ELEC-E8417 Switched-Mode Power Supplies **Exam 24.5.2018**

Question 1.

Calculate the Fourier series of the unfiltered output voltage of the buck converter. What are the highest Fourier components in the input and output voltage of the filter. In the converter input voltage U_d = 40 V, output voltage U_o = 5V, output power P_o = 5W, and switching frequency f_s = 50 kHz. In the filter the components are L = 43,75 μH, C = 470 μF.

Fourier series:

$$h(x) = h_0 + \sum_{n=1}^{+\infty} (a_n \cos(nx) + b_n \sin(nx))$$

$$h_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} h(x)dx \quad , \quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} h(x)\cos(nx)dx \quad , \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} h(x)\sin(nx)dx$$

Question 2.

In a Flyback-converter turns ratios $N_1{:}N_2$ = 5:1, output voltage U_o = 3 V, supply voltage U_d = 48 V, output power P_o = 60 W and switching frequency f_s = 200 kHz. The magnetizing inductance of the magnetic core is 0,2 mH and converter operates in continuous area, i.e. the magnetization of the core is always higher than zero. Derive equations for the maximum current and voltage ratings of the switch used in the converter and calculate their numerical values

Question 3.

The rectifier of a switched-mode power supply is equipped with an active power factor correction circuit. It has been realized with a single-phase diode bridge and step-up converter. RMS value of the supply voltage is 230 V and frequency 50 Hz. The rectifier is loaded with 1000 W, and the dc-voltage is 370 V and filtering capacitor is 100 μF.

a) Draw the equivalent circuit of the rectifier and explain its operating principle shortly.
b) Calculate the ac component in the output dc-voltage of the rectifier. The output current can be assumed to be ideal dc, efficiency of the rectifier 100 % and the switching frequency of the step-up converter large.

Question 4.

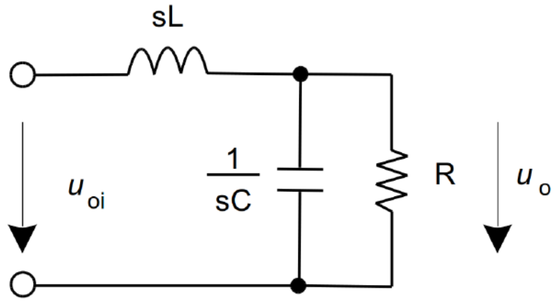
Why resonant topologies are used in switched-mode power supplies. Describe shortly their general operating principles. Give an example of one resonant topology, and discuss its operation shortly and describe its positive and negative features.

Question 5.

What are the basic principles of feedback control in switched-mode power supplies? Why feedback is needed? What types of feedback systems are used?

QUESTION 1.

The LC-filter damping the harmonics of the converter is



and its transfer function

$$G(s) = \frac{u_o}{u_{oi}} = \frac{\frac{R/sC}{R+1/sC}}{sL + \frac{R/sC}{R+1/sC}} \qquad G(s) = \frac{1}{LC\left(s^2 + \frac{1}{RC}s + \frac{1}{LC}\right)}$$

can also be written in form $G(s) = \dfrac{\omega_o^2}{s^2 + 2\xi\omega_o s + \omega_o^2}$ where $\omega_o = \dfrac{1}{\sqrt{LC}}$ and $\xi = \dfrac{1}{2R}\sqrt{\dfrac{L}{C}}$

Fourier-series of the unfiltered voltage

$$h_o = \frac{1}{2\pi}\int_{-D\pi}^{D\pi} dx = D \quad a_n = \frac{1}{\pi}\int_{-D\pi}^{D\pi} \cos(nx)dx = \frac{2}{n\pi}\sin(nD\pi)$$

$$h(x) = D + \frac{2}{\pi}\sum_{n=1}^{+\infty}\frac{1}{n}\sin(nD\pi)\cos(nx)$$

The voltage is

$$u_{o1} = h(x)U_d = U_d\left(D + \frac{2}{\pi}\sum_{n=1}^{+\infty}\frac{1}{n}\sin(nD\pi)\cos(nx)\right)$$ and its first harmonic

$$\hat{u}_{o1} = \frac{2U_d}{n\pi}\sin(nD\pi)$$

Duty cycle is $D = \dfrac{U_o}{U_d} = 0,125$ and the peak value of the 50 kHz component is

$$\hat{u}_{o1} = \frac{2U_d}{\pi}\sin(D\pi) \approx 9,745\text{V}$$

In order to estimate the damping of the filter, its transfer function can be simplified because the damping at 50 kHz will already be large. By replacing $s = jn\omega = jn\omega_s$ we obtain

$$u_{o,n} = G(s = jn\omega_s)u_{o1,n} = \frac{u_{o1,n}\omega_o^2}{-(n\omega_s)^2 + 2j\xi\omega_o n\omega_s + \omega_o^2}$$

and further simplifying

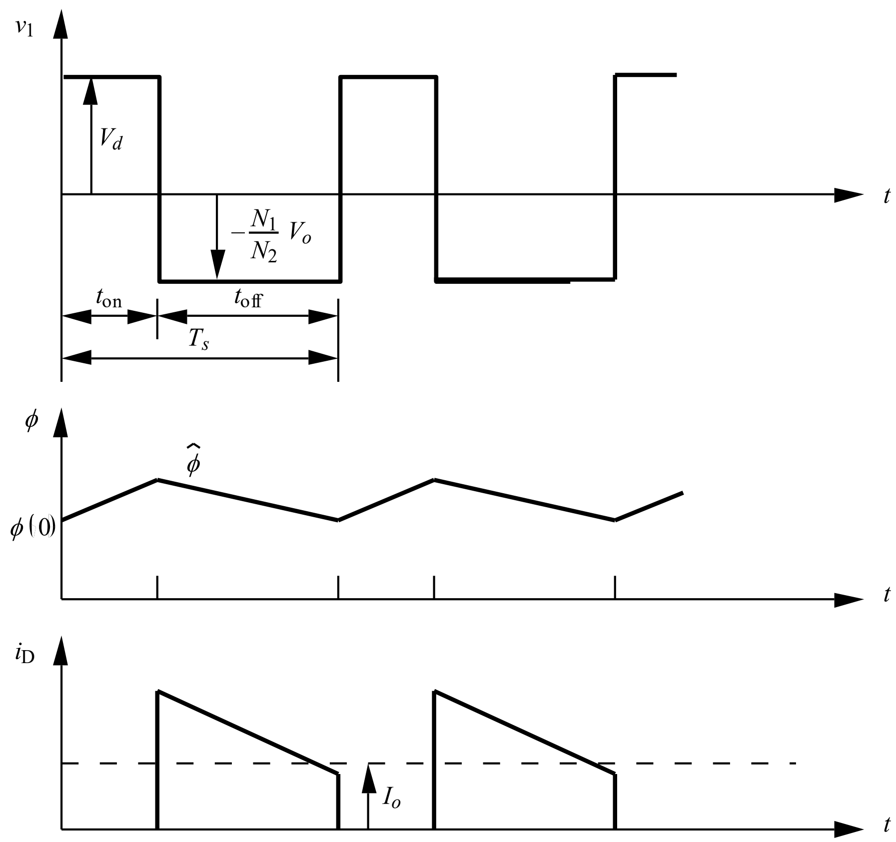
$$u_{o,n} = \frac{u_{o1,n}}{-(\frac{n\omega_s}{\omega_o})^2 + 2j\xi\omega_o n\frac{\omega_s}{\omega_o} + 1} \approx -\left(\frac{\omega_o}{n\omega_s}\right)^2 u_{oi,n}$$

With the numerical values, the 50 kHz component in the output voltage of the converter is

$$\hat{u}_{o1} = 4,8018\text{mV}$$

Question 2

a)



Peak value of the flux is at the end of conduction period

$$\hat{\phi}(t) = \phi(t_{on}) = \phi(0) + \frac{U_d}{N_1} t_{on}$$

In steady state integral of the voltage is zero

$$\phi(T_s) = \hat{\phi} - \frac{U_o}{N_2}(T_s - t_{on}) = \left(\phi(0) + \frac{U_d}{N_2} t_{on} - \frac{U_o}{N_2}(T_s - t_{on})\right) = \phi(0)$$ and therefore $$\frac{U_o}{U_d} = \frac{N_2}{N_1}\frac{D}{1-D}$$

$$\frac{N_2}{N_1} = 1 \Rightarrow D = \frac{U_o}{U_d + U_o} = \frac{1}{1 + U_d/U_o} \approx 0{,}058$$ when the turns ratio is 5/1 the result is 0,238

After turn-on of the switch, current in the primary increases linearly and it is equal to the magnetizing current. Peak value of the current is

$$\hat{I}_m = \hat{I}_{sw} = I_m(0) + \frac{U_d}{L_m} t_{on}$$

When switch is turned off, secondary voltage $-U_o$ is seen in the primary when multiplied by the turns ratio. Current decreases linearly

$$i_m(t) = \hat{I}_m - \frac{U_o(N_1/N_2)}{L_m}(t - t_{on}) \qquad (t_{on} < t < T_s)$$

This could also be used to derive the voltage ratio shown already above. The diode current is equal to the magnetizing current when transferred to the secondary, i.e.

$$i_D(t) = \frac{N_1}{N_2} i_m(t) = \frac{N_1}{N_2}\left[\hat{I}_m - \frac{U_o(N_1/N_2)}{L_m}(t - t_{on})\right] \qquad (t_{on} < t < T_s)$$

Peak value of this current can be calculated when the output current is known. The average value of the diode current is equal to the output current $I_o = P_o/U_o = 20$ A. Based on this we obtain the peak value as

$$\hat{I}_m = \hat{I}_{sw} = \frac{N_2}{N_1}\frac{1}{1-D} I_o + \frac{N_1}{N_2}\frac{(1-D)T_s}{2L_m} U_o \approx 21{,}25\ \text{A} + 70{,}65\ \text{mA} \approx 21{,}32\ \text{A}$$

With the values of the question, the result is 5,249 A + 142,9 mA = 5,39 A

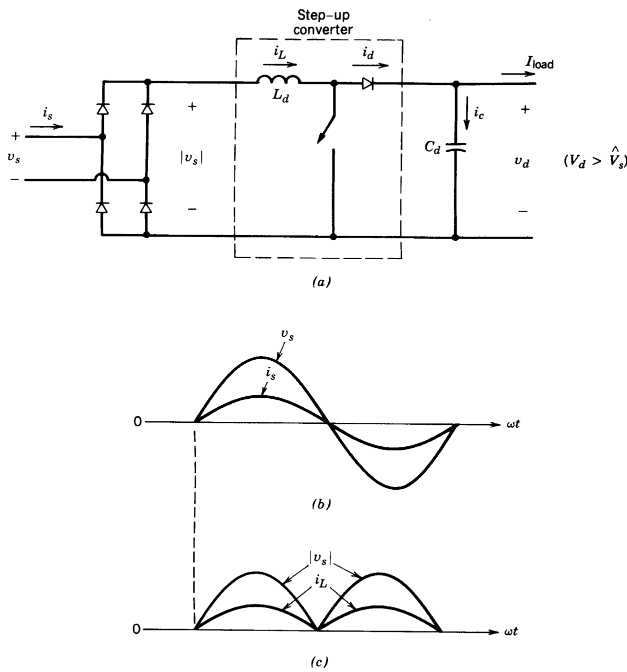
This is obtained by calculating the diode current at the end of a cycle and with help of this calculating the area of I_d shown in the figure. Current comprises of two terms. The first one depends on the output current seen in the primary. The latter part shows the effect of the magnetizing current.

Voltage over the switch, when it is not conducting is

$$u_{sw} = U_d + \frac{N_1}{N_2} U_o = \frac{U_d}{1-D} = 48\ \text{V} + 3\ \text{V} = 51\ \text{V}$$ With the values of the question, result is 48 + 5*3 = 63 V.

Question 3.

a) Detailed explanation is in the book



b) In PFC circuit input voltage and current can be assumed to be in phase

$$p_{in} = \sqrt{2}U_s\left|\sin\omega t\right|\left|\sqrt{2}I_s\sin\omega t\right| = U_sI_s - U_sI_s\cos 2\omega t = i_dU_d \qquad (1.1)$$

and further 100 % efficiency has been assumed. DC-current contains a dc-component and an ac component flowing through the capacitor

$$i_d = I_d + i_C = \frac{U_sI_s}{U_d} - \frac{U_sI_s}{U_d}\cos 2\omega t = I_d - I_d\cos 2\omega t \qquad (1.2)$$

Based on this capacitor current is

$$i_C = -\frac{U_sI_s}{U_d}\cos 2\omega t = -I_d\cos 2\omega t \qquad (1.3)$$

and voltage ripple over capacitor can be integrated

$$u_{d,ripple} = \frac{1}{C_d}\int i_C dt = -\frac{I_d}{2\omega C_d}\cos 2\omega t \qquad (1.4)$$

Average value of the dc current is $I_d = \frac{P_d}{U_d} = \frac{1000}{370}$ A and ripple voltage is thus

$$\hat{u}_{d,ripple} = \frac{I_d}{2\omega C_d} \approx 43 \text{ V} \qquad (1.5)$$

Question 4.
-reduction of losses, zero voltage or zero current switching, compared to hard switching waveforms are smoother => reduced EMI. Some topology example from textbook. Many papers missed this and therefore only 2,5 points. In some topologies peak values of current are exceeding the load current => increased conduction losses, also voltage peaks higher than input voltage is many topologies. Also, more complex are require exact timings for the gate driving.

Question 5.

Some required aspects,.

-Accurate and controlled output voltage with good dynamic although supply voltage or output current changes => tuning of controllers and model of the system
-State-space averaging, some basic description of this, only valid on frequencies below half of the switching frequency
-CCM and DCM area, transfer functions are different
-Input voltage feedforward
-Current mode control, inner current loop is faster than the slower outer voltage loop, limits peak current, automatic input voltage feedforward, transfer functions are simpler than in VMC, different current control methods (constant frequency, hysteresis), slope compensation needed when duty cycle is larger than 0,5

More detailed in the textbook