## Chapter 7

## DC-DC Switch-Mode Converters

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- dc-dc converters for switch-mode dc power supplies and dc-motor drives


## Block Diagram of DC-DC Converters



Figure 7-1 A dc-dc converter system.

- Functional block diagram


## Stepping Down a DC Voltage



(b)

Figure 7-2 Switch-mode dc-dc conversion.

- A simple approach that shows the evolution


## Pulse-Width Modulation in DC-DC Converters <br>  <br> (a)


(b)

Figure 7-3 Pulse-width modulator: (a) block diagram; (b) comparator signals.

- Role of PWM


## Duty cycle

- Switching frequency given by the sawtooth
- Control voltage $u_{\text {control }}$ is the diffence between reference and measured voltages amplified by the controller
- $u_{\text {control }}$ gives the on-time of the switch $t_{\text {on }}$
- Relative on time, duty cycle

$$
D=\frac{t_{o n}}{T_{s}}=\frac{u_{o h j}}{\hat{U}_{s t}}
$$

## Step-Down DC-DC Converter

- Pulsating input to the lowpass filter
- The larger the difference between the corner frequency of the filter $f_{c}$ and switching frequency $f s$ the better filtering is


(b)


Figure 7-4 Step-down dc-dc converter.

## Output voltage

- In continuous conduction mode (CCM)

$$
V_{\mathrm{o}}=\frac{1}{T_{\mathrm{s}}} \int_{0}^{T_{\mathrm{s}}} v_{\mathrm{o}}\left(\hat{\mathrm{t}} \mathrm{~d} t=\frac{t_{\mathrm{on}}}{T_{\mathrm{s}}} V_{\mathrm{d}}=D V_{\mathrm{d}}\right.
$$

- Output depends linearly from the
- Duty cycle
- and therefore also from control voltage $u_{\text {control }}$
- Discontinuous conduction mode (DCM)
- Will be discussed in next slides
- Also output current (power) has an effect on voltage average
- Nonlinear dependence on the duty cycle


## Step-Down DC-DC Converter: Waveforms



Figure 7-5 Step-down converter circuit states (assuming $i_{L}$ flows continuously): (a) switch on; (b) switch off.

- Steady state; inductor current flows continuously


## Continuous Conduction Mode, CCM

- Voltage integral over the inductor must be zero at steady state

$$
\begin{gathered}
\left(V_{\mathrm{d}}-V_{\mathrm{o}}\right) t_{\mathrm{on}}=V_{\mathrm{o}}\left(T_{\mathrm{s}}-t_{\mathrm{on}}\right) \\
\quad \Rightarrow \frac{V_{\mathrm{o}}}{V_{\mathrm{d}}}=\frac{t_{\mathrm{on}}}{T_{\mathrm{s}}}=D
\end{gathered}
$$

- If there are no losses in the converter

$$
V_{\mathrm{d}} I_{\mathrm{d}}=V_{\mathrm{o}} I_{\mathrm{o}} \quad \Rightarrow \frac{I_{\mathrm{o}}}{I_{\mathrm{d}}}=\frac{V_{\mathrm{d}}}{V_{\mathrm{o}}}=\frac{1}{D}
$$

## "Transformer"

- In CCM operates like transformer without galvanic isolation
- Duty cycle D acts like a portabless turns ratio in a transformer, changes from 0 to 1
- Input current $i_{d}$ changes also as the switch operates
- In many cases filtering needed in the input side too, depends on the supplying voltage source


## Filtering

- Filter inductance ensures that output is current source as input normally is voltage source
- Filter corner frequency $f_{\mathrm{C}}$
- Selected to be much smaller than $f_{s}$


Figure 7-4 Step-down dc-dc converter.

## Step-Down DC-DC Converter: Waveforms at the boundary of Cont./Discont. Conduction



Figure 7-6 Current at the boundary of continuous-discontinuous conduction: (a) current waveform; (b) $I_{L B}$ versus $D$ keeping $V_{d}$ constant.

- Critical current below which inductor current becomes discontinuous

$$
I_{L B}=\frac{i_{\text {Lpeak }}}{2}=\frac{U_{d}-U_{o}}{2 L} t_{o n}=\frac{D T_{S}}{2 L}\left(U_{d}-U_{o}\right)=I_{o B}
$$

## Step-Down DC-DC Converter: Discontinuous Conduction Mode



Figure 7-7 Discontinuous conduction in step-down converter.

- Steady state; inductor current discontinuous


## DCM with constant Ud

- Typical in motor drives
- Replacing $U_{o}=D U_{d} w e$ end up to

$$
I_{L B}=\frac{D U_{d} T_{s}}{2 L}(1-D)=I_{o B}
$$

- Highest output current for $C C M$ needed at $D=$ 0,5


## $U_{o} / U_{d}$ in DCM with constant Ud

- Voltage integral over the inductor

$$
\left(U_{d}-U_{o}\right) D T_{s}-U_{o} \Delta_{1} T_{s}=0 \Rightarrow \frac{U_{o}}{U_{d}}=\frac{D}{D+\Delta_{1}}
$$

- Average of output current

$$
\begin{aligned}
& I_{o}=i_{\text {Lpeak }} \frac{D+\Delta_{1}}{2}=\frac{U_{o}}{L} \Delta_{1} T_{s} \frac{D+\Delta_{1}}{2}=\frac{U_{d} T_{s}}{2 L} D \Delta_{1} \\
& \Rightarrow \Delta_{1}=\frac{1}{4} \frac{I_{o}}{I_{L B \max } D}
\end{aligned}
$$

- Replacing we obtain

$$
\frac{U_{o}}{U_{d}}=\frac{D^{2}}{D^{2}+\frac{1}{4} \frac{I_{o}}{I_{L B \max }}}
$$

## Step-Down DC-DC Converter: Limits of Cont./Discont. Conduction



Figure 7-8 Step-down converter characteristics keeping $V_{d}$ constant.

- The duty-ratio of 0.5 has the highest value of the critical current


## DCM with constant $U_{\text {。 }}$

- Typical in power supplies
- Replacing $U_{d}=U_{o} / D$

$$
I_{L B}=\frac{i_{\text {Lpeak }}}{2}=\frac{U_{d}-U_{o}}{2 L} t_{o n}=\frac{T_{s} U_{o}}{2 L}(1-D)
$$

- Maximum required current for CCM when
$D=0$

$$
I_{L B \max }=\frac{T_{s} U_{o}}{2 L}
$$

## Duty cycle in DCM with constant $U_{\text {o }}$

- Using previous equations

$$
\frac{U_{o}}{U_{d}}=\frac{D}{D+\Delta_{1}}=\frac{D}{D+\frac{2 L I_{o}}{U_{d} D T_{s}}}=\frac{D}{D+\frac{U_{o}}{U_{d} D} \frac{I_{o}}{I_{L B \max }}} \Leftrightarrow \frac{U_{o}}{U_{d}}\left(D^{2}+\frac{U_{o}}{U_{d}} \frac{I_{o}}{I_{L B \max }}\right)=D^{2}
$$

- Duty cycle is $D=\frac{U_{o}}{U_{d}} \sqrt{\frac{I_{o} / I_{L B \max }}{1-U_{o} / U_{d}}}$
- Ratio of voltages is $\frac{U_{o}}{U_{d}}=\frac{-D^{2}+D \sqrt{D^{2}+4 I_{o} / I_{L B \max }}}{2 I_{o} / I_{L B \max }}$
nonlinear


## Step-Down DC-DC Converter: Limits of Cont./Discont. Conduction



Figure 7-9 Step-down converter characteristics keeping $V_{o}$ constant.

- Output voltage is kept constant


## Step-Down Conv.: Output Voltage Ripple



Figure 7-10 Output voltage ripple in a step-down converter.

- ESR is assumed to be zero


## Ripple (sykkeisyys in Finnish)

- Peak-to-peak

$$
\begin{gathered}
\Delta U_{o}=\frac{\Delta Q}{C}=\frac{1}{2 C} \frac{\Delta I_{L}}{2} \frac{T_{s}}{2} \quad \Delta I_{L}=\frac{U_{o}}{L} t_{\text {off }}=\frac{U_{o}}{L}\left(T_{s}-t_{o n}\right)=\frac{U_{o}}{L}(1-D) T_{s} \\
\frac{\Delta U_{o}}{U_{o}}=\frac{1}{8} \frac{T_{s}^{2}}{L C}(1-D)=\frac{\pi^{2}}{2}(1-D)\left(\frac{f_{c}}{f_{s}}\right)^{2} \text { jossa } f_{c}=\frac{1}{2 \pi \sqrt{L C}}
\end{gathered}
$$

- Ripple reduces when $f_{\mathrm{C}}$ « $f_{\mathrm{s}}$
- Compare to Fig 7-4
- In CCM doesn't depend on output power
- Allowed ripple often $<1 \%$
$-u_{0}(t)=U_{0}$ can be assumed to be constant


## Step-Up DC-DC Converter (Boost)



Figure 7-11 Step-up dc-dc converter.

- Output voltage must be greater than the input


## Step-Up DC-DC Converter Waveforms

$$
U_{d} t_{o n}+\left(U_{d}-U_{o}\right) t_{\text {off }}=0 \Leftrightarrow \frac{U_{o}}{U_{d}}=\frac{T_{s}}{t_{o f f}}=\frac{1}{1-D}
$$


$U_{d} I_{d}=U_{o} I_{o} \Leftrightarrow \frac{I_{o}}{I_{d}}=\frac{U_{d}}{U_{o}}=1-D$


Figure 7-12 Continuous-conduction mode: (a) switch on; (b) switch off.

- Continuous current conduction mode


## Step-Up DC-DC Converter: Limits of Cont./Discont. Conduction, constant Uo



Figure 7-13 Step-up dc-dc converter at the boundary of continuous-discontinuous conduction.
$I_{L B}=\frac{i_{\text {Lpeak }}}{2}=\frac{U_{d}}{2 L} t_{o n}=\frac{T_{s} U_{o}}{2 L} D(1-D) \quad I_{o B}=I_{L B}(1-D)=\frac{T_{s} U_{o}}{2 L} D(1-D)^{2}$

- Note $I_{\mathrm{L}} \neq I_{0}$

$$
I_{o B \max }=\frac{2}{27} \frac{T_{S} U_{O}}{L}
$$

## Step-Up DC-DC Converter: DCM, constant Uo



Figure 7-14 Step-up converter waveforms: (a) at the boundary of continuousdiscontinuous conduction; (b) at discontinuous conduction.

$$
\begin{gathered}
U_{d} D T_{s}+\left(U_{d}-U_{o}\right) \Delta_{1} T_{s}=0 \Leftrightarrow \frac{U_{o}}{U_{d}}=\frac{D+\Delta_{1}}{\Delta_{1}} \quad \frac{I_{o}}{I_{d}}=\frac{\Delta_{1}}{D+\Delta_{1}} \\
I_{d}=\frac{U_{d}}{2 L} D T_{s} \frac{D+\Delta_{1}}{1} \text { and } I_{o}=\frac{T_{s} U_{d}}{2 L} D \Delta_{1} \quad D=\frac{U_{o}}{U_{d}} \Delta_{1}-\Delta_{1}=\sqrt{\frac{4}{27} \frac{U_{o}}{U_{d}}\left(\frac{U_{o}}{U_{d}}-1\right) \frac{I_{o}}{I_{o B \max }}}
\end{gathered}
$$

## Step-Up DC-DC Converter: Limits of Cont./Discont. Conduction



Figure 7-15 Step-up converter characteristics keeping $V_{o}$ constant.

- The output voltage is held constant


## Step-Up DC-DC Converter: Effect of Parasitics



Figure 7-16 Effect of parasitic elements on voltage conversion ratio (step-up converter).

- The duty-ratio is generally limited before the parasitic effects become significant


## Effect of Parasitics

- Especially nonidealities, i.e. resistive losses in L and C
- At large D, peak value of current is large when compared to the average value
- Major part of input voltage is needed to overcome the voltage drop of parasitics
- Losses are increasing too
- Output voltage ripple and capacitor current increase too


## Effect of Parasitics, $U_{0} / U_{d}$

- Losses of inductor and capacitor, $r_{\mathrm{L}}, r_{\mathrm{c}}$
- Resistors are percentages of load resistor $R$
- It can be shown that

$$
\frac{U_{o}}{U_{d}}=\frac{1}{1-D} \frac{(1-D)^{2} R}{R^{\prime}} \text { where } R^{\prime}=r_{L}+\frac{R r_{C}}{R+r_{C}}(1-D)+\frac{R^{2}(1-D)^{2}}{R+r_{C}}
$$

- Analysis of transfer functions in Chapter 10 gives this


## $U_{\mathrm{o}} / U_{\mathrm{d}}$ when parasitics are $1 \%$



## Effect of Parasitics, $U_{0} / U_{d}$

- Up to $D \approx 0,6$ ideal and practical curve match well
- Maximum increase is about 4,7 with $1 \%$ resistors
- In practice always less than 10, i.e. infinite gain impossible
- E.g. operating at $D \approx 0,88$
- Supply voltage reduces
- Feedback increases $D$ in order to increas $U_{0}$ Because of parasitics $U_{0}$ actually drops and finally $D=1$ and short circuit
- $D$ needs to be limited to some practical value


## Efficiency with parasitics

- For simplicity parasitics of $L$ considered only

$$
\frac{U_{o}}{U_{d}}=\frac{1-D}{R / r_{L}+(1-D)^{2}}=\frac{1-D}{\alpha+(1-D)^{2}}, \quad \alpha=\frac{R}{r_{L}}=r_{L}+R(1-D)^{2}
$$

## Efficiency with parasitics

- Only inductor losses $\quad \eta=\frac{P_{o}}{P_{o}+P_{r_{L}}}=\frac{U_{o}^{2} / R}{U_{o}^{2} / R+I_{L}^{2} r_{L}}=\frac{1}{1+\alpha /(1-D)^{2}}, \quad \alpha=\frac{R}{r_{L}}$
- The higher $D$ the smaller efficiency is



## Step-Up DC-DC Converter Output Ripple

- ESR is assumed to be zero

- Inductance has no effect as it is in the input side


Figure 7-17 Step-up converter output voltage ripple.

$$
\Delta U_{o}=\frac{\Delta Q}{C}=\frac{I_{o} D T_{s}}{C}=\frac{U_{o}}{R} \frac{D T_{s}}{C} \Rightarrow \frac{\Delta U_{o}}{U_{o}}=\frac{D T_{s}}{R C}=\frac{D T_{s}}{\tau}
$$

## Step-Down/Up DC-DC Converter, Buck-Boost

- How to derive Buck-Boost
- Series connection of Buck and Boost

- Buck, K2 is always open
- Boost, K1 is always closed


## Common current source

- Same current source, Boost turned around
- D1 needs to be removed, it only shortcircuits the current source



## Buck-Boost

- K2 can be removed too, it is only shortcircuiting current source



## Step-Down/Up DC-DC Converter, BuckBoost



Figure 7-18 Buck-boost converter.

- The output voltage can be higher or lower than the input voltage


## Step-Up DC-DC Converter: Waveforms

$$
\begin{aligned}
& U_{d} t_{o n}-U_{o} t_{o f f}=0 \\
& \Leftrightarrow U_{d} D T_{s}-U_{o}(1-D) T_{s}=0 \\
& \Rightarrow \frac{U_{o}}{U_{d}}=\frac{D}{1-D} \quad \text { ja } \frac{I_{o}}{I_{d}}=\frac{1-D}{D}
\end{aligned}
$$




(a)

(b)

Figure 7-19 Buck-boost converter ( $i_{L}>0$ ): (a) switch on; (b) switch off.

- CCM, Continuous conduction mode


## Step-Up DC-DC Converter: Limits of Cont./Discont. Conduction, Vo constant

$I_{L B}=\frac{i_{\text {Lpeak }}}{2}=\frac{U_{d}}{2 L} D T_{S}=\frac{T_{s} U_{o}}{2 L}(1-D)$
$I_{o}=I_{L}-I_{d}$ ja $I_{o B}=I_{L B}-I_{d B}=(1-D) I_{L B}=\frac{T_{s} U_{o}}{2 L}(1-D)^{2}$
$I_{o B \max }=\frac{T_{s} U_{o}}{2 L}$


(b)

Figure 7-20 Buck-boost converter: boundary of continuous-discontinuous conduction.

## Step-Up DC-DC Converter: Discontinuous Conduction Mode, Vo constant

$$
\begin{gathered}
U_{d} D T_{S}-U_{o} \Delta_{1} T_{S}=0 \Leftrightarrow \frac{U_{o}}{U_{d}}=\frac{D}{\Delta_{1}} \mathrm{ja} \frac{I_{o}}{I_{d}}=\frac{\Delta_{1}}{D} \\
I_{L}=\frac{U_{d}}{2 L} D T_{S} \frac{D+\Delta_{1}}{1}
\end{gathered}
$$

Figure 7-21 Buck-boost converter waveforms in a discontinuous-conduction mode.

$$
I_{d}=I_{o}=\frac{U_{o}}{2 L} \Delta_{1} T_{s} \frac{\Delta_{1} T_{s}}{T_{s}} \Rightarrow \Delta_{1}=\sqrt{\frac{2 L}{U_{o} T_{s}} I_{o}} \text { eli } D=\frac{U_{o}}{U_{d}} \Delta_{1}=\frac{U_{o}}{U_{d}} \sqrt{\frac{I_{o}}{I_{o B \max }}}
$$

## Step-Up DC-DC Converter: Limits of Cont./Discont. Conduction



Figure 7-22 Buck-boost converter characteristics keeping $V_{o}$ constant.

- The output voltage is held constant


## Buck-Boost: Effect of Parasitics



Figure 7-23 Effect of parasitic elements on the voltage conversion ratio in a buck-boost converter.

- The duty ratio is limited to avoid these parasitic effects from becoming significant, same as boost


## Step-Up DC-DC Converter: Output Voltage Ripple

- ESR is assumed to be zero
-Inductance not part of ripple equation
- Note $I_{L} \neq I_{0} \neq I_{0}$



Figure 7-24 Output voltage ripple in a buck-boost converter.

$$
\Delta U_{o}=\frac{\Delta Q}{C}=\frac{I_{o} D T_{S}}{C}=\frac{U_{o}}{R} \frac{D T_{s}}{C} \Rightarrow \frac{\Delta U_{o}}{U_{o}}=\frac{D T_{s}}{R C}=\frac{D T_{S}}{\tau}
$$

## Cuk DC-DC Converter



Figure 7-25 Cúk converter.

- Name is based on the family name of the invertor
-The output voltage can be higher or lower than the input voltage, dualism with buck-boost
- Capacitor C1 acts as intermediate energy storage


## Cuk DC-DC Converter: Waveforms

## - The capacitor voltage is assumed constant

\[

\]

$$
\stackrel{c|c| c}{v_{L 2}} \stackrel{v_{1}}{\left(V_{C 1}-V_{o}\right)}
$$

Figure 7-26 Cúk converter waveforms: (a) switch off; (b) switch on.

$$
\begin{aligned}
& U_{d} D T_{s}+\left(U_{d}-U_{C 1}\right)(1-D) T_{s}=0 \Rightarrow U_{C 1}=\frac{U_{d}}{1-D} \\
& \left(U_{C 1}-U_{o}\right) D T_{s}+\left(-U_{o}\right)(1-D) T_{s}=0 \Rightarrow U_{C 1}=\frac{U_{o}}{D} \\
& \frac{U_{o}}{U_{d}}=\frac{D}{1-D} \quad \frac{I_{o}}{I_{d}}=\frac{U_{d}}{U_{o}}=\frac{1-D}{D}
\end{aligned}
$$

## Sepic Converter

- Single-Ended Primary Inductance Converter
- $L_{1}$ :n correspons to Boost
- $L_{2}:$ n correspons to Buck-Boost



## Sepic comparison

- Advantages
- Continuous input current as in boost
- Step up is possible
- L1 and L2 can be connected magnetically, possible to compensate ripple in one of the inductors
- Reduces start and short-circuit currents
- Galvanic isolation by adding a second winding to L2
- Disadvantage
- Switch and diode current and voltage higher than in Boost


## Sepic, voltages

## - Similar to Cúk

$$
\begin{aligned}
& L_{1} \quad U_{d} D=(1-D)\left(U_{o}+U_{C 1}-U_{d}\right) \\
& L_{2} \quad U_{C 1} D=(1-D) U_{o}=0 \Rightarrow U_{C 1}=\frac{1-D}{D} U_{o} \\
& U_{d} D=(1-D)\left(U_{o}+\frac{1-D}{D} U_{o}-U_{d}\right) \Rightarrow \frac{U_{o}}{U_{d}}=\frac{D}{1-D}
\end{aligned}
$$

## Sepic, currents

- In steady state average current of C1 needs to be zero

$$
i_{C 1}=i_{L 2} \quad \text { Switch conducts }
$$

$$
i_{C 1}=i_{L 1} \quad \text { Switch is not conducting }
$$

$$
\Rightarrow I_{L 2} D=(1-D) I_{L 1}=(1-D) I_{d} \Rightarrow I_{L 2}=\frac{1-D}{D} I_{d}=I_{o}
$$

- $I_{\mathrm{L} 1}=I_{\mathrm{d}}$ and $I_{\mathrm{L} 2}=I_{\mathrm{o}}$


## Full-bridge dc-dc converter

- Converter for UPS and DC-Motor Drives
- Either dc or ac output depending on modulation
- Four quadrant operation is possible



Figure 7-27 Full-bridge dc-dc converter.

## Bi-polar voltage switching

- Switches
- TA+ and TB-
- TA- and TB+
-Are controlled simultaneously
-Output changes between $+U_{\mathrm{d}}: \mathrm{n}$ and $-\mathrm{U}_{\mathrm{d}}: \mathrm{n}$


Figure 7-28 PWM with bipolar voltage switching.

## Control of output

- Triangle

$$
u_{t r i}=\hat{U}_{t r i} \frac{t}{T_{s} / 4} \quad 0<t<T_{s} / 4 \quad t_{1}=\frac{u_{\text {control }}}{\hat{U}_{\text {tri }}} \frac{T_{s}}{4}
$$

- On time of $\mathrm{T}_{\mathrm{A}_{+}}$and $\mathrm{T}_{\mathrm{B}-}$

$$
t_{\text {on }}=2 t_{1}+\frac{T_{s}}{2} \quad D_{1}=\frac{t_{o n}}{T_{s}}=\frac{1}{2}\left(1+\frac{u_{\text {control }}}{\hat{U}_{\text {tri }}}\right)
$$

- On time of $\mathrm{T}_{\mathrm{A}_{-}}$and $\mathrm{T}_{\mathrm{B}_{+}}$

$$
\begin{aligned}
D_{2} & =1-D_{1} \\
U_{o} & =U_{A N}-U_{B N}=\left(D_{1}-D_{2}\right) U_{d}=\left(2 D_{1}-1\right) U_{d} \\
& =\frac{U_{d}}{\hat{U}_{\text {tri }}} u_{\text {control }}=\text { Const. } \times u_{\text {control }}
\end{aligned}
$$

## Uni-polar voltage switching

## -Two control voltages with opposite signs

$$
\begin{gathered}
D_{1}=\frac{t_{\text {on }}}{T_{s}}=\frac{1}{2}\left(1+\frac{u_{\text {control }}}{\hat{U}_{\text {tri }}}\right) \\
U_{o}=\left(2 D_{1}-1\right) U_{d}=\frac{U_{d}}{\hat{U}_{\text {tri }}} u_{\text {control }}=\text { vakio } \times u_{\text {control }^{\text {coo }}}
\end{gathered}
$$



Figure 7-29 PWM with unipolar voltage switching.

## RMS of output voltage

- Bipolar

$$
U_{o, r m s}=\sqrt{\frac{1}{T_{s}} \int_{0}^{T_{s}} u_{o}^{2} d t}=U_{d}
$$

$$
U_{r, r m s}=\sqrt{U_{o, r m s}^{2}-U_{o}^{2}}=U_{d} \sqrt{1-\left(2 D_{1}-1\right)^{2}}=2 U_{d} \sqrt{D_{1}-D_{1}^{2}}
$$

- Unipolar

$$
\begin{gathered}
U_{o, r m s}=\sqrt{\frac{1}{T_{s}} \int_{0}^{T_{s}} u_{o}^{2} d t}=\sqrt{\frac{4 t_{1}}{T_{s}} U_{d}^{2}}=U_{d} \sqrt{\frac{u_{\text {control }}}{\hat{U}_{\text {tri }}}}=U_{d} \sqrt{2 D_{1}-1} \\
U_{r, r m s}=\sqrt{U_{o, r m s}^{2}-U_{o}^{2}}=U_{d} \sqrt{\left(2 D_{1}-1\right)-\left(2 D_{1}-1\right)^{2}}=U_{d} \sqrt{6 D_{1}-4 D_{1}^{2}-2}
\end{gathered}
$$

## Output Ripple in Converters for DCMotor Drives



Figure 7-30 $\quad V_{r, \text { rms }}$ in a full-bridge converter using PWM:
(a) with bipolar voltage switching; (b) with unipolar voltage switching.

- bi-polar and uni-polar voltage switching


## Switch Utilization in DC-DC Converters

- Ratio
- $U_{T}=$ maximum voltage over switch
- $I_{T}=$ maximum current

$$
\frac{P_{o}}{P_{T}}=\frac{U_{o} I_{o}}{U_{T} I_{T}}
$$

- It varies significantly in various converters


Figure 7-31 Switch utilization in dc-dc converters.

## Equivalent Circuits in DC-DC Converters

- replacing inductors and capacitors by current and voltage sources, respectively

(a)
(b)


(c)

(d)

(e)

Figure 7-32 Converter equivalent circuits: (a) step-down; (b) step-up; (c) step-down/step-up; (d) Cúk; (e) full-bridge.

## Reversing the Power Flow in DC-DC Conv.



Figure 7-33 Reversible power flow with reversible direction of the output current $\boldsymbol{i}_{\boldsymbol{o}}$.

- For power flow from right to left, the input current direction should also reverse


## Efficiency and losses of power semiconductor devices

- Losses of power semiconductor devices
- Conduction losses
- Switching losses, turn-on and turn-off
- For simplicity following investigation is on Buck converter
- Can be generalized to other converters


## Conduction losses

- On-state voltage drop
- Assumed to be the same in the switch and diode
- Losses

$$
P_{D C}=P_{Q}+P_{D}=U_{o n} I_{o} \frac{t_{o n}}{T_{s}}+U_{o n} I_{o} \frac{t_{o f f}}{T_{s}}=U_{o n} I_{o}, \quad t_{o n}+t_{o f f}=T_{s}
$$

- Efficiency

$$
\eta=\frac{P_{o}}{P_{o}+P_{D C}}=\frac{U_{o} I_{o}}{U_{o} I_{o}+U_{o n} I_{o}}=\frac{U_{o}}{U_{o}+U_{o n}}
$$

- Example
$-U_{0}=3 \mathrm{~V}, U_{\text {on }}=0,5 \mathrm{~V}=>\eta=85,7 \%$


## Switching losses

- Infinite rise and fall time when switches are turning on and off
- An ideal switch would
- Conduct full load current immeadiately without voltage drop
- Current and voltage are assumed to changing linearly during turn-on and -off


## Switching inductive current



(b)

## Turning off and on

- T-conducts $I_{0}$ and it is turned off
- Voltage over it increases and when it is $U_{d}$ diode D+ starts to conduct
- Because of parasitic inductances voltage exceeds $U_{d}$
- D+ conducts $I_{0}$ and $T$ - is turned on
- Current inceares and exceeds $I_{0}$ because of diode reverse recovery current
- After recovery of the diode voltage over T- drops to nearly zero


## Switching losses, best case

- It is assumed that voltage and current are changing simultaneously

$$
\begin{aligned}
& P_{o n}=\frac{t_{o n}}{T_{s}} \int_{0}^{t_{o n}} u i d t=\frac{t_{o n}}{T_{s}} \frac{U_{d} I_{o}}{6} \\
& P_{o f f}=\frac{t_{o f f}}{T_{s}} \int_{0}^{t_{o f f}} u i d t=\frac{t_{o f f}}{T_{s}} \frac{U_{d} I_{o}}{6}
\end{aligned}
$$



## Conduction and switching losses, best case

- It is assumed that $t_{\text {on }}=t_{\text {off }} \quad P_{A C}=P_{o n}+P_{\text {off }}=\frac{t_{o n}}{T_{s}} \frac{U_{d} I_{o}}{3}$
- Efficiency including conduction and switching losses
$\eta=\frac{P_{o}}{P_{o}+P_{D C}+P_{A C}}=\frac{U_{o} I_{o}}{U_{o} I_{o}+U_{o n} I_{o}+U_{d} I_{o} t_{o n} / 3 T_{s}}=\frac{U_{o}}{U_{o}+U_{o n}+U_{d} t_{o n} / 3 T_{s}}$
- Example (cont.)
- Lets assume $U_{\mathrm{d}}=48 \mathrm{~V}, U_{o}=3 \mathrm{~V}, U_{\text {on }}=0,5 \mathrm{~V}, t_{\mathrm{s}}=50 \mathrm{kHz}$ and $t_{\mathrm{on}}$ $=t_{\text {off }}=0,3 \mu \mathrm{~s}$ and $T_{\mathrm{s}}=20 \mu \mathrm{~s}$
- With these numbers

$$
\eta=\frac{3}{3+0,5+48 \cdot 0,3 / 3 \cdot 20} 100 \%=80,2 \%
$$

## Conduction and switching losses, worst case

- Current reaches final value before voltage drops

$$
\begin{aligned}
& P_{o n}=\frac{t_{R i}}{T_{s}} \frac{U_{d} I_{o}}{2}+\frac{t_{F U}}{T_{s}} \frac{U_{d} I_{o}}{2} \\
& P_{o f f}=\frac{t_{R U}}{T_{s}} \frac{U_{d} I_{o}}{2}+\frac{t_{F i}}{T_{s}} \frac{U_{d} I_{o}}{2}
\end{aligned}
$$



## Conduction and switching losses, worst case

- $t_{\mathrm{Ri}}=t_{\mathrm{FU}}=t_{\mathrm{RU}}=t_{\mathrm{Fi}}$

$$
\eta=\frac{P_{o}}{P_{o}+P_{D C}+P_{A C}}=\frac{U_{o}}{U_{o}+U_{o n}+2 U_{d} t_{R i} / T_{s}}
$$

- Other values same as before, additionally, $t_{\mathrm{Ri}}=$ 0,3 $\mu \mathrm{s}$

$$
\eta=\frac{3}{3+0,5+2 \cdot 48 \cdot 0,3 / 20} 100 \%=60,7 \%
$$

- Corresponding linear power supply

$$
\eta=\frac{U_{o}}{U_{d}} 100 \%=\frac{3}{48} 100 \%=6,25 \%
$$

