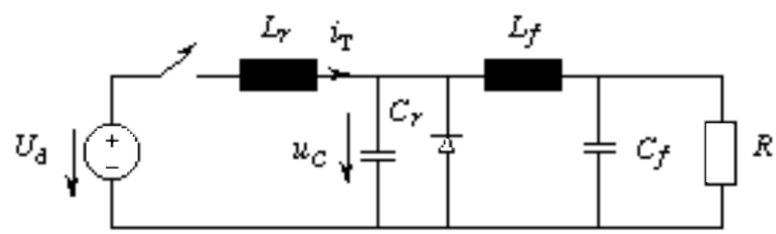
ELEC-E8417 Switched-Mode Power Supplies **Exam 29.5.2019**

1. In a Buck converter output voltage $U_o = 5$ V and supply voltage changes as $10\text{ V} \le U_d \le 15$ V. Output power $P_o \ge 50$ W, switching frequency $f_s = 300$ kHz and output capacitor $C = 47$ µF. Derive equations and calculate the value of the required filtering inductance so that the operation is always at continuous conduction mode. Calculate ripple component in the output voltage.

2. In the Zero Current Switching converter shown below $f_0 = 1$ MHz, $Z_0 = 10\ \Omega$, $P_o = 10$ W, $U_d = 15$ V ja $U_o = 10$ V. At the time of closing the switch inductance L_r has no current, C_r no voltage and the constant load current flows through the diode. Sketch the waveform of the current i_T and capacitor voltage u_C and write their equations in time domain. Calculate the important corner time values of these waveforms. Calculate the maximum values of the current and voltage.



Current and voltage of the resonant circuit with initial values I_{L0} and U_{C0} and output current I_o can be calculated from:

$$i_L = I_o + (I_{L0} - I_o)\cos\omega_0 t + \frac{U_d - U_{C0}}{Z_0}\sin\omega_0 t$$

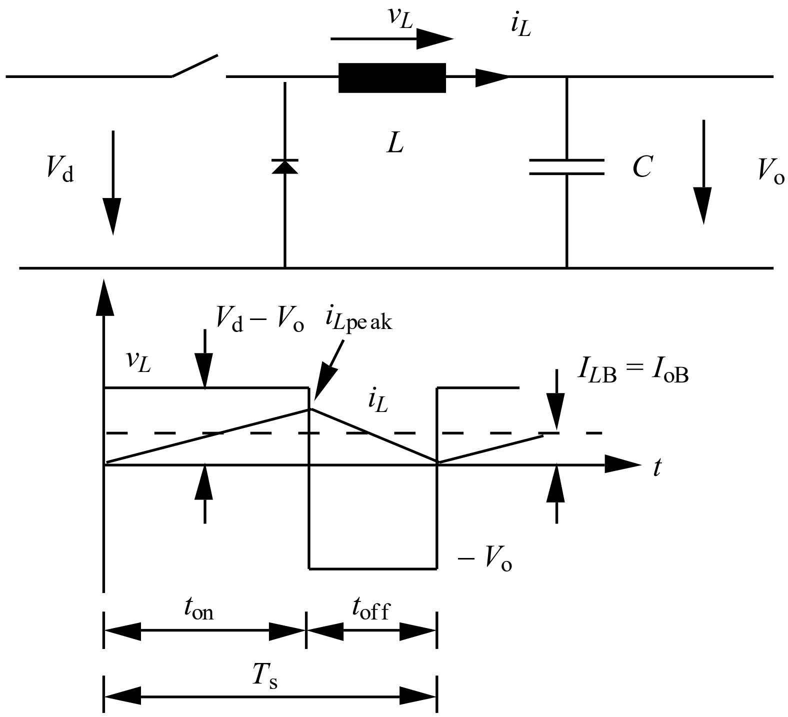
$$u_C = U_d - (U_d - U_{C0})\cos\omega_0 t + Z_0 (I_{L0} - I_o)\sin\omega_0 t \qquad \omega_0 = 2\pi f_0 t = \frac{1}{\sqrt{L_r C_r}} \qquad Z_0 = \sqrt{\frac{L_r}{C_r}}$$

3. In a Flyback-converter turns ratios $N_1{:}N_2 = 5{:}1$, output voltage $U_o = 3$ V, supply voltage $U_d = 48$ V, output power $P_o = 60$ W and switching frequency $f_s = 200$ kHz. The magnetizing inductance of the magnetic core is 0,2 mH and converter operates in continuous area, i.e. the magnetization of the core is always higher than zero. Derive equations for the maximum current and voltage ratings of the switch used in the converter and calculate their numerical values

4. You are designing ac/dc rectifier to be used in a switched-mode power supply. What aspects should be taken into consideration when designing the rectifier?

5. Which factors are having an effect on the efficiency of a switched-mode power supply? How efficiency can be improved?

Question 1.



Waveforms in the borderline between CCM and DCM are shown above. Inductor current is derived form

$$i_L = \frac{1}{L}\int u_L dt \tag{1.1}$$

and when the switch is conducting

$$i_L = \frac{U_d - U_O}{L} t \qquad 0 < t \ < t_{ON} \tag{1.2}$$

A nd in the borderline

$$I_{LB} = \frac{i_{Lpeak}}{2} = \frac{U_d - U_o}{2L} t_{ON} = \frac{1-D}{2L} U_O T_S \tag{1.3}$$

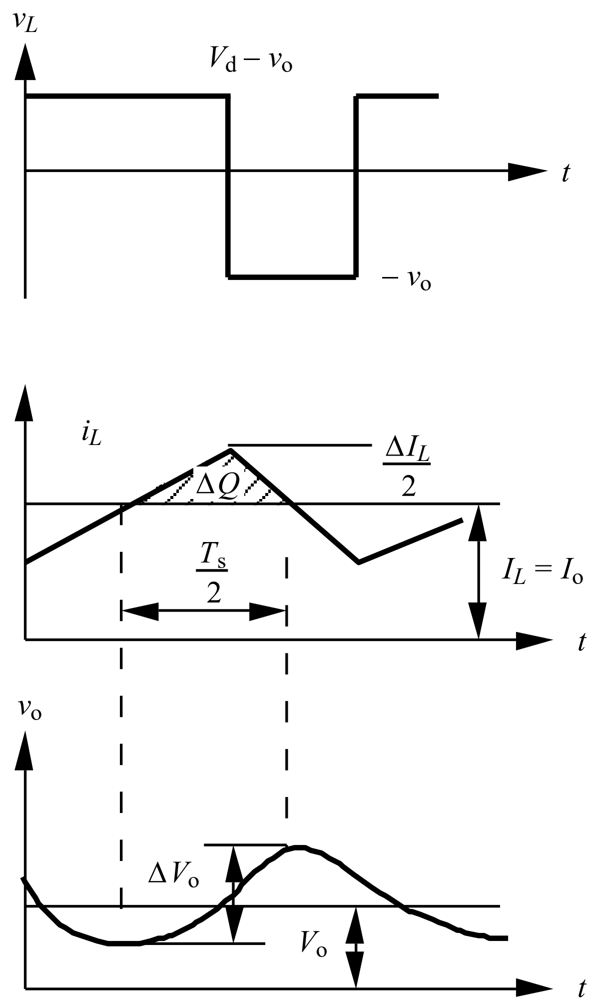
If current average is below this it is discontinuos. For the dimensioning of the inductance

$$L = \frac{1-D}{2I_{LB}} U_O T_S \tag{1.4}$$

The most demanding situation is, when output current I_{LB} is smallest possible and the duty cycle has its minimum value.

$$L_{\min} = \frac{1-D_{\min}}{2I_{LB\min}} U_O T_S = \frac{1-\frac{5}{15}}{2*10} * 5 * \frac{1}{300*10^3} \text{ H} \approx 0,556\mu\text{H} \tag{1.5}$$

Inductance value would be even smaller with the maximum duty cycle but then operation would be DCM when input voltage is greater than 10 V.



Inductor current and output voltage vaweforms are shown above. Output voltage change can be calculated from the change in charge, ΔQ. Based on the area of the triangle

$$\Delta U_O = \frac{\Delta Q}{C} = \frac{1}{2C}\frac{\Delta I_L}{2}\frac{T_S}{2} \tag{2.1}$$

It is best to calculate the current change with output voltage as it remains constant. When the switch is not conducting voltage is $-U_o$ peak-to-peak change in current is

$$\Delta I_L = \frac{U_O}{L}t_{off} = \frac{U_O}{L}(T_S - t_{on}) = \frac{U_o}{L}(1-D)T_S\,, \tag{2.2}$$

and inserted in Eq. (2.1)

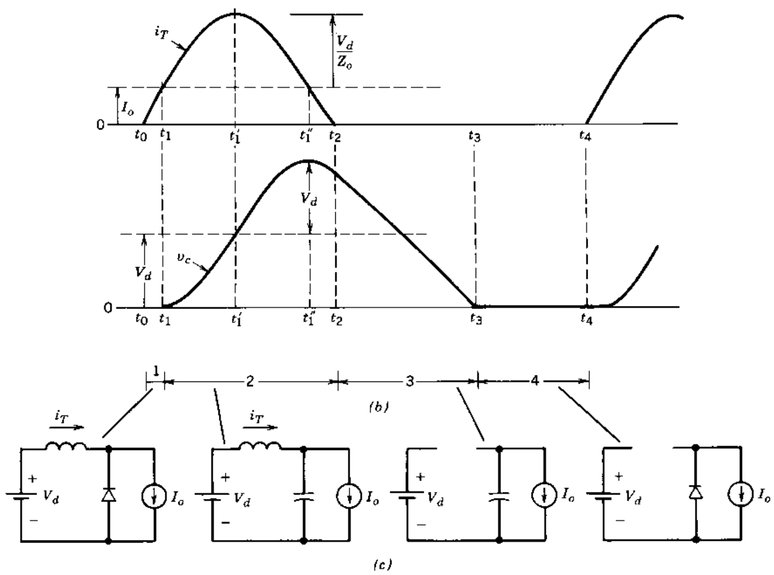
$$\Delta U_O = \frac{T_S}{8C}\frac{U_O}{L}(1-D)T_S = U_O\frac{\pi^2}{2}(1-D)(\frac{f_C}{f_S})^2 \approx 133 \text{ mV} \tag{2.3}$$

When supply voltage is 10 V and with 15 V $\Delta U_O \approx 177$ mV

Question 2

$$Z_0 = \sqrt{\frac{L_r}{C_r}} \Leftrightarrow L = Z_0^2 C_r, \omega_0 = \frac{1}{\sqrt{L_r C_r}} = \frac{1}{Z_0 C_r} \Leftrightarrow C_r = \frac{1}{Z_0 \omega_0} \approx 159\mu\text{F} , L_r \approx 1,59\mu\text{H}$$

Operation of the ZCS circuit has been presented in the textbook and course slides. Inductor current and capacitor voltage waveforms are presented in the figures below.



At time t_0 switch is closed and input voltage U_d is over inductor L_r and its current increases linearly

$$\frac{di_T}{dt} = \frac{U_d}{L_r} \Rightarrow t_1 - t_0 = \frac{L_r}{U_d} I_O \approx 0{,}106\,\mu s$$

As the switch current i_T increases the diode current decreases untill $i_T = I_O$, when diode stops to conduct at t_1. After that, the current of the resonant circuit increases further and the difference $i_T - I_O$ charges capacitor C_r, whose voltage increases as part of the resonant circuit. During the interval $t_2 - t_1$ the given equations

$$i_L(t) = I_0 + (I_{L0} - I_0)\cos\omega_0 t + \frac{U_d - U_{C0}}{Z_O}\sin\omega_O t$$

$$u_C(t) = U_d - (U_d - U_{C0})\cos\omega_O t + Z_O(I_{L0} - I_O)\sin\omega_O t$$

can be used. With the initial values $I_{L0} = I_O$ ja $U_{C0} = 0$

$$i_L(t) = I_0 + \frac{U_d}{Z_O}\sin\omega_O t$$

$$u_C(t) = U_d - U_d\cos\omega_O t$$

Maximum value of current is at $\sin\omega_0 t = \frac{\pi}{2} \Rightarrow t_1' - t_1 = \frac{\pi}{2\omega_0} \approx 0,25\ \mu\text{s}$ and

$$i_{T,\max} = I_0 + \frac{U_d}{Z_O} = 1A + \frac{15}{10}A \approx 2{,}5A$$

Capacitor voltage at $t_1^{'}$

$$u_C(t') = U_d(1 - \cos\frac{\pi}{2}) = U_d = 15V$$

and the maximum value at $\omega_0 t = \pi$

$$u_{C,\max} = U_d(1 - \cos\pi) = 2U_d = 30V$$

when $t_1^{''} - t_1 = \frac{\pi}{\omega_0} \approx 0{,}5\ \mu\text{s}$ and inductor current has reduced to be $i_L(t'') = I_0 + \frac{U_d}{Z_O}\sin\pi = I_0$.

Resonance period $t_2 - t_1$ is obtained from

$$i_T = I_0 + \frac{U_d}{Z_O}\sin\omega_O t = 0 \Leftrightarrow \omega_O t = \arcsin\frac{-I_O Z_O}{U_d}$$

$$\Rightarrow \omega_O t = \arcsin\frac{-10}{15} \approx 0{,}616\,\mu s$$

Inductor current is zero at t_2 and capacitor voltage

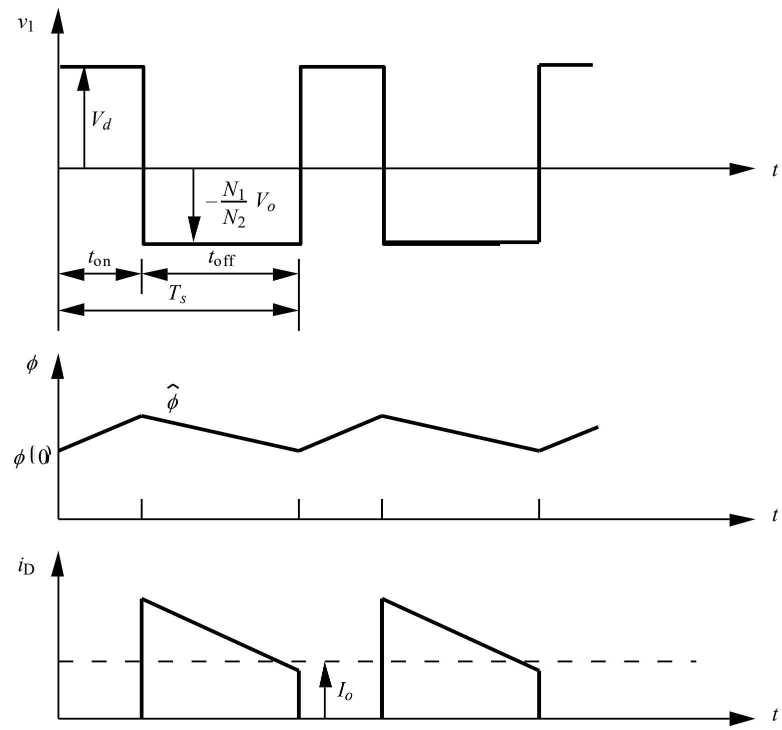
$$u_C(t_2) = U_d(1 - \cos(\pi + 0{,}729) \approx 1{,}745U_d \approx 26{,}2V$$

After this capacitor is discharged with constant current and the voltage drops to zero in time

$$dt = t_3 - t_2 = C\frac{du}{I_o} \approx C\frac{26{,}2\text{ V}}{1\text{ A}} \approx 0{,}417\mu\text{s}$$

Duration of the whole cycle is $t_3 - t_0 \approx 0{,}106\ \mu\text{s} + 0{,}616\ \mu\text{s} + 0{,}417\mu\text{s} \approx 1{,}139\ \mu\text{s}$

Question 3
a)



Peak value of the flux is at the end of conduction period

$$\hat{\phi}(t) = \phi(t_{on}) = \phi(0) + \frac{U_d}{N_1}t_{on}$$

In steady state integral of the voltage is zero

$$\phi(T_s) = \hat{\phi} - \frac{U_o}{N_2}(T_s - t_{on}) = \left(\phi(0) + \frac{U_d}{N_2}t_{on} - \frac{U_o}{N_2}(T_s - t_{on})\right) = \phi(0)$$ and therefore $$\frac{U_o}{U_d} = \frac{N_2}{N_1}\frac{D}{1-D}$$

$$\frac{N_2}{N_1} = 1 \Rightarrow D = \frac{U_o}{U_d + U_o} = \frac{1}{1 + U_d/U_o} \approx 0,058$$ **when the turns ratio is 5/1 the result is 0,238**

After turn-on of the switch, current in the primary increases linearly and it is equal to the magnetizing current. Peak value of the current is

$$\hat{I}_m = \hat{I}_{sw} = I_m(0) + \frac{U_d}{L_m}t_{on}$$

When switch is turned off, secondary voltage $-U_o$ is seen in the primary when multiplied by the turns ratio. Current decreases linearly

$$i_m(t) = \hat{I}_m - \frac{U_o(N_1/N_2)}{L_m}(t - t_{on}) \qquad (t_{on} < t < T_s)$$

This could also be used to derive the voltage ratio shown already above. The diode current is equal to the magnetizing current when transferred to the secondary, i.e.

$$i_D(t) = \frac{N_1}{N_2} i_m(t) = \frac{N_1}{N_2}\left[\hat{I}_m - \frac{U_o(N_1/N_2)}{L_m}(t - t_{on})\right] \quad (t_{on} < t < T_s)$$

Peak value of this current can be calculated when the output current is known. The average value of the diode current is equal to the output current $I_o = P_o/U_o = 20$ A. Based on this we obtain the peak value as

$$\hat{I}_m = \hat{I}_{sw} = \frac{N_2}{N_1}\frac{1}{1-D} I_o + \frac{N_1}{N_2}\frac{(1-D)T_s}{2L_m} U_o \approx 21,25 \text{ A} + 70,65 \text{ mA} \approx 21,32 \text{ A}$$

With the values of the question the result is 5,25 A + 142,8 mA = 5,39 A

This is obtained by calculating the diode current at the end of a cycle and with help of this calculating the area of I_d shown in the figure. Current comprises of two terms. The first one depends on the output current seen in the primary. The latter part shows the effect of the magnetizing current.

Voltage over the switch, when it is not conducting is

$$u_{sw} = U_d + \frac{N_1}{N_2} U_o = \frac{U_d}{1-D} = 48 \text{ V} + 5*3 \text{ V} = 63 \text{ V}$$

Question 4

Topics that were expected to be discussed in the answer
-single-phase versus three-phase, differences in line current harmonics and filtering of the dc-voltage
-in power supplies power is normally from the source to the load, i.e. one-directional
-diode bridge + filtering dc-capacitor => line-current harmonics
-power factor correction, single-phase bridge + boost converter
-energy storage => large dc-capacitor stores energy, high voltage better, also dc-filtering
-high frequency disturbances, beyond line-current harmonics, EMC, filtering
-something about selecting the power semiconductor devices, current- and voltage ratings, efficiency etc

Question 5.

Topics that should have been discussed in the answer.

-switching losses, increasing switching frequency increases losses, they can be reduced by using faster power semiconductor devices with smaller rise and fall times.
-resonant technologies, zero current or zero voltage switching reduces switching losses in theory to zero
-snubber circuits can reduce switching losses
-conduction losses, voltage drop in semiconductors can be modelled often as a voltage are resistance, using better devices, replacing diodes with MOSFET (so called synchronous rectification), Schottky diodes. Wiring losses can be reduced by using thicker traces and wires, Litz wire

-Losses in magnetic components, inductors and transformers, winding losses and core losses (hysteresis), materials with less losses, lower switching frequency,

-Also topology has an effect on losses, voltage and current over the switches and number of switches can change

-temperature is having on effect on many components and changes losses