ELEC-E8417 Switched-Mode Power Supplies **Exam 26.5.2016**

Question 1.

In step-up converter (Boost) the output voltage U_o = 48 V and supply voltage changes between 10 V $\leq U_d \leq$ 15 V. Output power $P_o \geq$ 15 W and switching frequency f_s = 330 kHz, C = 47 μF. Calculate the needed inductance so that the operation is always in continuous conduction mode. Calculate the output voltage ripple ΔU_o when this inductance value is used.

Question 2.

In a Flyback-converter turns ratios $N_1{:}N_2$ = 5:1, output voltage U_o = 3 V, supply voltage U_d = 48 V, output power P_o = 60 W and switching frequency f_s = 200 kHz. The magnetizing inductance of the magnetic core is 0,2 mH and converter operates in continuous area, i.e. the magnetization of the core is always higher than zero. Derive equations for the maximum current and voltage ratings of the switch used in the converter and calculate their numerical values

Question 3.

The rectifier of a switched-mode power supply is equipped with an active power factor correction circuit. It has been realized with a single-phase diode bridge and step-up converter. RMS value of the supply voltage is 230 V and frequency 50 Hz. The rectifier is loaded with 1000 W and the dc-voltage is 370 V and filtering capacitor is 100 μF.

a) Draw the equivalent circuit of the rectifier and explain its operating principle shortly.
b) Calculate the ac component in the output dc-voltage of the rectifier. The output current can be assumed to be ideal dc, efficiency of the rectifier 100 % and the switching frequency of the step-up converter large.

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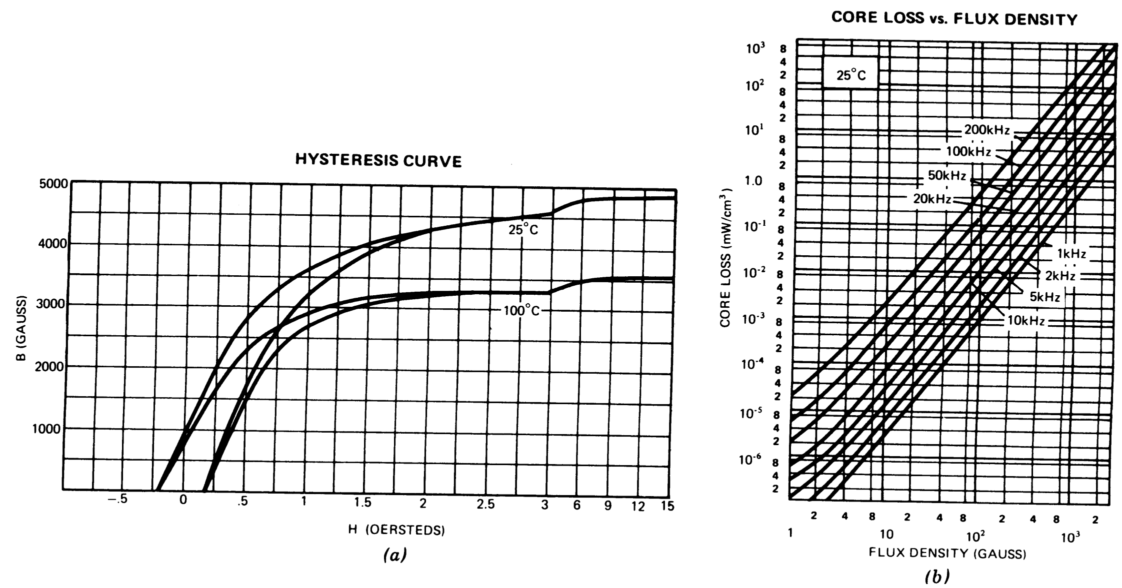
Question 4.

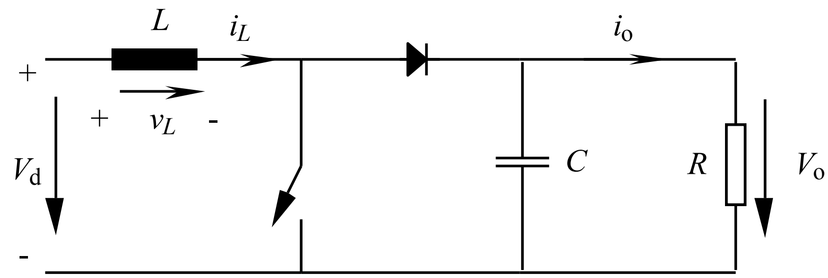
Transformer of a full-bridge converter is built using ferrite material, which magnetizing and loss waveforms are shown below (Örstedt = 1/(4π)·10³ A/m). Supply voltage U_d = 180 V, duty cycle $D = 0{,}5$, f_s = 100 kHz, ΔB_{max} = 0,2 T (Wb/m²) = 2000 Gauss and the measured peak value of the magnetizing current is 0,5 A. Calculate the losses of the transformer core when temperature is 25°C. Inductance of the magnetic circuit can be calculated from $L_m = N^2/R_m = N^2\mu A_C/l_m$ where N is number of turns, μ permeability of the ferrite, A_C surface area of the core, and l_m average length of the magnetic circuit.

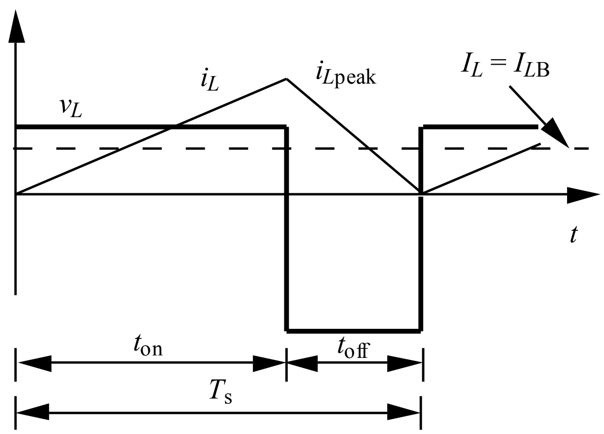


Question 5.

Why feedback control is needed in switched-mode power supplies? What basic principles need to be taken into account when designing feedback control?

QUESTION 1.





In CCM

$$\frac{U_O}{U_d} = \frac{1}{1-D}$$
$$\Rightarrow D = 1 - \frac{U_d}{U_O} = 1 - \frac{15}{48} \ldots 1 - \frac{10}{48} = 0,687 \ldots 0,791 \tag{1.1}$$

Converter is assumed to be lossless

$$U_d I_d = U_O I_O \Leftrightarrow \frac{I_O}{I_d} = \frac{U_d}{U_O} = 1 - D\,, \tag{1.2}$$

At the boundary of CCM and DCM.

$$I_{LB} = \frac{i_{Lpeak}}{2} = \frac{U_d}{2L} t_{ON} = \frac{T_S U_O}{2L} D(1-D) \tag{1.3}$$

Based on this and Eq. (1.2) output current at boundary is

$$I_{OB} = I_{LB}(1-D) = \frac{T_S U_O}{2L} D(1-D)^2\,. \tag{1.4}$$

Duty cycle when this current is at maximum is obtained by derivation.

$$\frac{dI_{OB}}{dD} = \frac{T_S U_O}{2L}\left[(1-D)^2 - 2D(1-D)\right] = 0$$
$$\Rightarrow 3D^2 - 4D + 1 = 0 \Leftrightarrow D = \frac{4 \pm \sqrt{16-12}}{6} = 1 \vee \frac{1}{3} \qquad (1.5)$$

Eli maksimi saadaan ohjaussuhteella 1/3. When I_{oBmax} is equal to the minimum of output current, converter operates always in CCM.

$$I_{OB\max} = \frac{2}{27}\frac{T_S U_O}{L} = I_{O\min} \Leftrightarrow L_{\min} = \frac{2}{27}\frac{T_S U_O}{I_{O\min}} \qquad (1.6)$$

Tarvittavan kuristimen arvo saadaan laskettua lähtövirran minimiarvon avulla. Lähtötietoina on annettu; lähtöteho $P_O \geq 15$ W ja lähtöjännite $U_O = 48$ V => $I_O \geq 15/48$ A.

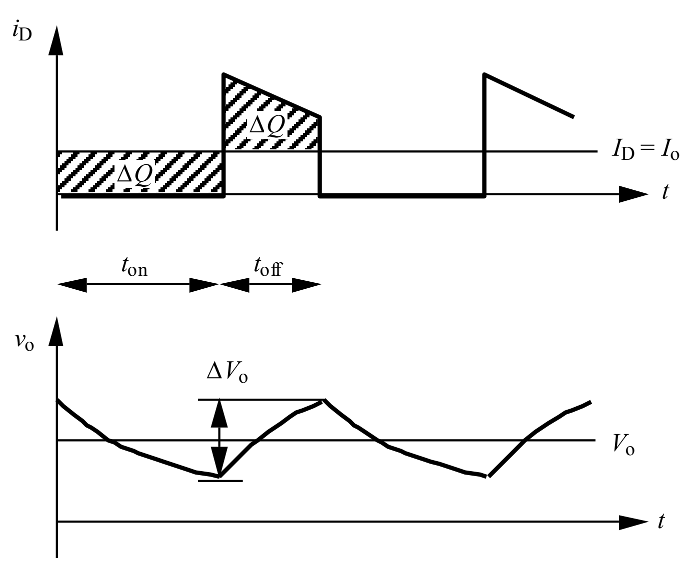
$$L_{\min} = \frac{2}{27}\frac{T_S U_O}{I_{o\min}} = \frac{2}{27}\frac{\frac{1}{300kHz}48V}{5/48V} \approx 0{,}113\text{mH} \qquad (1.7)$$

With the values of the question, L_{min} is 0,0344 mH.

If L is higher than this, operation is always in CCM. However, in this question D = 0,687 …. 0,791 and 1/3 is out from this., Therfore, L can be even smaller than above. With D = 0,687

$$L_{\min} = \frac{T_S U_O}{2I_{OB}} D(1-D)^2 \approx 51{,}6\mu\text{H} \qquad (1.8)$$

The value is 15,6 μH. D = 0,79 would give even a smaller value for L but this cannot be used as operation with other D:s would be in DCM.

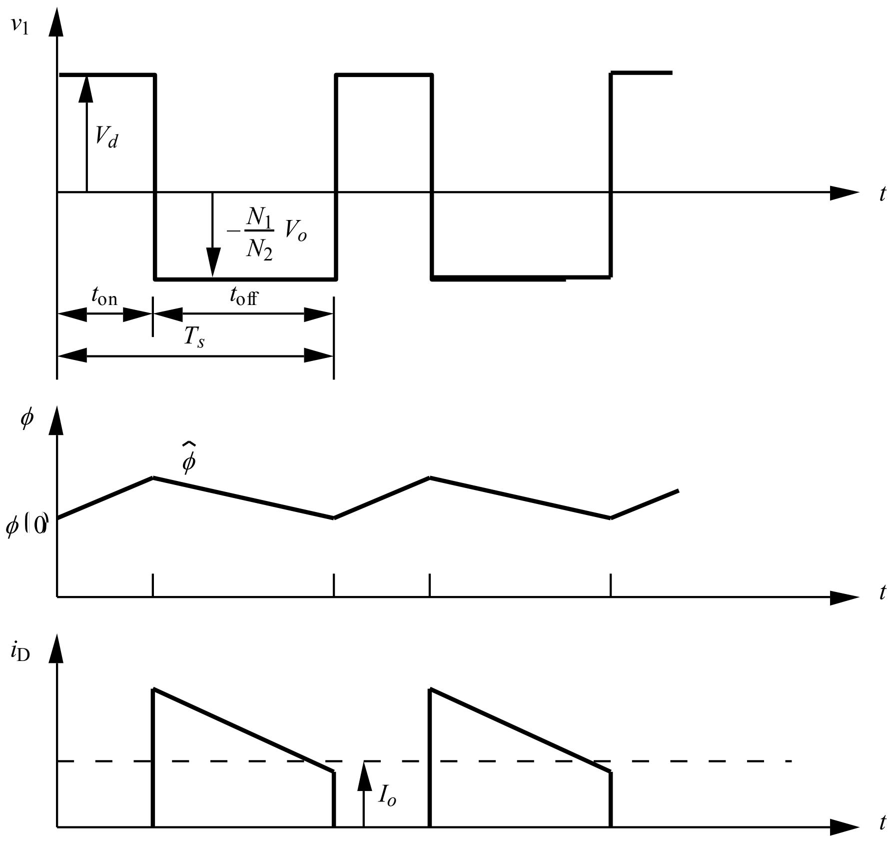


$$\Delta U_O = \frac{\Delta Q}{C} = \frac{I_O D T_S}{C} = \frac{U_O}{R}\frac{D T_S}{C} \tag{1.9}$$

$$\Delta U_O = \frac{\Delta Q}{C} = \frac{I_O D T_S}{C} = \frac{\frac{5}{48}\ \text{A}\ 0{,}79 \frac{1}{300\text{kHz}}}{47\mu\text{F}} = 5{,}8\ \text{mV} \mathrel{\hat{=}} 0{,}12\% \tag{1.10}$$

With the values of the question the result is 15,96 mV, which is equal to 0,03 %.

Question 2
a)



Peak value of the flux is at the end of conduction period

$$\hat{\phi}(t) = \phi(t_{on}) = \phi(0) + \frac{U_d}{N_1}t_{on}$$

In steady state integral of the voltage is zero

$$\phi(T_s) = \hat{\phi} - \frac{U_o}{N_2}(T_s - t_{on}) = \left(\phi(0) + \frac{U_d}{N_2}t_{on} - \frac{U_o}{N_2}(T_s - t_{on})\right) = \phi(0)$$ and therefore $$\frac{U_o}{U_d} = \frac{N_2}{N_1}\frac{D}{1-D}$$

$$\frac{N_2}{N_1} = 1 \Rightarrow D = \frac{U_o}{U_d + U_o} = \frac{1}{1 + U_d/U_o} \approx 0{,}058$$ when the turns ratio is 5/1 the result is 0,238

After turn-on of the switch, current in the primary increases linearly and it is equal to the magnetizing current. Peak value of the current is

$$\hat{I}_m = \hat{I}_{sw} = I_m(0) + \frac{U_d}{L_m}t_{on}$$

When switch is turned off, secondary voltage $-U_o$ is seen in the primary when multiplied by the turns ratio. Current decreases linearly

$$i_m(t) = \hat{I}_m - \frac{U_o(N_1/N_2)}{L_m}(t - t_{on}) \qquad (t_{on} < t < T_s)$$

This could also be used to derive the voltage ratio shown already above. The diode current is equal to the magnetizing current when transferred to the secondary, i.e.

$$i_D(t) = \frac{N_1}{N_2}i_m(t) = \frac{N_1}{N_2}\left[\hat{I}_m - \frac{U_o(N_1/N_2)}{L_m}(t - t_{on})\right] \qquad (t_{on} < t < T_s)$$

Peak value of this current can be calculated when the output current is known. The average value of the diode current is equal to the output current $I_o = P_o/U_o = 20$ A. Based on this we obtain the peak value as

$$\hat{I}_m = \hat{I}_{sw} = \frac{N_2}{N_1}\frac{1}{1-D}I_o + \frac{N_1}{N_2}\frac{(1-D)T_s}{2L_m}U_o \approx 21{,}25\ \text{A} + 70{,}65\ \text{mA} \approx 21{,}32\ \text{A}$$

With the values of the question the result is 5,25 A + 142,8 mA = 5,39 A

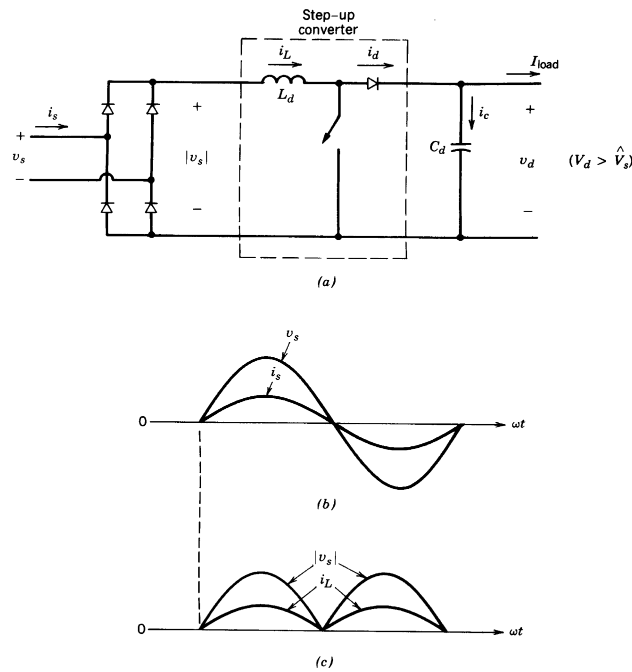
This is obtained by calculating the diode current at the end of a cycle and with help of this calculating the area of I_d shown in the figure. Current comprises of two terms. The first one depends on the output current seen in the primary. The latter part shows the effect of the magnetizing current.

Voltage over the switch, when it is not conducting is

$$u_{sw} = U_d + \frac{N_1}{N_2}U_o = \frac{U_d}{1-D} = 48\ \text{V} + 3\ \text{V} = 51\ \text{V}$$

Question 3.

a) Detailed explanation is in the book



b) In PFC circuit input voltage and current can be assumed to be in phase

$$p_{in} = \sqrt{2}U_s \left|\sin\omega t\right| \left|\sqrt{2}I_s \sin\omega t\right| = U_s I_s - U_s I_s \cos 2\omega t = i_d U_d \qquad (2.1)$$

and further 100 % efficiency has been assumed. DC-current contains a dc-component and an ac component flowing through the capacitor

$$i_d = I_d + i_C = \frac{U_s I_s}{U_d} - \frac{U_s I_s}{U_d}\cos 2\omega t = I_d - I_d \cos 2\omega t \qquad (2.2)$$

Based on this capacitor current is

$$i_C = -\frac{U_s I_s}{U_d}\cos 2\omega t = -I_d \cos 2\omega t \qquad (2.3)$$

and voltage ripple over capacitor can be integrated

$$u_{d,ripple} = \frac{1}{C_d}\int i_C dt = -\frac{I_d}{2\omega C_d}\cos 2\omega t \qquad (2.4)$$

Average value of the dc current is $I_d = \frac{P_d}{U_d} = \frac{1000}{370}$ A and ripple voltage is thus

$$\hat{u}_{d,ripple} = \frac{I_d}{2\omega C_d} \approx 43 \text{ V} \qquad (2.5)$$

Question 4.

Magnetization in full-bridge is two-directional, and $\Delta B_{\max}$ = 0,2 T = 20 000 Gaussia is the highest flux. From the core loss curve at 100 kHz it can be seen that loss density is 400 mW/cm3. We need to calculate the volume of the core $V_m = A_C l_m$ in order to obtain the losse..

When resistive voltage drops are ignored, supply voltage is equal to the flux change

$$U_d = \frac{d\Psi}{dt} = N\frac{d\phi}{dt} = NA_C\frac{dB}{dt} = NA_C\frac{2\Delta B_{\max}}{DT_S} \tag{2.6}$$

Core area is thus

$$A_C = \frac{U_d DT_S}{2N\Delta B_{\max}} = \frac{U_d}{4N\Delta B_{\max} f_s} \tag{2.7}$$

Number of winding turns is still unknown. Magnetizing current

$$\hat{i}_m = \frac{U_d DT_S}{2L_m} \Rightarrow L_m = \frac{U_d DT_S}{2\hat{i}_m} \tag{2.8}$$

Magnetizing inductance depends on core area and length of the flux path

$$L_m = \frac{N^2}{R_m} = \frac{N^2 \mu A_C}{l_m} \tag{2.9}$$

From this we obtain

$$L_m = \frac{U_d DT_S}{2\hat{i}_m} = \frac{N^2 \mu A_C}{l_m} \Rightarrow l_m = \frac{N^2 \mu A_C}{U_d DT_S} 2\hat{i}_m \tag{2.10}$$

Core volume can now be calculated, when we use the core are in (4.2) two times

$$\begin{aligned} V_m = A_C l_m &= \frac{U_d}{4N\Delta B_{\max} f_s}\frac{N^2 \mu A_C}{U_d DT_S} 2\hat{i}_m = NA_C\frac{\mu 2\hat{i}_m}{4\Delta B_{\max} D} \\ &= N\frac{U_d}{4N\Delta B_{\max} f_s}\frac{\mu 2\hat{i}_m}{4\Delta B_{\max} D} = \frac{U_d \mu 2\hat{i}_m}{\left(4\Delta B_{\max}\right)^2 Df_s} \end{aligned}$$

For the numerical calculation, permeability of the ferrite is needed. It can be estimated from the given B-H loop. As flux density is less than 0,2 T operating area is quite linear. We use the higher 25°C slope when the volume and also losses are highest.

Then

$$\mu \approx \frac{2000\mathrm{G}}{(0{,}25 - -0{,}25)\mathrm{örstedt}} = \frac{0{,}2\mathrm{T}}{0{,}5\frac{1}{4\pi}10^{3}\,\mathrm{A/m}} \approx 5{,}0*10^{-3}\,\frac{\mathrm{Vs}}{\mathrm{Am}}$$

core volume

$$V_m \approx \frac{U_d \mu 2\hat{i}_m}{(4\Delta B_{\max})^2 D f_s} = \frac{180*5{,}0*10^{-3}*2*0{,}5}{(4*0{,}2)^2*100*10^3*0{,}5} \approx 28{,}12\mathrm{cm}^3$$

losses

$$P_v = V\,400\mathrm{mW/cm}^3 * 28{,}12\mathrm{cm}^3 \approx 11{,}3\mathrm{W}$$

Question 5.

Some required aspects,.

-Accurate and controlled output voltage with good dynamic although supply voltage or output current changes => tuning of controllers and model of the system
-State-space averaging, some basic description of this, only valid on frequencies below half of the switching frequency
-CCM and DCM area, transfer functions are different
-Input voltage feedforward
-Current mode control, inner current loop is faster than the slower outer voltage loop, limits peak current, automatic input voltage feedforward, transfer functions are simpler than in VMC, different current control methods (constant frequency, hysteresis), slope compensation needed when duty cycle is larger than 0,5

More detailed in the textbook