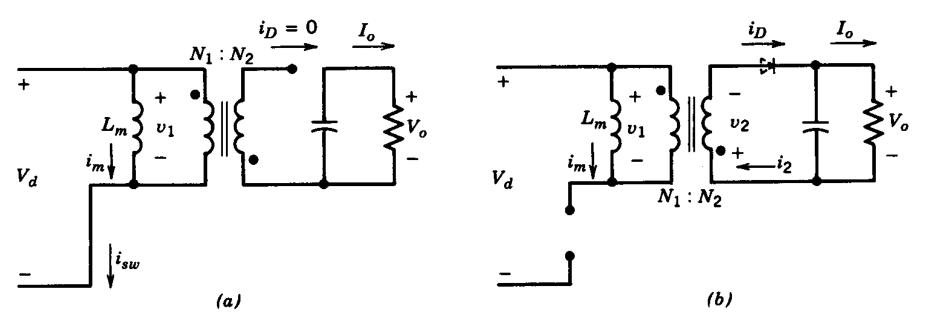
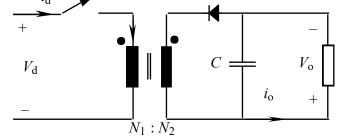


Figure 10-7 Flyback converter circuit states: (a) switch on; (b) switch off.

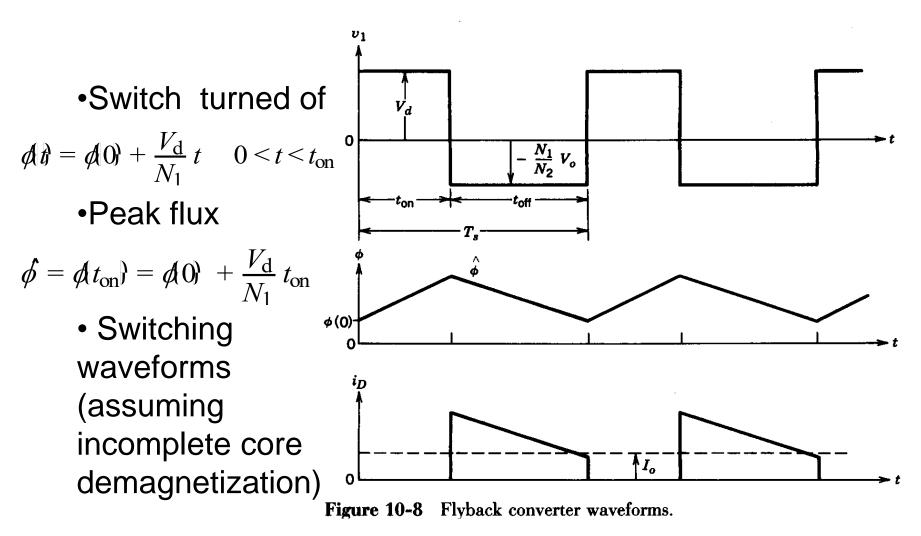
• Switch on and off states (assuming incomplete core demagnetization)

Operation

- Switch is turned on
 - Primary current increases but in secondary no current because of the diode
- Switch is turned off and primary current stops
 - Induced secondary voltage tries to keep flux constant
 - Secondary current flows and discharges energy stored in the magnetic circuit => flyback
- Flyback in Finnish "epäsuoraan tehonsiirtoon perustuva tasasähkönmuuttaja"



Flyback waveforms



Voltage ratio

- After tun-off
 - Energy stored in air gap keeps secondary current flowing and energy is transferred
 - Secondary voltage is $-U_{o}$ and flux decreases $\phi(t) = \phi - \frac{V_{o}}{N_{2}} \{t - t_{on}\}$ $t_{on} < t < T_{s}$
- In steady state flux change (voltage integral) must be zero $\int_{a}^{b} T_{x} = \int_{a}^{b} - \frac{V_{o}}{T_{v}} \{T_{v} = t_{v}\}$

Currents

- During on-time current increases $i_{\rm m}(t) = i_{\rm sw}(t) = I_{\rm m}(0) + \frac{V_{\rm d}}{L_{\rm m}}t \quad 0 < t < t_{\rm on} \qquad \hat{I}_{\rm m} = \hat{I}_{\rm sw} = I_{\rm m}(0) + \frac{V_{\rm d}}{L_{\rm m}}t_{\rm on}$
- After turn-off output voltage $-U_o$ is reflected into the primary and magnetizing current decreases

$$i_{\rm m}(t) = \hat{I}_{\rm m} - \frac{V_{\rm o} (N_1 / N_2)}{L_{\rm m}} (t - t_{\rm on}) \quad t_{\rm on} < t < T_{\rm s}$$

 Setting *i*_m(*T_s*) = *i*_m(0) the same voltage equations as in the previous page is obtained

Voltage and current ratings

Diode current

$$i_{\rm D}(t) = \frac{N_1}{N_2} i_{\rm m}(t) = \frac{N_1}{N_2} \left[\hat{I}_{\rm m} - \frac{V_{\rm o} (N_1 / N_2)}{L_{\rm m}} (t - t_{\rm on}) \right] t_{\rm on} < t < T_{\rm s}$$

Average of diode current is equal to I_o
 Poak value of switch and magnetizing current

$$\hat{I}_{\rm m} = \hat{I}_{\rm sw} = \frac{N_2}{N_1} \frac{1}{1-D} I_{\rm o} + \frac{N_1}{N_2} \frac{(1-D) T_{\rm s}}{2L_{\rm m}} V_{\rm o}$$

Voltage over switch when not conducting

$$v_{\rm sw} = V_{\rm d} + \frac{N_1}{N_2} V_{\rm o} = \frac{V_{\rm d}}{1 - D}$$

Demagnetizing area

- At low loads magnetizing current goes to zero before next cycle
- It can be shown that in this area

$$\frac{V_o}{V_d} = D_{\sqrt{\frac{R}{2L_m f_s}}}$$

- $-R = equvalent load resistance, U_o/I_o$
- $L_{\rm m}$ = magnetizing inductance
- Output voltage is not only dependent on *D*, equal to DCM in dc-dc converters

Other Flyback Converter Topologies

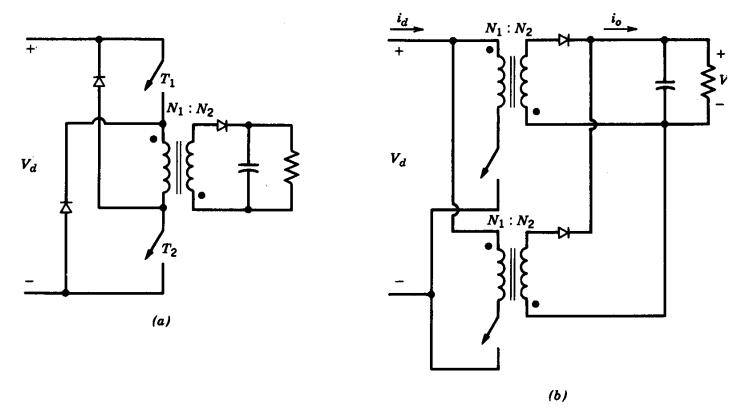


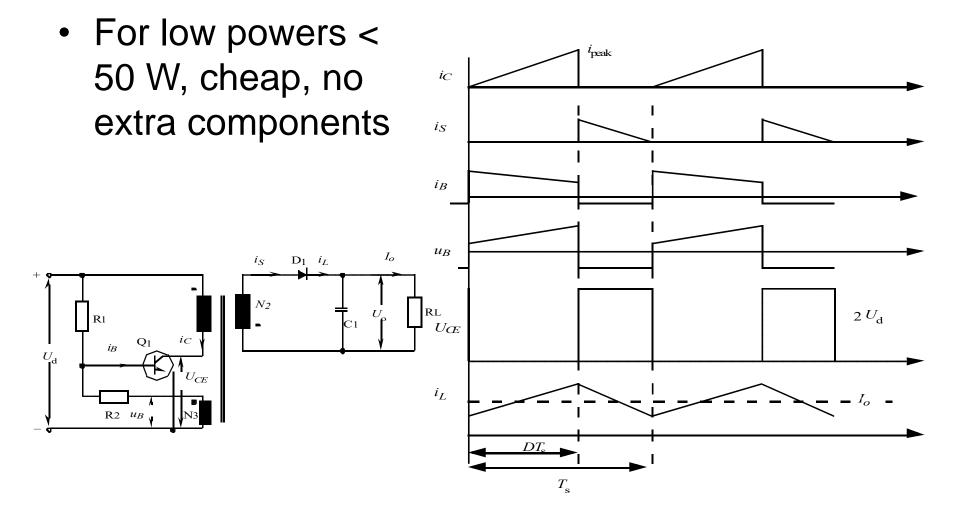
Figure 10-9 Other flyback topologies: (a) two-transistor flyback converter; (b) parallelled flyback converters.

Not commonly used

Comparison

- Two-transistor flyback
 - Transistor turned on and off simultaneously
 - Voltage rating half
 - No snubber needed in primary
 - Energy of leakage inductances discharges through diodes
- Parallel connected
 - More reliable, redundancy
 - Efective switching frequency increased when 180 ° phase-shift between PWM's
 - Standard modules, several parallel connections, load sharing with current control

Blocking oscillator or ringing choke converter



Turn-on

- Transistor Q₁ receives base current through R₁ and starts to conduct
- Peak value of primary current

$$I_{\rm pp} = \hat{I}_{\rm sw} = \frac{V_{\rm d}}{L_{\rm m}} D_{\rm max} T_{\rm s} = \frac{V_{\rm d}}{L_{\rm m}} t_{\rm on}$$

- Voltage induced over N₃ tries to prevent flux increase and together with R₂ creates base current to Q₁ $I_{\rm B} = \frac{V_{\rm B}}{R_2} = \frac{N_3}{N_1} \frac{V_{\rm d}}{R_2}$
 - At some point base current is not enough and transistor turns off
- Switching frequency changes with load

Forward Converter, Buck derived

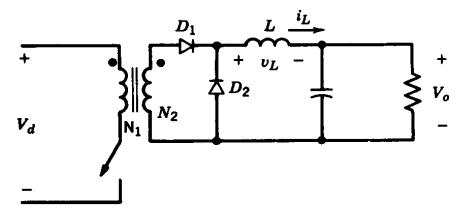


Figure 10-10 Idealized forward converter.

• Derived from Buck; idealized to assume that the transformer is ideal (not possible in practice)

Operation

- After turn-on primary current inceases
 - Secondary induced voltage tries to cancel increase in flux => D2 blocking and D1 conducting
- Voltage over inductance L

$$v_L = \frac{N_2}{N_1} V_d - V_o \quad 0 < t < t_{on}$$

- After turn-off diode D2 conducts $v_L = -V_o$ $t_{on} < t < T_s$
- Voltage integrals

$$\left(\frac{N_2}{N_1} V_d - V_o\right) DT_s = V_o (1 - D) T_s$$

$$\Longrightarrow \frac{V_{\rm o}}{V_{\rm d}} = \frac{N_2}{N_1} D$$

Forward Converter: in Practice

- In practice magnetizing current needs to be taken into account
- Demagnetizing winding N3
 - •Magnetizing energy is discharged to supply
- •Switching waveforms (assuming incomplete core demagnetization)

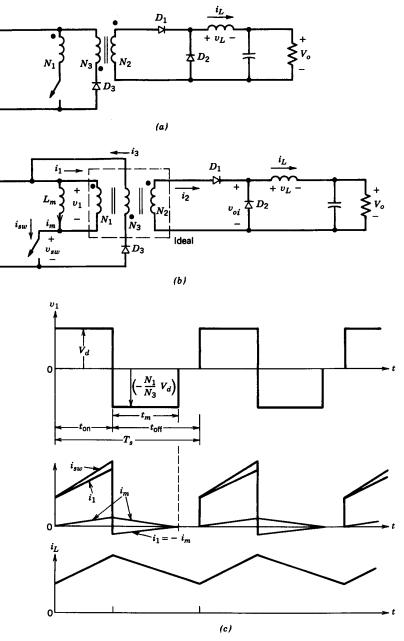


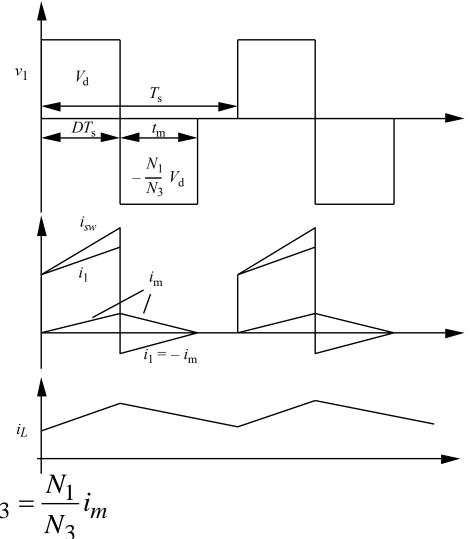
Figure 10-11 Practical forward converter.

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 V_d

Forward, waveforms

- During turn-on $v_1 = V_d$ $0 < t < t_{on}$
- And magnetizing current increaseslinearly
- When switched off
 *i*₁ = - *i*_m and it
 discharges
 magnetic energy



$$N_1i_1 + N_3i_3 = N_2i_2 = 0 \quad \Rightarrow \quad i_3 =$$

Chapter 10 Switching DC Power Supplies

Demagnetization

• During demagnetization primary voltage

$$u_1 = -\frac{N_1}{N_3} U_d \qquad t_{on} < t < t_{on} + t_m$$

• Demagnetization time t_m can be calculated from voltage integral over L_m

$$U_d t_{on} - \frac{N_1}{N_3} U_d t_m = 0 \Longrightarrow \frac{t_m}{T_s} = \frac{N_3}{N_1} D$$

• Time to demagnetize is $(1 - D)T_s$ and therefore maximum duty cycle $1 - D_{max} = \frac{N_3}{D_{max}} \Rightarrow D_{max} = \frac{1}{1 - D_{max}}$

$$1 - D_{\max} = \frac{N_3}{N_1} D_{\max} \Longrightarrow D_{\max} = \frac{1}{1 + N_3/N_1}$$

• Often $N_3 = N_1$ because of bifilar winding, then $D_{max} = 0.5$

Forward Converter: Other Possible Topologies

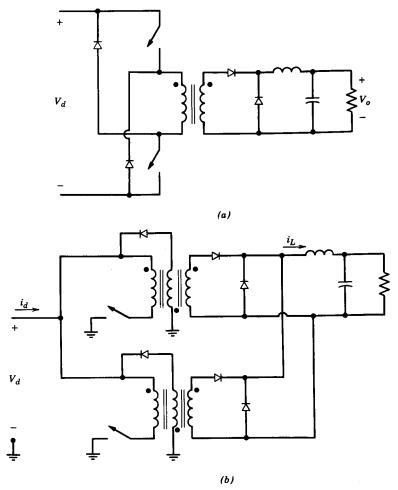


Figure 10-12 Other topologies of forward converter: (a) two-switch forward converter; (b) parallelled forward converters.

• Two-switch Forward converter is very commonly used

Comparison

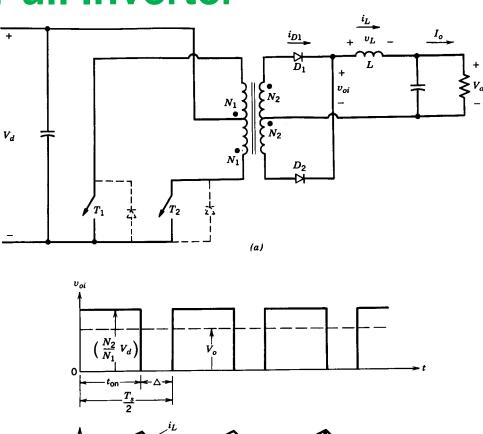
- Two-transistor foward
 - Voltage rating half
 - At turn-off magnetizing energy flows through diodes
 - No demagnetizing winding needed
- Parallel connected
 - Same advantages as in flyback

Push-Pull Inverter

Derived from
Buck
(2*Forward)

•Square-wave ac produced in primary

•Leakage inductances become a problem



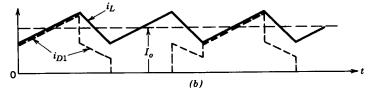


Figure 10-13 Push-pull converter.

Voltages

D1 conducts as T1 conducts
 Voltage over output inductor

$$u_L = \frac{N_2}{N_1} U_d - U_o \qquad 0 < t < t_{on}$$

- During ∆ no switch conducting, current of L is split equally between secondaries
- During second half-cycle T2 conducts

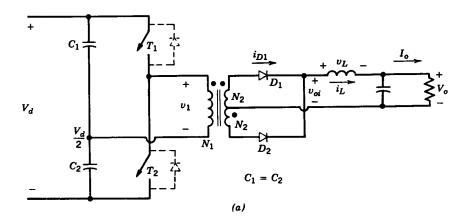
$$u_L = -U_o \qquad t_{on} < t < t_{on} + \Delta = \frac{T_s}{2}$$

Voltage integral

$$\left(\frac{N_2}{N_1}U_d - U_o\right) DT_s = U_o \Delta \qquad \frac{U_o}{U_d} = \frac{N_2}{N_1} \frac{DT_s}{DT_s + \Delta} = \frac{N_2}{N_1} \frac{DT_s}{DT_s + T_s/2 - DT_s}$$

$$\Rightarrow \frac{U_o}{U_d} = 2\frac{N_2}{N_1} D \qquad 0 < D < 0,5$$

Half-Bridge Converter



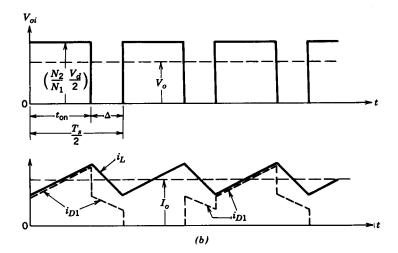


Figure 10-14 Half-bridge dc-dc converter.

• Derived from Buck

Voltages

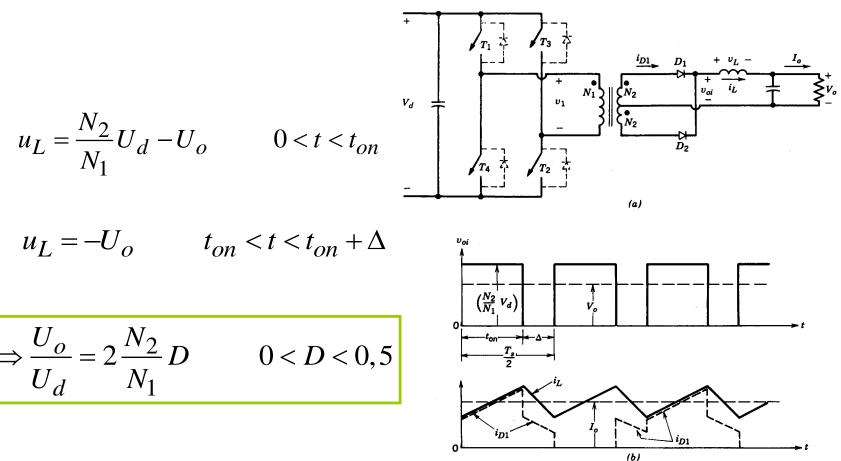
- During turn-on $u_L = \frac{N_2}{N_1} \frac{U_d}{2} U_o \qquad 0 < t < t_{on}$
- When no switch conducts

$$u_L = -U_o \qquad t_{on} < t < t_{on} + \Delta = \frac{T_s}{2}$$

Voltage integral

$$\begin{pmatrix} \frac{N_2}{N_1} \frac{U_d}{2} - U_o \end{pmatrix} DT_s = U_o \Delta \qquad \frac{U_o}{U_d} = \frac{1}{2} \frac{N_2}{N_1} \frac{DT_s}{DT_s + \Delta} = \frac{1}{2} \frac{N_2}{N_1} \frac{DT_s}{DT_s + T_s/2 - DT_s}$$
$$\Rightarrow \frac{U_o}{U_d} = \frac{N_2}{N_1} D \qquad 0 < D < 0,5$$

Full-Bridge Converter





• Used at higher power levels (> 0.5 kW)

Full bridge vs. Half

 Ratio of windings in Full Bridge (FB) and Halfbridge (HB)

$$\left(\frac{N_2}{N_1}\right)_{HB} = 2\left(\frac{N_2}{N_1}\right)_{FB}$$

- If output current is assumed ideal and magnetizing current zero $(I_{SW})_{HB} = 2(I_{SW})_{FB}$
- FB should be used in high power applications

Current-Source Converter

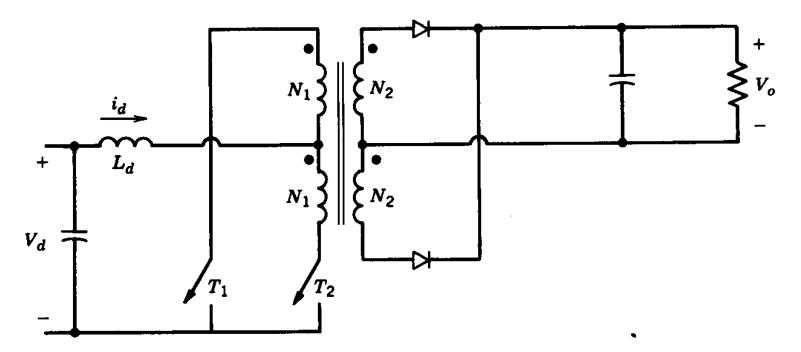


Figure 10-16 Current source converter (D > 0.5).

• More rugged (no shoot-through) but both switches must not be open simultaneously

Operation

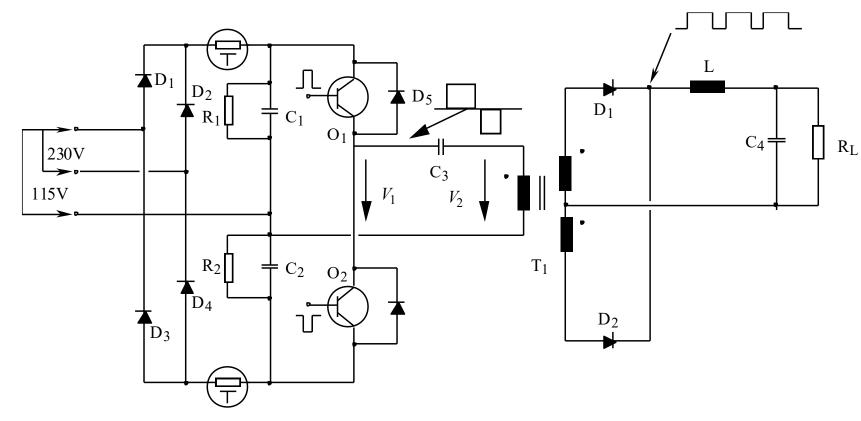
- Push-Pull with added input inductor
 - Current sourced
- When both switches are conduction current in inductor increases
 - Energy is stored and when only one switch conducts energy is transferred to secondary
- Equivalent to boost $\frac{U_o}{U_d} = \frac{N_2}{N_1} \frac{1}{2(1-D)} \qquad D > 0,5$
- Power/weigth ratio lower than in voltage sourced converters

Compensation of nonsymmetric voltage

- Topologies with two quadrant magnetization
 - Positive and negative voltage pulses in primary must be equal
 - Otherwise every cycle increases voltage integral and transformed flux => so called flux walking
 - Transformer saturates
- One solution is to add a series capacitor
 - Removes dc-component from supply
 - Position is same as in resonant converter but here C is larger
 - ESR, equivalent series resistance must be small as all current flows through it

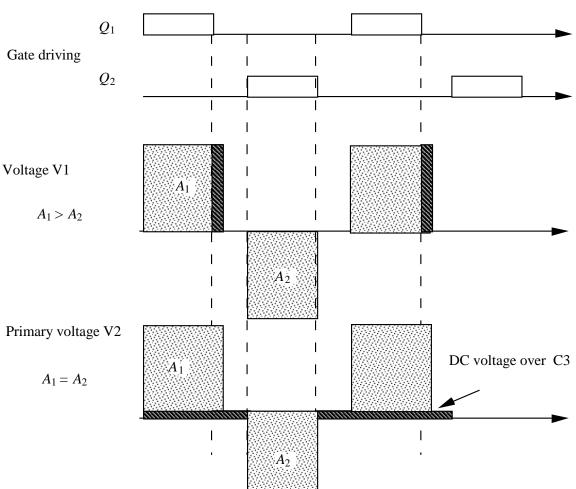
Series dc capacitor

- Capacitor C3 added in series with primary
- D5 and D6 to discharge leakage inductances



Waveforms

- Unsymmetry e.q. due to different delays in gate, Q1 slower
- a) voltage without capacitor
- b) capacitor removes dc component



Resonance?

Capacitor and output inductance, resonance frequency

$$f_R = \frac{1}{2\pi\sqrt{L_RC}}$$
, jossa $L_R = \left(\frac{N_1}{N_2}\right)^2 L$

- LR output inductance in primary
- Capacitor charging should be linear
- Resonance frequency must be low enough when compared to $f_{\rm s}$
- Suitable value e.q. $0,25 \cdot f_s$

Example

• E.q. $f_s = 20$ kHz, output inductance 20 mH and turns ration 10

$$C = \frac{1}{2\pi f_R^2 \left(N_1 / N_2 \right)^2 L} \approx 0.5 \ \mu \text{F}$$

- Capacitor's dc-component has an effect on primary voltage
 - Positive cycle, it is removed
 - Negative cycle, it is added (or vice versa)
 - If capacitor is too small dc component is "too" large

Linear charging

• Capacitor voltage is

$$U_C = \frac{I}{C}dt = \frac{I}{C}\frac{T_s}{2}D_{\max}$$

I is average of primary current

• And it needs to be small enough when compared to supply voltage, 10-20 %

Example, cont.

- Continuing the previous example
- Supply voltage 320 V_{dc}
- Average primary current in worst case 2,3 A
- Maximum duty cycle 80 %
- Capacitor voltage

$$U_C = \frac{I}{C} \frac{T_s}{2} D_{\text{max}} = \frac{2,3}{0,5 \cdot 10^{-6}} \frac{0,8}{2 \cdot 20 \cdot 10^3} \text{ V} = 92 \text{ V}$$

Size of capacitor

- It can be concluded that 92 V is too high when compared to input 320/2 = 160
 - Deteriates SMPS operation especially at lower limit of input
- E.q. if 30 V is upper limit

$$C = \frac{I}{U_C} \frac{T_s}{2} D_{\text{max}} = \frac{2.3}{30} \frac{0.8}{2 \cdot 20 \cdot 10^3} \text{ F} = 1,53 \text{ }\mu\text{F}$$

- In principle 30 V capacitor is enough but using e.q.
 200V component is safer
- At higher power current rating of the capacitor becomes a problem

Ferrite Core Material

CORE LOSS vs. FLUX DENSITY

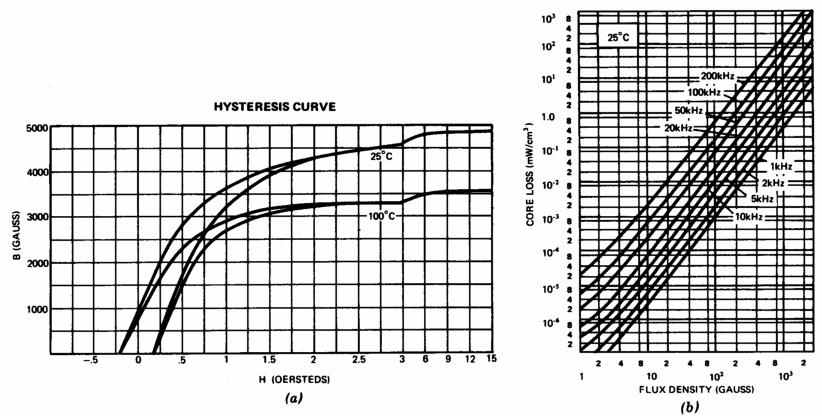


Figure 10-17 3C8 ferrite characteristic curves: (a) B-H loop; (b) core loss curves. (Courtesy of Ferroxcube Division of Amperex Electronic Corporation.)

• Several materials to choose from based on applications

Units

- Often non-standard units are used with magnetic circuits
- In SI-system

 $- 1 \text{ gauss} = 0,1 \text{ mT} = 0,1 \cdot 10^{-3} \text{ Vs/m}^2$

- 1 örstedt =
$$(1/4\pi) \cdot 10^3$$
 A/m

Core Utilization in Various Converter Topologies

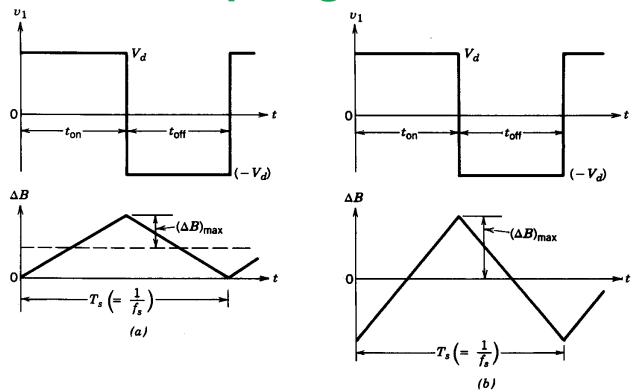


Figure 10-18 Core excitation: (a) forward converter, D = 0.5; (b) full-bridge converter, D = 0.5.

• At high switching frequencies, core losses limit excursion of flux density

Maximum flux density

• When $N_1 = N_3$ $\Delta B_{\text{max}} = \frac{U_d}{4N_1A_cf_s}$ kun D = 0,5

 $-A_c =$ Area of the cross section of transformer

- In Forward converter magnetization in one quadrant $\Delta B_{\text{max}} < \frac{B_m B_r}{2}$
- Half- and Full-bridge, magnetization in two quadrants

$$\Delta B_{\max} < B_m$$

Flux density (1/2)

- High saturation flux density B_m
 - => high ΔB_{max}
 - smaller cross section and transformer
- Small switching frequency
 - $B_{\rm m}$ limits $\Delta B_{\rm max}$
 - Increasing switching frequency reduces transformer size
 - However at high frequencies B_m must be reduced to reduce losses or it is necessary to use better ferrite material with lower losses

Flux density (2/2)

- One quadrant magnetization
 - Remanence must be small
 - In practice air gap is used
- Two quadrant magnetization
 - Air gap prevents saturation at start up and transients
 - Unsymmetry of primary voltage in steady state needs to be removed

Flyback

- Magnetic circuit is used as energy storage
 - It has to have an air gap
 - Stores energy
 - Prevents saturation

Transformer losses (1/2)

• General equation for core losses

$$\frac{P_{core}}{m^3} = kf^a \left(\Delta B\right)^b$$

- ΔB = maximum flux swing during f_s - Constants *k*, *a*, *b* depend on core material

$$P_w = R_{ac} I_{rms}^2$$
, $R_{ac} = F_r R_{dc}$ = winding ac resistance

- Copper losses are due to winding resistance
 - *F*_r accounts for temperature and eddy currents

Transformer losses (2/2)

- It follows that
 - *P*core increases as core size and ΔB increase
 - P_W reduces as core size and ΔB increase
- Previous changes are opposite
- When designing a transformer
 - Target is to minimize losses when output power P_0 is given
 - It can be shown that optimum case is when winding and core loss densities are equal

Transformer dimension for different topologies

- Next pages, summary of transformer dimensioning, output power
 - Reference is /Pressman, 1998 pages. 277-294/
- In the following equations
 - Filling factor of winding is assumed to be 0,4
 - When calculating primary quantities efficiency is assumed to be 0,8

Quantities

- In the equations of following page units are
 - B_{max} = flux density in Gauss
 - $A_{\text{core}} = \text{core area in } \text{cm}^2$
 - ^A_W = winding area in cm²
 - ^D_{cma} = inverse of current density when area is expressed in circular mills

 $D_{cma} = \frac{\text{Area of wire in circular mills}}{I_{RMS}}$

• circular mill = area of a circle when diameter is 1 mm.

circular mill
$$\approx 5,07 \cdot 10^{-6} = \text{cm}^2$$

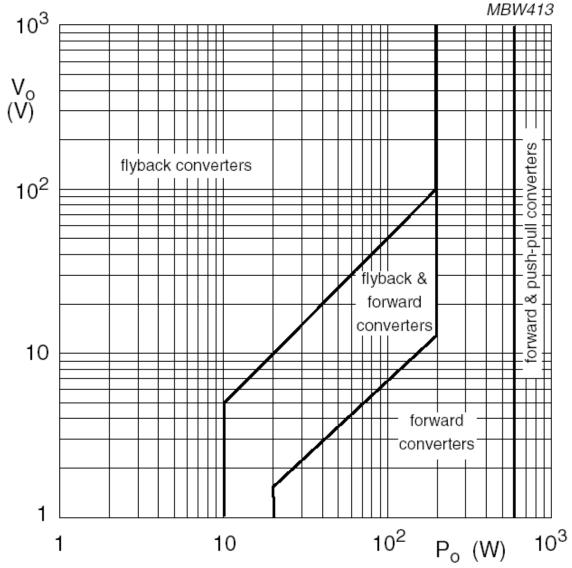
Topologies

- Forward $P_o = \frac{0,0005B_{\text{max}} fA_{\text{core}} A_{\text{w}}}{D_{\text{cma}}}$
- Push-pull $P_o = \frac{0,001B_{\text{max}} fA_{\text{core}} A_{\text{w}}}{D_{\text{cma}}}$
- Half-bridge $P_o = \frac{0,0014B_{\text{max}} fA_{\text{core}} A_{\text{w}}}{D_{\text{cma}}}$
- Full-bridge, same as half-bridge
- Output power increases from the same core as topology is changed toward bridge converters
 - At the same time more power semiconductor devices are needed



Summary

 Data Handbook Soft Ferrites and Accessories

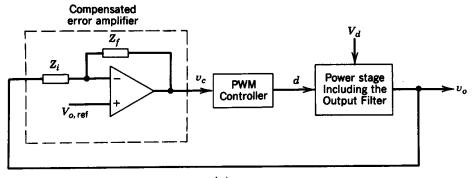


Feedback Control

Content

- Linearisation of the converter and modulation
- PWM-modulator
- Optimizing the controller
- Voltage feed forward
- Current control

Control to Regulate Voltage Output





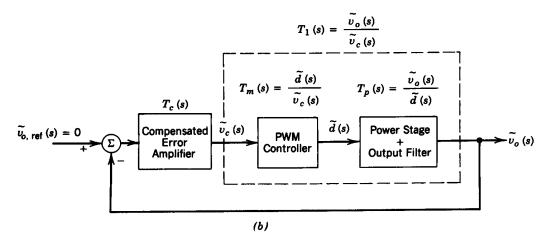


Figure 10-19 Voltage regulation: (a) feedback control system; (b) linearized feedback control system.

Linearized representation of the feedback control system

Methods

- Middlebrook, Cúk
 - state-space averaging
 - Linear model in some operating point
- Vorperian
 - average PWM switch
 - Newer method and used often, a bit easier
 - Only the non-linear part is averaged, i.e. switch and diode

State-space averaging

- R. D. Middlebrook, S. Cúk A; A General Unified Approach to modelling Switching-Converter Power Stages, IEEE PESC 1976
- State-Space Averaging, SSA
- Target is transfer function

 $\tilde{u}_o(s)/\tilde{d}_o(s)$

In CCM

Step 1

- State variables are *x* (matrix)
 - Current of inductance
 - Capactor voltage
 - E.g. in Cúk more variables are needed
- Also losses of components taken into account
- Two set of equations, on and off

State equations and output voltage

$$\dot{\mathbf{x}} = A_1 \mathbf{x} + B_1 u_d \quad dT_s \qquad u_o = C_1 \mathbf{x} \quad dT_s$$

 $\dot{\mathbf{x}} = A_2 \mathbf{x} + B_2 u_d \quad (1 - d) T_s \qquad u_o = C_2 \mathbf{x} \quad (1 - d) T_s$

STEP 2 Averaging

Previous equations

 $\dot{\boldsymbol{x}} = \boldsymbol{A}_{1}\boldsymbol{x} + \boldsymbol{B}_{1}\boldsymbol{u}_{d} \qquad dT_{s}$ $\dot{\boldsymbol{x}} = \boldsymbol{A}_{2}\boldsymbol{x} + \boldsymbol{B}_{2}\boldsymbol{u}_{d} \qquad (1 - d)T_{s}$ $\boldsymbol{u}_{o} = \boldsymbol{C}_{1}\boldsymbol{x} \qquad dT_{s}$ $\boldsymbol{u}_{o} = \boldsymbol{C}_{2}\boldsymbol{x} \qquad (1 - d)T_{s}$

are time averaged

$$\dot{\mathbf{x}} = \left[\mathbf{A}_{1}d + \mathbf{A}_{2} \left(1 - d \right) \right] \mathbf{x} + \left[\mathbf{B}_{1}d + \mathbf{B}_{2} \left(1 - d \right) \right] u_{d}$$
$$u_{o} = \left[\mathbf{C}_{1}d + \mathbf{C}_{2} \left(1 - d \right) \right] \mathbf{x}$$

STEP 3 Linearisation, ac-component in variables

$$\mathbf{x} = \mathbf{X} + \tilde{\mathbf{x}}$$
 $u_o = U_o + \tilde{u}_o$ $d = D + \tilde{d}$

- Supply voltage is assumed constant
- Inserted in the averaged equations and further $\dot{X} = 0$

$$\dot{\tilde{x}} = AX + BU_d + A\tilde{x} + \left[\left(A_1 - A_2 \right) X + \left(B_1 - B_2 \right) U_d \right] \tilde{d}$$

Terms containing multiplications of two ac components, small and neglected

$$\boldsymbol{A} = \boldsymbol{A}_1 \boldsymbol{D} + \boldsymbol{A}_2 (1 - \boldsymbol{D})$$
$$\boldsymbol{B} = \boldsymbol{B}_1 \boldsymbol{D} + \boldsymbol{B}_2 (1 - \boldsymbol{D})$$

Steady state solution

- In steady state derivatives and ac-components are zero $AX + BU_d = 0$
- From the averaged equation remains $\dot{\tilde{x}} = A\tilde{x} + [(A_1 - A_2)X + (B_1 - B_2)U_d]\tilde{d}$
- Similarly, output voltage is $u_o = U_o + \tilde{u}_o = CX + C\tilde{x} + (C_1 - C_2)X\tilde{d}$ $C = C_1D + C_2(1-D)$
- Steady state output voltage is $\tilde{u}_o = C\tilde{x} + (C_1 C_2)X\tilde{d}$

$$U_o = CX$$

Steady state voltage ratio

$$\frac{U_o}{U_d} = -CA^{-1}B$$

Step 4 Laplace transformation

• Laplace of $\dot{\tilde{x}} = A\tilde{x} + [(A_1 - A_2)X + (B_1 - B_2)U_d]\tilde{d}$

 $s\tilde{\boldsymbol{x}}(s) = A\tilde{\boldsymbol{x}}(s) + \left[(A_1 - A_2)\boldsymbol{X} + (B_1 - B_2)\boldsymbol{U}_d \right] \tilde{\boldsymbol{d}}(s)$ $\tilde{\boldsymbol{x}}(s) = \left[s\boldsymbol{I} - \boldsymbol{A} \right]^{-1} \left[(A_1 - A_2)\boldsymbol{X} + (B_1 - B_2)\boldsymbol{U}_d \right] \tilde{\boldsymbol{d}}(s)$

• Output voltage equation $\tilde{u}_o = C\tilde{x} + (C_1 - C_2)X\tilde{d}$

Laplace transformed and inserted => transfer function is

$$T_{p}(s) = \frac{\tilde{u}_{o}(s)}{\tilde{d}(s)} = C[sI - A]^{-1} [(A_{1} - A_{2})X + (B_{1} - B_{2})U_{d}] + (C_{1} - C_{2})X$$

Forward Converter: An Example

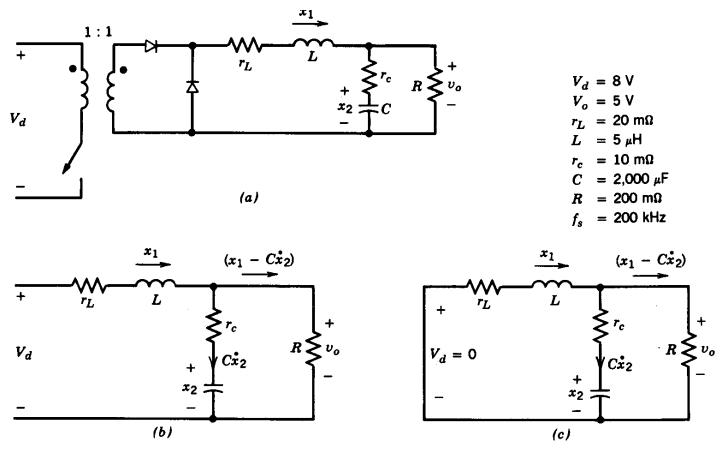


Figure 10-20 Forward converter: (a) circuit; (b) switch on; (c) switch off.

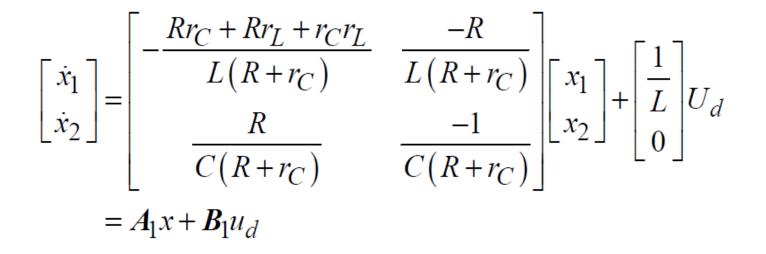
• The switch and the diode are assumed to be ideal

Switch conducts

$$-U_d + L\dot{x}_1 + r_L x_1 + R(x_1 - C\dot{x}_2) = 0$$

$$-x_2 - Cr_C \dot{x}_2 + R(x_1 - C\dot{x}_2) = 0 \Leftrightarrow \dot{x}_2 = \frac{Rx_1 - x_2}{C(R + r_C)}$$

In matrix form



Switch not conducting

- Only difference is that $U_d = 0$
- Matrixes are then

 $A_2 = A_1$ ja $B_2 = 0$

• Output voltage in both cases

$$u_o = R\left(x_1 - C\dot{x}_2\right) = \frac{Rr_C}{R + r_C} x_1 + \frac{R}{R + r_C} x_2$$
$$= \left[\frac{Rr_C}{R + r_C} \quad \frac{R}{R + r_C}\right] \begin{bmatrix}x_1\\x_2\end{bmatrix} = C_1 x = C_2 x$$

Transfer function

• After some steps

$$T_p(s) = \frac{\tilde{u}_o(s)}{\tilde{d}(s)} = U_d \frac{1 + sr_C C}{LC \left[s^2 + s\left(\frac{1}{RC} + \frac{r_C + r_L}{L}\right) + \frac{1}{LC}\right]}$$

Characteristic polynomial

$$s^{2} + 2\xi\omega_{o}s + \omega_{o}^{2} \qquad \qquad \omega_{o} = \frac{1}{\sqrt{LC}} \qquad \qquad \xi = \frac{\frac{1}{RC} + \frac{r_{C} + r_{L}}{L}}{2\omega_{o}}$$

$$T_p(s) = \frac{\tilde{u}_o(s)}{\tilde{d}(s)} = U_d \frac{\omega_o^2}{\omega_z} \frac{s + \omega_z}{s^2 + 2\xi\omega_o s + \omega_o^2} \qquad \qquad \omega_z = \frac{1}{r_C C}$$

Forward Converter: Transfer Function Plots

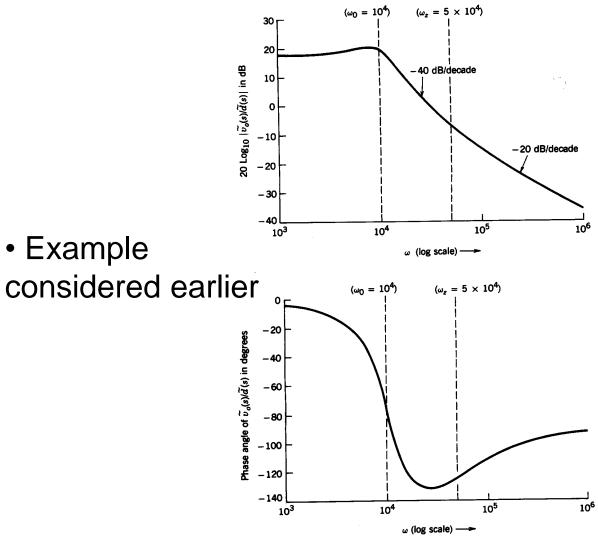


Figure 10-21 (a) Gain plot of the forward converter in Fig. 10-20a. (b) Phase plot of the forward converter in Fig. 10-20a.

Flyback Converter: Transfer Function Plots

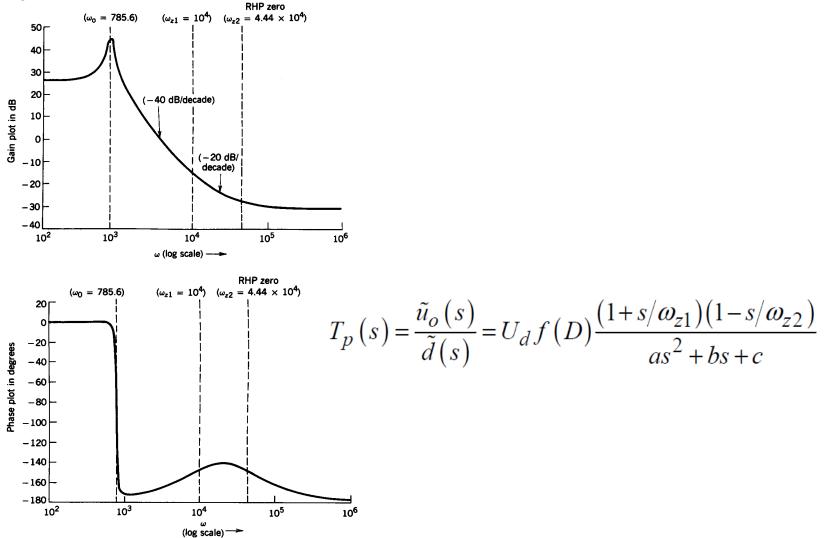


Figure 10-22 (a) Gain plot for a flyback converter. (b) Phase plot for a flyback converter.

Rigth half plane zero

- Flyback has zero in right half plane
 - This is seen in the phase of the transfer function
- In steady state duty cycle is increased
 - Time $(1 d)T_s$ is reduced
 - Time dT_s increases and output capacitor is discharged longer
 - Immediately after the increase of *d* output voltage decreases and it doesn't increase
- In steady state increase of *d* increases also output voltage

Linearizing the PWM Block

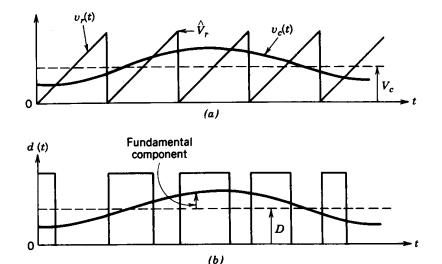


Figure 10-23 Pulse-width modulator.

• The transfer function is essentially a constant with zero phase shift

Gain of the PWM IC

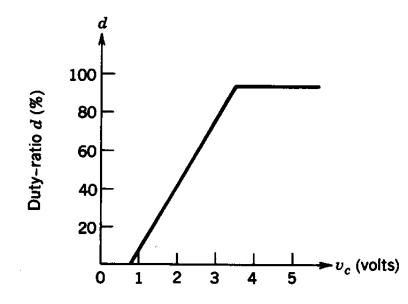
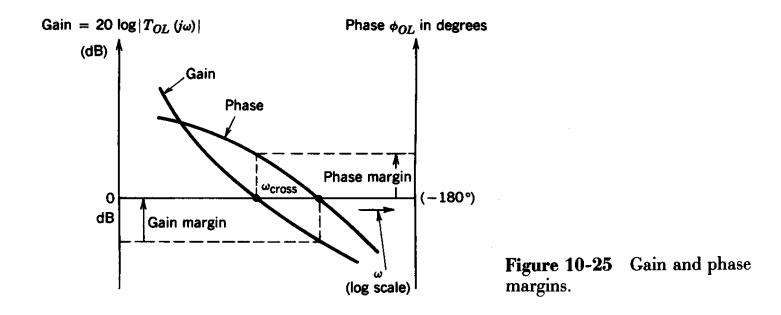


Figure 10-24 Pulse-width modulator transfer function.

• It is slope of the characteristic

Typical Gain and Phase Plots of the Open-Loop Transfer Function



• Definitions of the crossover frequency, phase and gain margins

A General Amplifier for Error Compensation

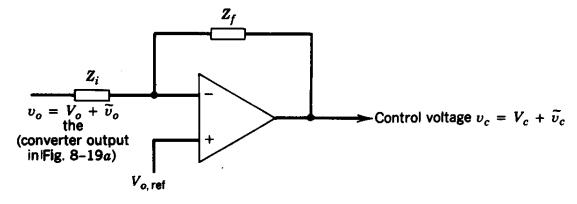


Figure 10-26 A general compensated error amplifier.

Can be implemented using a single op-amp

Type-2 Error Amplifier

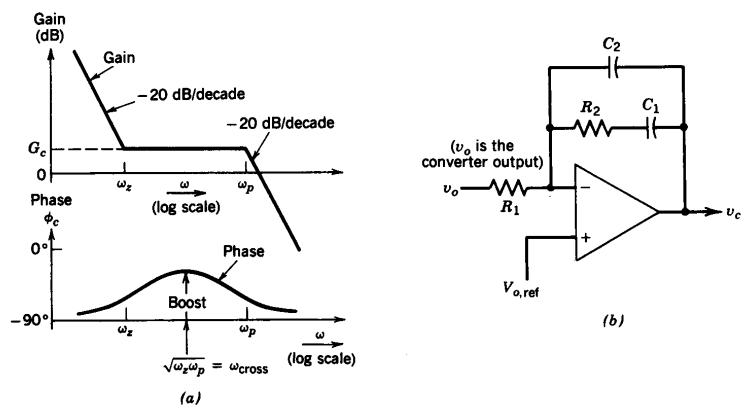


Figure 10-27 Error amplifier.

Shows phase boost at the crossover frequency

Coefficents of the controller

- Simple and often in SMPS used method is so called K-factor
 - H. Dean Venable; The K-Factor: A New Mathematical Tool for Stability Analysis and Synthesis, Proceedings of Powercon 10, March 22-24, 1983.
- Select ω_{cross} and with *K*-factor

$$\omega_z = \frac{\omega_{\text{cross}}}{K} \qquad \omega_p = K\omega_{\text{cross}}$$

• It can be shown the the required phase Boost give K as

$$K = \tan\left(45^o + \frac{Boost}{2}\right)$$

Boost and phase margin

• Phase margin $PM = 180^{\circ} + \phi_1 + \phi_c$

 $-\phi_c$ is phase of controller T_c at ω_{cross} $-\phi_1$ is sum of phases of power stage T_p and modulator T_m

Required phase boost is

$$PM = 180^{\circ} + \phi_{1} + \phi_{c} = 180^{\circ} + \phi_{1} - 90^{\circ} + Boost$$
$$\Rightarrow Boost = PM - \phi_{1} - 90^{\circ}$$

PM is normally selected to be 45° - 60°,
 - => Boost => K

Gain

• Amplifier gain is selected so that the open loop gain of $T_{OL}(G_1G_c)$ at cross over frequency is one

$$G_{c}(\omega = \omega_{\text{cross}}) = \frac{1}{G_{1}(\omega = \omega_{\text{cross}})} = \frac{1}{|T_{1}(\omega = \omega_{\text{cross}})|} = \frac{1}{|T_{p}(\omega = \omega_{\text{cross}})T_{m}(\omega = \omega_{\text{cross}})|}$$

This gives $G_{c} = \frac{1}{KR_{1}C_{2}\omega_{\text{cross}}} = \frac{1}{G_{1}}$

Resistor R₁ can be selected freely and other components are

$$C_2 = \frac{G_1}{KR_1\omega_{\text{cross}}} \qquad C_1 = C_2\left(K^2 - 1\right) \qquad R_2 = \frac{K}{C_1\omega_{\text{cross}}}$$

Phase boost

- With the previous controller structure phase can be boosted only 90 degrees
- E.g. flyback has zero in rigth half plane
 - Controller with two zeros and poles can boost the phase 180 degrees

Voltage Feed-Forward

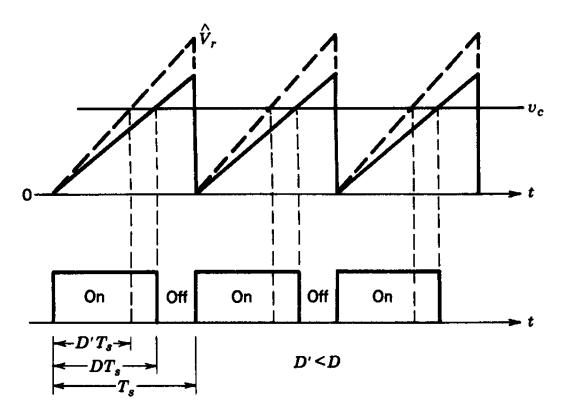
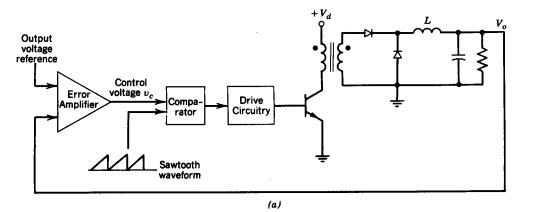


Figure 10-28 Voltage feed-forward: effect on duty ratio.

- Makes converter more immune for input voltage variations, duty cycle is changed immediately
- Overcompensation should be avoided

Voltage versus Current Mode Control



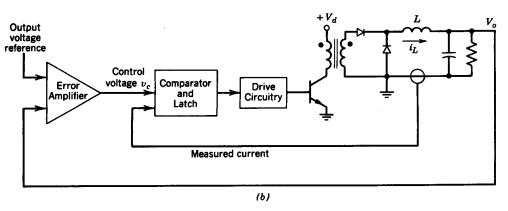


Figure 10-29 PWM duty ratio versus current-mode control: (a) PWM duty ratio control; (b) current-mode control.

• Regulating the output voltage is the objective in both modes of control

Chapter 10 Switching DC Power Supplies

10-100

Various Types of Current Mode Control

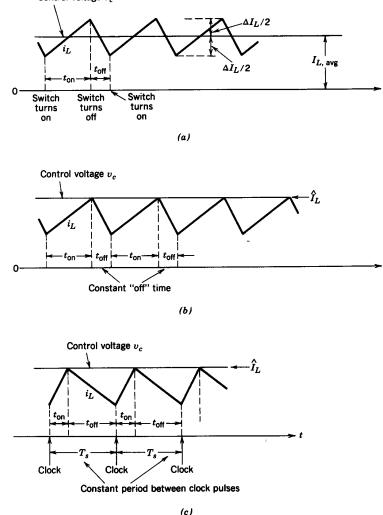


Figure 10-30 Three types of current-mode control: (a) tolerance band control; (b) constant-off-time control; (c) constant frequency with turnon at clock time.

Chapter 10 Switching DC Power Supplies

Constant frequency, peak-

current mode control is used

most frequently

10-101

Peak Current Mode Control

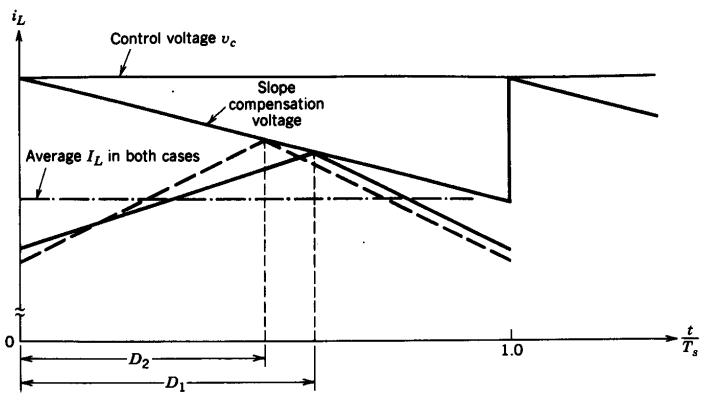
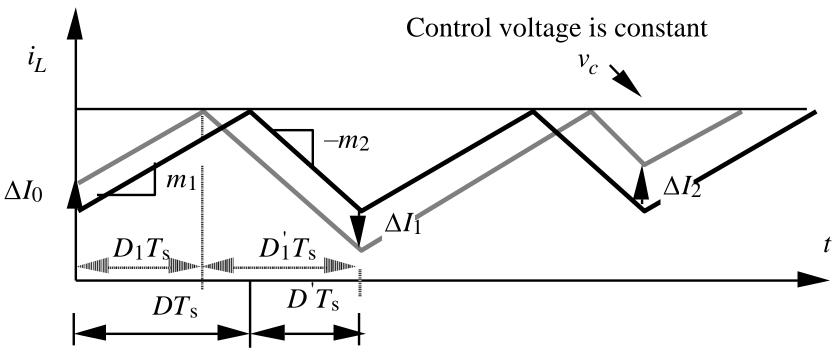


Figure 10-31 Slope compensation in current-mode control $(D_2 \text{ is smaller for a higher input voltage with a constant <math>V_o$).

• Slope compensation is needed

Instability in current control

- E.g. noise in measurement and current is according to the dashed line, slope still *m*¹
- The disturbance ∆*l*₀ increases after every cycle when duty cycle is larger than 0,5



Change in current

• Current increase with slope m_1 and decrease with slope m_2 . In steady state

$$\frac{m_2}{m_1} = \frac{D}{D'} = \frac{D}{1-D}$$

- Based on the previous figure $\Delta I_o = (D - D_1)T_s m_1$ $\Delta I_1 = (D - D_1)T_s m_2 = (1 - D - 1 + D_1)T_s m_2 = -(D - D_1)T_s m_2$
- After first pulse $\Delta I_1 = -\frac{m_2}{m_1} \Delta I_o = -\frac{D}{1-D} \Delta I_o$

Geometric series

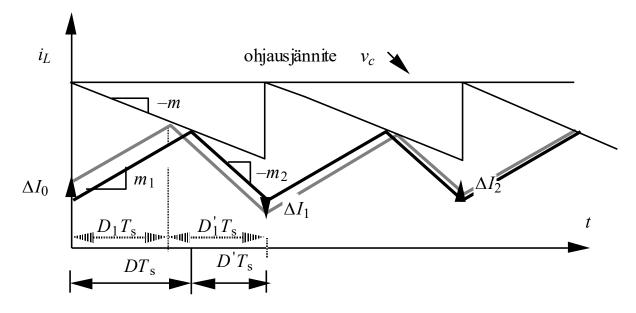
• With recursion, after the *n*:th pulse

$$\Delta I_n = \left(-\frac{D}{1-D}\right)^n \Delta I_o$$

- If duty cycle *D* is larger than 0,5
 - Geometric series where constant is larger than one
 - => disturbance is amplified as shown in the figure => unstable

Slope compensation

- Instability is removed if vc is not constant
- Compensating voltage used in vc or added in the measured current



Slope compensation guarantees stability

Disturbance with slope compensation

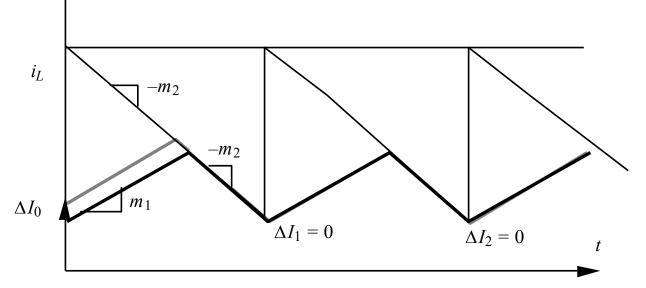
• It can be shown that after *n*:n cycles

$$\Delta I_n = \left(-\frac{m_2 - m}{m_2 + m}\right)^n \Delta I_o$$

- When *m* is selected properly disturbance decreases even if *D* larger than 0,5
- Fastest response when $m = m_2$.

Optimal compensation

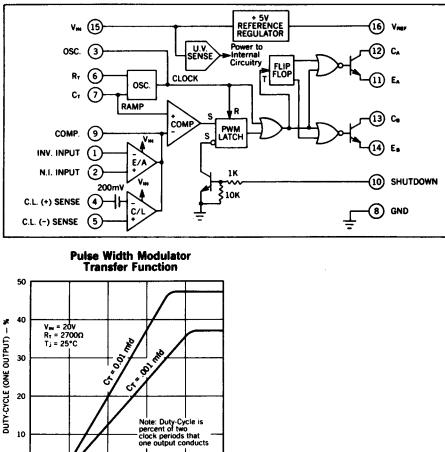
Disturbance is removes already after the first cycle



- Is usefull even if duty cycle is less than 0,5
- Optimal slope changes with operating point, normally slope kept constant

A Typical PWM Control IC





 Many safety control functions are built in



Figure 10-32 Pulse-width modulator UC1524A: (a) block diagram; (b) transfer function. (Courtesy of Unitrode Integrated Circuits Corp.)

5

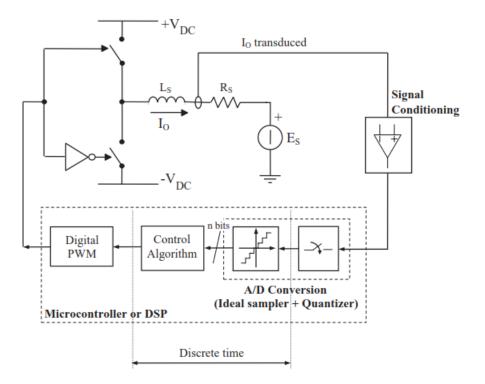
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Digitalization

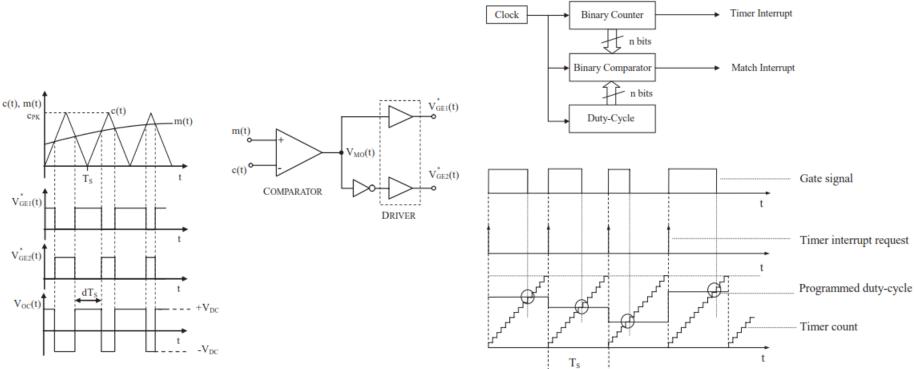
- Digitalization has several levels, often can be understood as digital control
 - Digital power, also the inner control loop should be digital
- Motor drives, UPS and high power supplies have already been for long totally digital
 - In low power supplies still inner loop is most often analog and microcontroller is used for interfacing
- Digital Control Advantages:
 - Flexibility, Ease of upgrade, Man to Machine Interface (MMI)
 - Sophisticated control techniques
 - possibility to change control laws, switching frequency and improve efficiency and/or dynamics
 - Reduced number of components, Unsensitivity to components' ageing
- Digital Control Disadvantages:
 - Design complexity, Cost, Dynamic performance (sampling frequency, quantization...)

Digital Control in Power Electronics

• Typical organization of a digital current controller



Analog vs. Digital Pulse Width Modulation

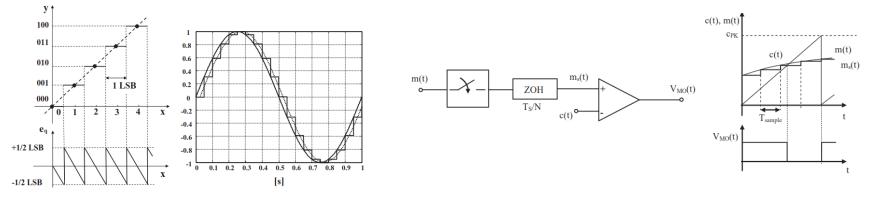


Analog implementation of a PWM modulator. Analog comparator determines the state of the switches by comparing the carrier signal c(t) and the modulating signal m(t)

Simplified organization of a digital PWM. At the beginning of the counting period, the gate signal is set to high and goes low at the match condition occurrence, i.e., any time the binary counter value is equal to the programmed duty-cycle. Binary counter triggers an interrupt request for the microprocessor at the beginning of each modulation period.

Sampling delay

- Because of the sample and hold effect, the response of the modulator to any disturbance, e.g. to one requiring a rapid change in the programmed duty-cycle value, can take place only during the modulation period following the one where the disturbance actually takes place
- Double update or multi-sampling are ways to reduce the delay
- Multi-sampling presents some limitations as well
 - need for proper filtering of the switching noise
 - need for non conventional hardware
 - generation of dead bands

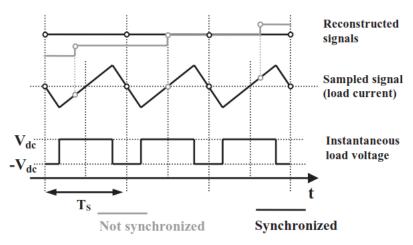


Quantization error eq Sample and hold delay effect

Multi-sampled PWM

Synchronized sampling and switching processes

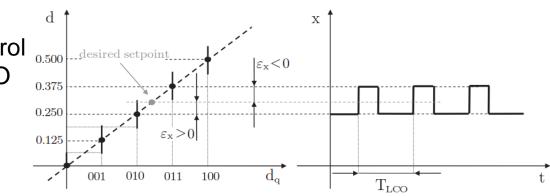
- Sampling takes place always at the beginning (or in the middle) of the modulation period, the average current value is automatically obtained and this is what we want to control
 - If the sampling frequency is lower than the switching one, an aliased, low frequency component appears on the reconstructed signal
- Shannon's theorem conditions will always be violated
 - The typically recommended high ratio between sampling frequency and sampled signal bandwidth will "never" be possible in power electronics



Limit Cycle Oscillations, LCO

- The desired set-point for the control variable d is not anyone of its possible values => system oscillates with period TLCO
- Converter output current
 - Is proportional to the integral of the inverter average output voltage, that is in turn, proportional to the duty-cycle

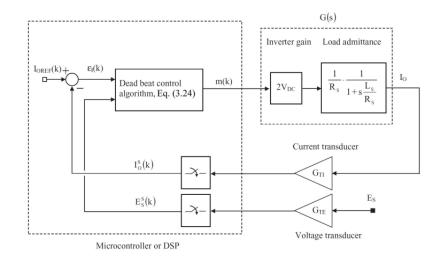
 - Integrator in current control reduces the effect of LCO

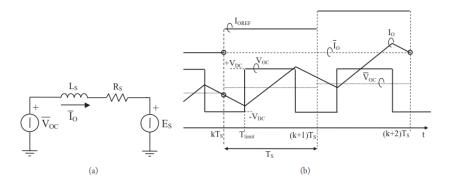


(Model) Predictive or dead beat current control

- At any given control iteration, we want to find the average inverter output voltage, that can make the average inductor current to reach its reference by the end of the modulation period
 - Requires detailed model of the system
 - Fast responses
- Hot topic in control theory and in power electronics control

(Model) Predictive or dead beat current control





$$m(k+1) = -m(k) + \frac{L_S}{T_S} \cdot \frac{1}{2 \cdot G_{TI} \cdot V_{DC}} \left[I_{OREF}^S(k) - I_O^S(k) \right] + 2 \cdot \frac{1}{2 \cdot G_{TE} \cdot V_{DC}} E_S^S(k), \qquad (3.24)$$

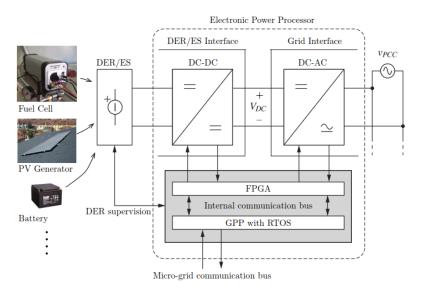
Custom designed digital control chips

- Software programmable devices, like microcontrollers or digital signal processors are the most commonly adopted approach to digital control in power electronics
- BUT flexibility is costly especially if perfomance requirements are high
- Custom designed digital control chips
 - Optimize the cost of the controller, tailoring the hardware to the exact functionality it is called to provide
 - Maximizing the performance level of a given power converter topology, by compressing computation times to the minimum and pushing switching frequency, small-signal control bandwidth and large-signal speed of response as high as possible.
- Some leading integrated circuit manufacturers are nowadays offering application specific digital control chips, designed and optimized for particular DC-DC converter topologies and applications

Example: Distributed Energy Source (DER) / Energy Storage (ES)

Subsystem	Functionality	Scope	Target
Inverter	PWM PLL Current loop Fault detection	Local Local Local Local	FPGA
	$\begin{array}{c} P & \text{loop} \\ Q & \text{loop} \\ \hline Grid & \text{voltage control} \\ Grid & \text{characterization} \\ Global & \text{optimization} \\ \end{array}$	Local/Distributed Local/Distributed Local/Distributed Distributed	RT GPP
DC/DC	PWM Input V/I control Source/storage optimization	Local Local Local/Remote	FPGA RT GPP

Feature	Parameter	Value
Processor	Model Processor Speed	PowerPC 400 MHz
Memory	Nonvolatile System	512 MB 256 MB
FPGA	Model # Slices # DSP48s	Xilinx Spartan-6 LX45 6822 58
Network	Network interface	IEEE 802.3 Ethernet
Communication	Port	RS-232, RS-485 CAN, USB
Peripherals	Channel	16 AI, 12-bit, $\pm 10\mathrm{V},100\mathrm{kHz}$ 14 ch., 500 kHz gate drivers $^{(1)}$



Example: STLUX family from ST Microelectronics

- Especially designed for lighting applications
- SMED (state machine event driven) technology which allows the device to pilot six independently configurable PWM clocks with a maximum resolution of 1.3 ns
 - autonomous state machine, which is programmed to react to both external and internal events and may evolve without any software intervention
- Each SMED is configured via the STLUX internal microcontroller
- A set of dedicated peripherals complete the STLUX:
 - 4 analog comparators with configurable references and 50 ns max. propagation delay
 - It is ideal to implement zero current detection algorithms or detect current peaks
 - 10-bit ADC with configurable op amp and 8-channel sequencer
 - DALI: hardware interface that provides full IEC 60929 and IEC 62386 slave interface
 - 96 MHz PLL for high output signal resolution

Example: STLUX



STLUX

Digital controllers for lighting and power conversion applications with up to 6 programmable PWM generators, 96 MHz PLL, DALI

Datasheet - production data



Features

- Up to 6 programmable PWM generators (SMEDs - "State Machine Event Driven")
 - 10 ns event detection and reaction
 - Max.1.3 ns PWM resolution
 - Single, coupled and two coupled operational modes
 - Up to 3 internal/external events per SMED
 - DALI (digital addressable lighting interface)
 - Interrupt driven hardware encoder
 - Bus frequency: 1.2, 2.4 or 4.8 kHz
 - IEC 60929 and IEC 62386 compliant plus 24-bit frame extension
 - Configurable noise rejection filter
 - Reverse polarity on Tx/Rx lines
 - 4 analog comparators
 - 4 internal 4-bit references
 - 1 external reference
 - Less than 50 ns propagation time
 - Continuous comparison cycle

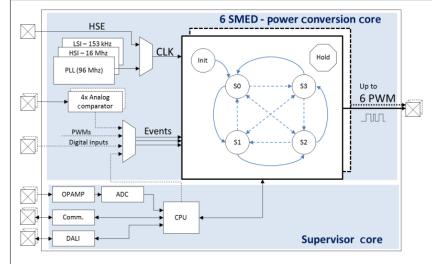
ADCs (up to 8 channels)

- 10-bit precision, with operational amplifier to extend resolution to 12-bit equivalent
- Sequencer functionality
- Input impedance: 1 MΩ
- Configurable gain value: x1 and x4
- Integrated microcontroller
 - Advanced STM8[®] core with Harvard architecture and 3-stage pipeline
 - Max. f_{CPU}: 16 MHz
 - Multiple low power modes

- Memories
 - Flash and E²PROM with read while write (RWW) and error correction code (ECC)
 - Program memory: 32 Kbytes Flash; data retention 15 years at 85 °C after 10 kcycles at 25 °C
 - Data memory: 1 Kbyte true data E²PROM; data retention:15 years at 85 °C after 100 kcycles at 85 °C
 - RAM: 2 Kbytes
- Clock management
 - Internal 96 MHz PLL
 - Low power oscillator circuit for external crystal resonator or direct clock input
 - Internal, user-trimmable 16 MHz RC and low power 153.6 kHz RC oscillators
 - Clock security system with clock monitor
- Basic peripherals
 - System and auxiliary timers
 - IWDG/WWDG watchdog, AWU, ITC
- I/O
 - GPIO with highly robust design, immune against current injection
 - Fast digital input DIGIN, with configurable pull-up
- Communication interfaces
 - UART asynchronous with SW flow control and boot loader support
 - I²C master/slave fast-slow speed rate
- Operating temperature: -40 °C up to 105 °C

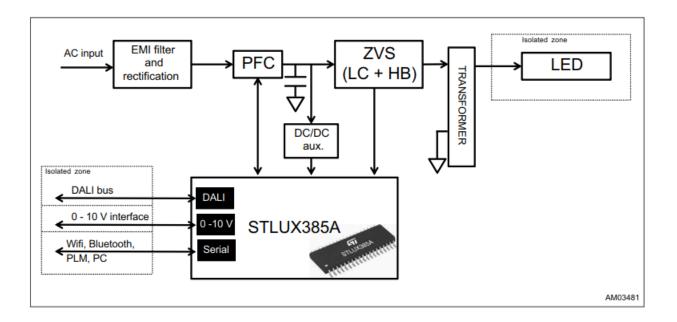
Table 1. Device summary

Part number	Package
STLUX385A, STLUX383A	TSSOP38
STLUX325A	VFQFPN32
STLUX285A	TSSOP28



Example: 100 W LED street lighting application using STLUX385A

- The application consists of a PFC regulator followed by a zero voltage switching (ZVS) LC resonant stage
- LED current is adjusted using a primary side regulation (PSR) control technique



References

- Simone Buso, Paolo Mattavelli, Digital Control in Power Electronics, Second Edition, Morgan& Claypool Publishers, 2015
- Tobias Geyer, Model Predictive Control of High Power Converters and Industrial Drives, Wiley, 2016.
- C. Buccella, C. Cecati, and H. Latafat, Digital Control of Power Converters A Survey, IEEE Transactions on Industrial Informatics, Vol. 8, No. 3, August 2012, pp. 437–447.
- StMicroelectronics, AN4461, Application note, 100 W LED street lighting application using STLUX385A

Current Limiting

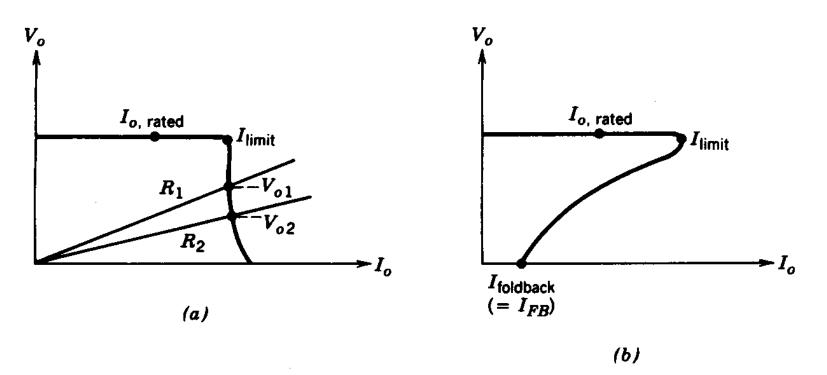
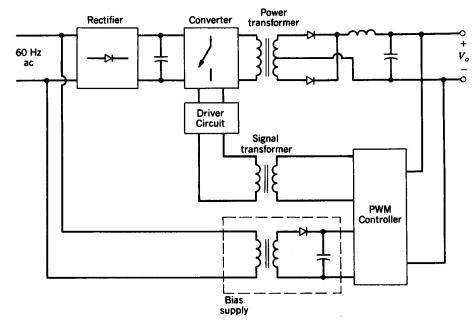


Figure 10-33 Current limiting: (a) constant current limiting; (b) foldback current limiting.

• Two options are shown

Implementing Electrical Isolation in the Feedback Loop



(a)

• Two ways are shown

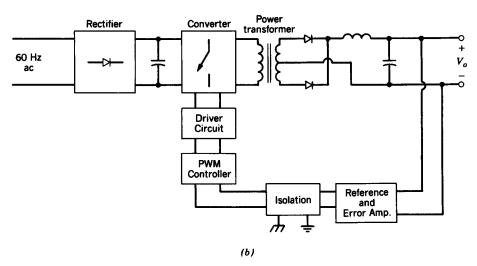


Figure 10-34 Electrical isolation in the feedback loop: (a) secondary-side control; (b) primary-side control.

Chapter 10 Switching DC Power Supplies

Implementing Electrical Isolation in the Feedback Loop

R.F. Transformer Coupled Feedback

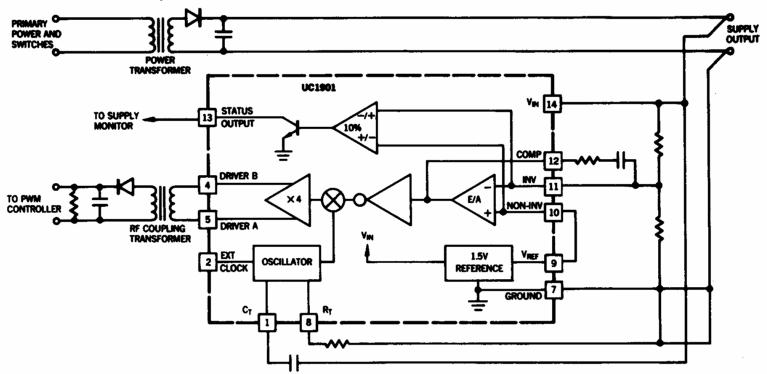


Figure 10-35 Isolated feedback generator UC1901. (Courtesy of Unitrode Integrated Circuits Corp.)

• A dedicated IC for this application is available

Chapter 10 Switching DC Power Supplies

Input Filter

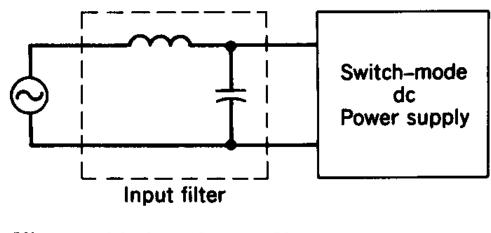


Figure 10-36 Input filter.

Needed to comply with the EMI and harmonic limits

Chapter 10 Switching DC Power Supplies

ESR of the Output Capacitor

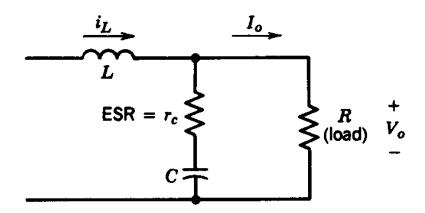


Figure 10-37 ESR in the output capacitor.

• ESR often dictates the peak-peak voltage ripple

Chapter 10 Switching DC Power Supplies